

Effect of Shape of Particles on Hydraulic Conductivity

M. A. Alam¹, A. M. Kalra², Rishav Sharma³

¹Associate Professor, PEC University of Technology, Sector 12, Chandigarh, India

²Professor, PEC University of Technology, Sector 12, Chandigarh, India

³M.E. Student, PEC University of Technology, Sector 12, Chandigarh, India

Abstract: *The possibility of occurrence of groundwater mainly depends upon two geological factors- porosity and permeability, which is also known as hydraulic conductivity. For maximum recharge of the ground water, hydraulic conductivity is an important parameter to be correlated in the design of ground water schemes. Hydraulic conductivity is highly dependent upon aquifer properties and fluid flow regime. Shape of particles is one of the criteria which dictate the hydraulic conductivity of an aquifer. To investigate the variation of Hydraulic conductivity with respect to shape factor of the particles, the present study has been conducted. On the basis of the results obtained a relationship has been developed which justifies the wide variation in the value of C in Hazen's equation. The data reflects that the shape of the particles is mainly responsible for these variations which affect the hydraulic conductivity of the aquifer material.*

Keywords: hydraulic conductivity, angularity, shape factor, permeameter

1. Introduction

Water is the most vital element needed for the survival of mankind. Optimal utilization of which necessitates the conjunctive use of surface and ground water resources. The use of ground water is primarily dependent upon replenishment by natural rainfall and induced recharge. A material is porous if it contains interstices and permeable if the interstices are interconnected. Permeability is defined as the property of a porous material, which permits the passage of water (or other fluids) through its interconnecting voids. It is a measure of ease with which water moves through an aquifer material[1]. The present investigation is an experimental work involving a study of the effect of the shape of various materials on their permeability. This paper presents the results of the experiments undertaken.

2. Formulation of the Problem

In the present study an attempt is made to develop the relationship between the permeability and angularity of granular materials for linear regime of flow based on dimensional analysis. For this, experiment was conducted on various materials to study the following relationships between- Friction factor and Reynold's number; and Size of particles and Angularity. A brief discussion regarding the formulation of the problem is given below:

2.1 Relationship between Friction Factor and Reynold's Number

The permeation of fluid through porous media depends on a number of variables. According to Rose [2], head loss across the porous bed, H may be expressed as,

$$H = F_1(v, l, d, D, \rho, \mu, g, e, n, Z, U) \quad (1)$$

Where, H is the head loss across porous bed; v, superficial velocity of flow; l, the length of bed; d, size of particle; D, diameter of the permeameter; ρ , mass density of fluid; μ , viscosity of fluid; g, acceleration due to gravity; e, surface roughness of particles; n, porosity of the bed; Z, dimensionless shape factor; U, dimensionless factor for size distribution.

By dimensional analysis equation 1 reduces to,

$$\frac{H}{d} = F_2\left(\frac{\rho v d}{\mu}, \frac{g d}{v^2}, \frac{L}{D}, \frac{D}{d}, n, Z, \frac{e}{d}, U\right) \quad (2)$$

If beds composed of uniform sizes are studied and it is assumed that surface roughness will have a comparatively small effect on porous media flow, equation (2) can be written as,

$$\frac{H}{d} = F_3\left(\frac{\rho v d}{\mu}, \frac{g d}{v^2}, \frac{L}{D}, n, Z\right) \quad (3)$$

Parameters H/d, L/d, and gd/v^2 can be combined to give a single dimensionless factor, conventionally known as the friction factor i.e.

$$F_3\left(\frac{H}{d}, \frac{g d}{v^2}, \frac{L}{D}, \frac{D}{d}, n, Z, \frac{\rho v d}{\mu}\right) = 0 \quad (4)$$

Equation 4 may be further solved as,

$$F_4\left(\frac{H}{d}, \frac{g d}{v^2}, \frac{\rho v d}{\mu}, \frac{L}{D}, n, Z\right) = 0 \quad (5)$$

Where, $i = H/L$ is the hydraulic gradient; $\frac{2igd}{v^2}$ is the

Friction factor and $\frac{\rho vd}{\mu}$ is the Reynold's number.

If all the experiments are conducted at a constant porosity and it is assumed that the D/d ratios involved in this study will have negligible effect on percolation. Equation 5 simplifies to

$$F_5 \left(\frac{igd}{v^2}, \frac{\rho vd}{\mu}, n, Z, \right) = 0 \quad (6)$$

2.2 Relationship between the Size of Particles and their Angularity

The angularity of particles, defined as the porosity of the material when it is compacted in a standard manner prescribed in BS: 812-1967, will depend on the shape of particles as well as on their size. Mathematically, this can be expressed as,

$$\eta = F_6(Z, d) \quad (7)$$

Where η is the porosity when the material is compacted in a standard manner by prescribed BS: 812-1967 [3].

Other terms have been defined earlier.

For a material belonging to the same shape group, Z will be constant in Equation 7. Therefore, Equation (7) can be modified as

$$\eta = F_7(d) \quad (8)$$

To study the functional relationship between the angularity and size of particles, tests were conducted on various sizes of different materials and the results analysed. An attempt has been made to correlate equation (6) and (7) [4].

3. Experimental Work

Figure 1 shows the experimental set up.

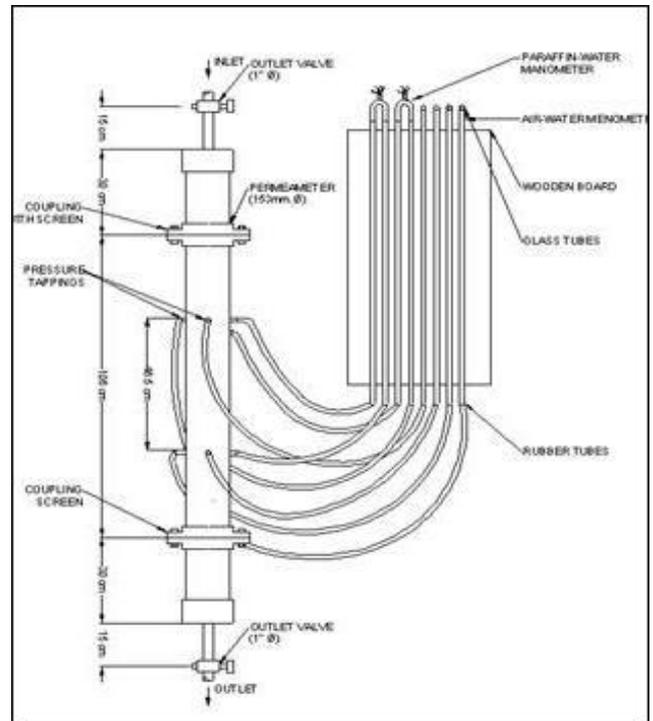


Figure 1: Details of permeameter set-up

3.1 Materials

The materials tested were:

- Glass marble : 1.905cm, 2.54cm
- Marble Chips : 0.4cm, 0.63cm, and 0.8cm
- Crushed Quartzite : 0.4cm, 0.63cm, 0.8cm
- Sand : 0.03cm, 0.085cm, 0.118cm

3.2 Experimental Equipment

The experimental equipment comprised

- (a) Source of supply
- (b) Permeameter
- (c) Discharge measuring device
- (d) Manometers
- (e) Pycnometer

- (a) Source of supply

The permeameter receives its water supply from an overhead tank at a height of 2.65m above the permeameter outlet. The tank receives its supply from a re-circulating tank so that a constant head is maintained in the overhead tank.

- (b) Permeameter

The constant head vertical flow type permeameter was used for hydraulic tests in this work. The main permeameter section consisted of a 10.16 cm internal diameter GI tube with a total length of 1.06 meter with a test length of 46.5 cm. Four pressures tapping making an angle of 90° to each

other were provided along the circumference of permeameter at the starting and ending points of the test length. This arrangement of tapping points was adopted to ensure the mean pressure at the section under consideration. The inlet to the permeameter was regulated with the help of an outlet sluice valve of 25.42mm diameter. I.S. 2.0 mm mesh screen was used in the filter for resting the porous media. For filling and removing of the material, the permeameter was detached from its supports each time.

(c) Discharge measurement

The discharge was measured by volumetric method. The water was collected in a bucket for a certain period, which was recorded with a stopwatch and collected water was then measured with the help of a 2000 cc capacity glass jar. Volume of water collected at a particular duration will give the discharge.

(d) Manometer

To cover the desired range of flow, two types of manometer were used

- Air-water manometer
Simple piezometers were used to measure the head losses of about 0.1 cm to 35.0 cm of water.
- Paraffin water manometer
Inverted U-tube paraffin water differential manometers were used to measure low head losses. The manometers were supported on a wooden board with a graduated scale in cm, giving a correct reading of manometer up to one mm.

(e) Pycnometer

I.S. pycnometer is used in the specific gravity test.

3.3 Experimental Procedure

The various tests conducted during the course of this study can be divided into two categories.

- (a) Preparation of samples
- (b) Hydraulic tests

- (a) Preparation of samples: The materials used in this work were graded into various samples of known proportion and size with the help of various tests such as the Sieve analysis test, Specific gravity test and Angularity test.
- (b) Hydraulic tests: The hydraulic tests were conducted to study the effect of resistance to flow of water in a given sample of material. The method of carrying out these tests was as follows.

Preparation of the Bed:

Before filling the permeameter with the material to be tested, the inlet portion of the permeameter was taken off. It was proposed in the present study to keep the porosity constant for all runs of the materials. Therefore, the weight of the material needed to fill the permeameter was calculated as:-

$$W_s = (1-n) V_T G_s \gamma_w \quad (9)$$

Where W_s is the weight of the material; n , porosity; V_T , volume of the tube; G_s , specific gravity of the material; γ_w , specific weight of the water

In order to get a uniformly packed bed throughout the length of the permeameter, the necessary quantity of material was divided into ten equal parts. Each part was thrown gently over the whole cross-section of the tube, and compacted by steel rod, after the surface was levelled. The packing method was same for all the runs. The number of blows required for compacting the material varied for various materials.

After packing the tube and levelling the top of the material, the coupling was fitted. After fixing the permeameter in the vertical position and connecting to the water supply system, the outlet and inlet valves were completely opened to make the material saturated for 5 to 6 minutes. Then the outlet valve was slowly closed so that the water entered the manometric tubes. Before starting the test, air in the permeameter tube as well as in manometer tubes was removed. After removing the air, the outlet valve was completely opened to start the test with maximum discharge.

3.4 Test Run

This involved three main operations:

- (a) Measuring the discharge through the permeameter.
- (b) Reading the pressure drop across the test length of the material.
- (c) Reading the temperature of water

In all the runs, observations were taken in receding order of magnitude of discharge and hydraulic gradient. The test was started with maximum discharge so that if there was any settlement of the bed due to the impact of the jet of incoming water, it was secured in the beginning. The bed was then checked at the end of each run for settlement, if any. The porosity of the bed was determined after making due allowances for settlement. The discharge was measured by collecting the water in a bucket for a certain period, which was recorded with a stopwatch. The flow was then reduced for the next observation of the run. This operation was continued till the discharge was decreased from maximum to a certain measurable minimum discharge. For each discharge two observations were recorded to make sure that flow was steady for each set of observation.

For higher discharge as the pressure drop was higher, the air-water manometer was used. And for low discharge paraffin-water manometer was used. The manometer readings from paraffin-water manometer were converted to head of water. The temperature of water was recorded at the beginning and the end of each run. After data was collected for one run, the procedure was repeated.

4. Analysis of Results

The present study investigates a relationship between the permeability (K) of the materials used, their angularity (η) and size of the particles (d_g - geometrical diameter of particle). To study the relationship between angularity (η) and size of particles (d_g), curve of logarithm of angularity ($\log \eta$) and logarithm of size ($\log d_g$) of particles is drawn. The results of the experimental investigation on the different materials used in the present study are presented as Friction factor (Fr) vs Reynold's number (Re) graph on log-log scale. A discussion of results in relation to the different aspects of the problem studied is given below.

4.1 Relationship between angularity and size of particles

The angularity of the particles can be used a measure of the shape of the particles; but this definition of shape factor applies to irregular shaped only. In addition, the angularity of any material depends on its size. The angularity tests, were therefore conducted in irregular material of three sizes apart from one regular material. Experimental results for angularity tests for these materials are shown below in Table 1.

Table 1: Variation of angularity with size of particles

S. No.	Material	Size(Mm)	Angularity (%)
1	Glass Marble	1.905	40.73
		2.54	40.26
2	Marble Chips	0.4	43.78
		0.63	42.99
		0.8	42.58
3	Crushed Quartzite	0.4	45.47
		0.63	44.68
		0.8	44.27
4	SAND	0.03	44.12
		0.085	42.32
		0.118	41.77

From a study of these results, it is seen that the angularity of any material decreases with an increase in its size. These results are plotted using log of angularity ($\log \eta$) against the corresponding log of geometric mean diameter ($\log d_g$) of the material (Figure 1). All the data appear to follow a straight-line law given by $\log \eta = n \log d_g + \log C'$ (10)

Equation 10 can be simplified as,

$$\eta = C' d_g^n \quad (11)$$

The value of index n in this equation was found to be constant at -0.0402 (from Figure1) for all the materials. The value of C' varies with shape of particles and its value, obtained from Figure 1, for various materials is shown in Table 2.

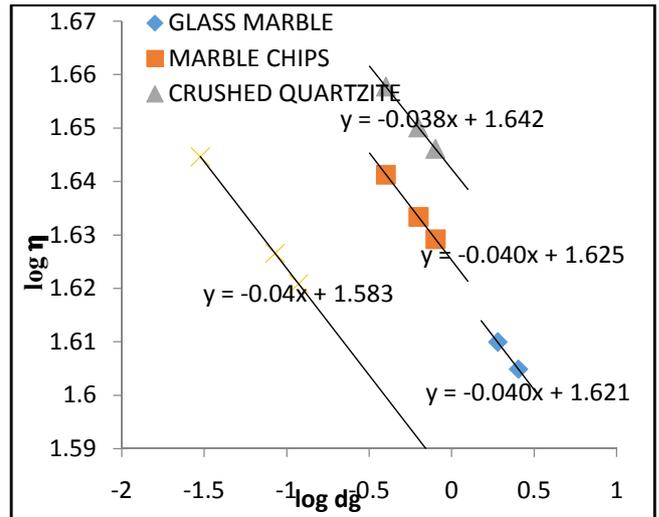


Figure 1: Curve of $\log \eta$ against $\log d_g$

The Equation 11 can therefore, be written as:

$$C' = \eta d^{0.0402} \quad (12)$$

Where,

C' = parameter defining the angularity and depends on shape of the particles.

4.2 Relationship between permeability and angularity of granular material

The permeation of fluids through porous masses depends on a number of variables as mentioned in Equation 1. For uniform granular media at a constant porosity, the law governing the seepage phenomenon is given by Equation 5 given earlier. The results of experimental investigation for different materials can therefore be plotted on log-log graph as Friction factor ($Fr = 2\eta d_g/v^2$) vs. Reynold's number ($Re = vd_g/v$) as shown in Figure 2-5.

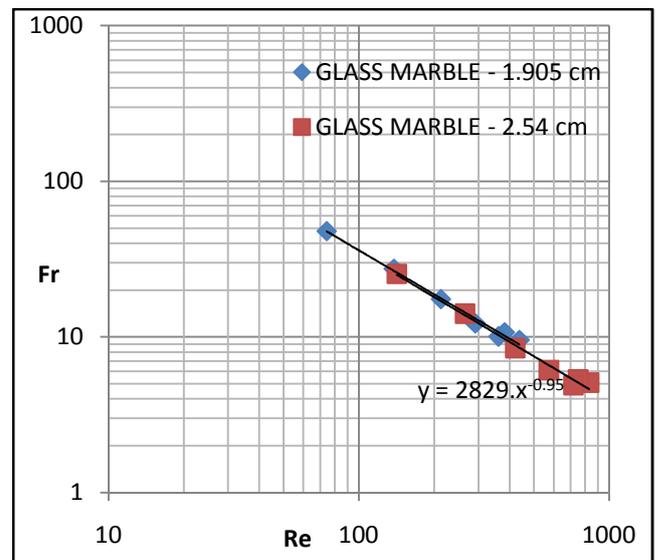


Figure 2: Fr-Re Curve for glass marble

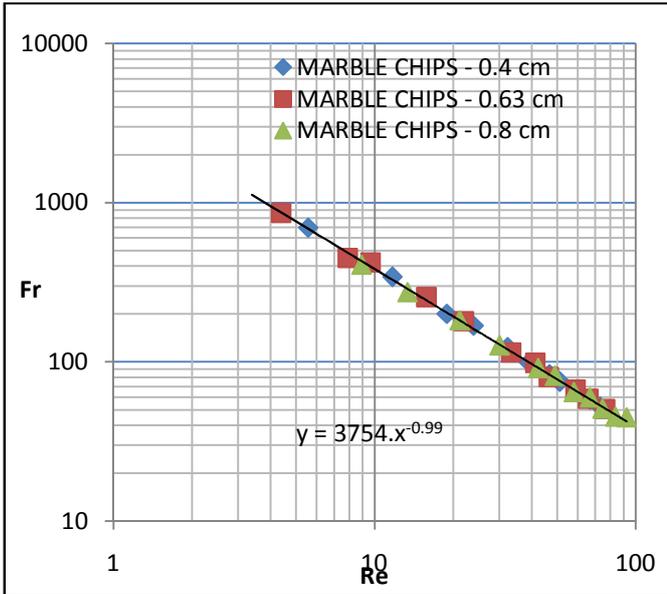


Figure 3: Fr-Re Curve for marble chips

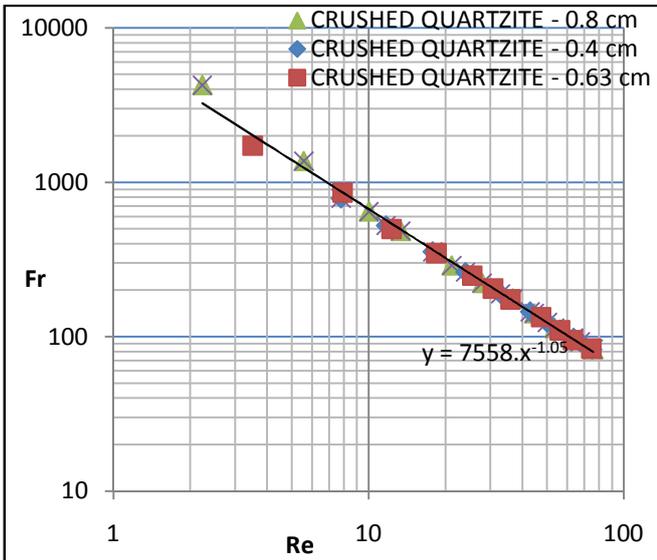


Figure 4: Fr-Re Curve for crushed quartzite

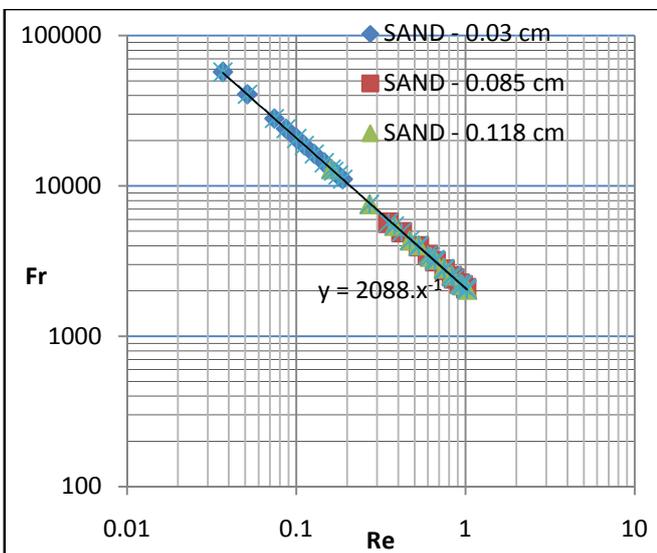


Figure 5: Fr-Re Curve for sand

The variation was found to be linear. Within linear regime of flow, these results follow an equation of the form:

$$Fr = \frac{C_1}{Re} \quad (13)$$

On substituting $F = 2igd / v^2$, $Re = vd / \nu$ and $V = Ki$ in equation 13,

$$\frac{K}{d_g^2} = \frac{2\rho g}{\mu C_1} \text{ or, } \frac{K}{d_g^2} = \frac{2g}{\nu C_1} = C \quad (14)$$

Where, C_1 is the constant which for a given porosity depends only on the shape of particles; C , parameter defining the permeability of the material; d_g , geometric mean diameter of particles; ν , kinematic viscosity of the fluid ($\nu = \frac{\mu}{\rho}$); other

parameters being as mentioned earlier.

For a constant porosity, the value of 'C' varies with shape of particles. The values of C_1 obtained from Figure 2-5 are shown in Table 1.

Table 2: Values of C and C_1

S. No.	Material	C_1 (at proposed porosity)	C (at 40% porosity)	LOG C	LOG C'
1	Glass Marble	3077.9	72.0279	1.8575	1.6211
2	Marble Chips	3754.9	59.0415	1.7712	1.6253
3	Crushed Quartzite	7558.4	29.3309	1.4673	1.6424
4	Sand	2094.6	105.8412	2.0247	1.5838

From equation 13 it is obvious that the coefficient C' depends on the shape of particles. Similarly, factor C in equation 14, which defines the permeability of materials, is also dependent on shape alone provided that the porosity and other factors are constant, it is therefore concluded that C' is a function of C .

Mathematically, this can be expressed as:

$$C' = f(C) \quad (15)$$

To study the relationship between C' and C , values of C' and C obtained from experimental results are plotted as $\log C$ vs $\log C'$ (Figure6).

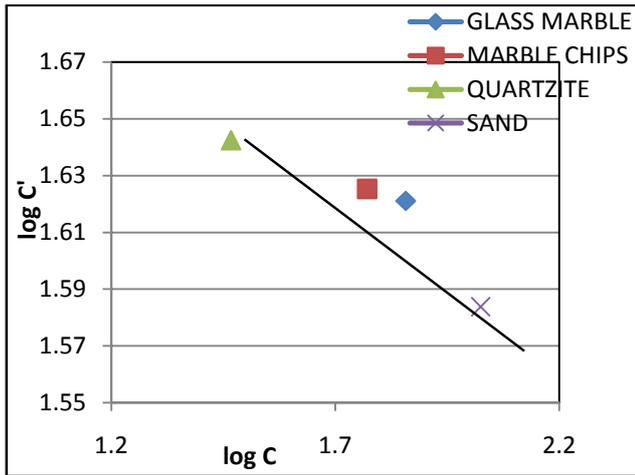


Figure 6: LogC vs LogC' at 40% porosity

To make C' independent of porosity, it was proposed to conduct all these tests at same porosity, i.e. 40% in each case. To achieve this it was found necessary to compact the material by crushing it with a heavy rammer, which could have crushed the material. No attempt was therefore made to compact the material with a heavy rammer to get the proposed porosity. Alternatively the tests had to be conducted at different suitable porosity for each material and by using Kozeny's formula, [5] all the values of C obtained from equation 13, have been corrected to those at 40% porosity. All the points in Figure 6 followed a straight line given by

$$\log C' = \log B + m \log C \quad (16)$$

Equation 16 can be simplified as

$$C' = BC^m \quad (17)$$

The values of B and m in the above equation were obtained with the help of Figure 6

The final equation can be written as,

$$C' = 59.5937 C^{-0.0859} \quad (18)$$

Or,

$$C^{+0.0859} = \frac{59.5937}{C'} \quad (19)$$

Substituting the values of C' and C from equations 12 and 14, respectively, the above equation can be expressed as:

$$\left[\frac{K}{d_g^2} \right]^{0.0859} = \frac{59.5937}{\eta d_g^{0.0402}} \quad (20)$$

Or,

$$K = \frac{4.63 \times 10^{20} \times d_g^{1.532}}{\eta^{11.64}} \quad (21)$$

Where K is the permeability of soil; d_g , geometric mean diameter of particles (cm); and η is the angularity of particles (%).

From the above equation, it is obvious that that by conducting angularity number tests and sieve analysis, an idea about the permeability of a given material can be obtained. Since Equation 20 is based on experimental results at 40% porosity, the above value of permeability is applicable to this porosity only. For other porosity a suitable porosity function can be used

5. Conclusion

From the discussion related to the analysis of results, an important conclusion drawn on the basis of this study reflects that the experimental results agreed closely with Hazen's formula $K = C D_{10}^2$ (where C varies from 41 to 146)[6]. It was found that the main cause of variation of the coefficient C in Hazen's formula is the shape of particles. It shows that the hydraulic conductivity is affected by the shape of the particle besides other factors related to it. The proposed formula (equation 21) is applicable for 40% porosity and 25°C temperature. It can be used to calculate the hydraulic conductivity of natural materials by conducting angularity tests and sieve analysis.

6. Scope for Further Studies

In the present study emphasis has been given to linear regime of flow. Therefore, it is suggested to investigate the problem in transition as well as turbulent regimes of flow.

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Author Profile



M. A Alam received the B.Sc. Engineering and M.Sc. Engineering (Hydraulic Structure) degrees in Civil Engineering from Z.H. College of Engineering and Technology in 1985 and 1987, respectively and research areas are in the field of Flow through Granular Media. Author of more than 25 research paper and guided more than 22 M.E Thesis.



Dr. A. M Kalra received the B.Sc. Engineering and M.E. (Irrigation & Hydraulics) degrees in Civil Engineering from Punjab Engineering College in 1974 and 1976, respectively and Doctorate in Modeling of Water Resources Systems. He has Authored more than 50 research paper and guided more than 25 M.E Thesis and 5 PhD projects.