

Optimum Replacement Time for a Deteriorating System

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Abstract: *The Preventive Replacement strategy for determining an optimal replacement time for an automobile system that deteriorates with time is proposed and investigated, while the part of the system is in the final stages of its life span. The failure time followed the weibull, distribution and hence, the optimum replacement time can be determined. This time minimizes total downtime and maximizes the proportion of time the parts of the system available.*

Keywords: Deteriorating system, Weibull distribution, Maintenance optimization, Preventive Replacement strategy

1. Introduction

All equipment during operation fail, degrade or age. Therefore maintenance is required to keep them running. The operation of a particular system in deteriorating condition will lead to a high machine downtime. This is due to the failure of parts of the system at unexpected time. One of the solutions to this matter is to use Preventive Replacement, it is one of the maintenance optimization strategies that can balance the failure cost in unexpected time and minimize downtime of a deteriorating system. From strategic or philosophical points of view most of the relevant maintenance practices can be classified into two general categories, Preventive maintenance and corrective maintenance. Normally, all equipment or component lifetime will go through three conditions, namely decreasing failure rate (DFR), constant failure rate (CFR) and increasing failure rate (IFR) stated by Ebeling[5]. The critical condition of the component lifetime usually in IFR conditions (deteriorate condition) that will increase the frequency of failures. The application of traditional maintenance or corrective maintenance will cause increasing failure cost and machine downtime. Preventive replacement refers to periodically replacing of non-repairable system in deteriorating condition. The main objective of Preventive replacement is to reduce the frequency of failures resulting in a balance of failure cost and maintenance benefits such as downtime, reliability, availability etc. The earliest Preventive replacement strategy model was developed by Barlow, R.E, Proschan, F [3] and it becomes a fundamental criterion for various replacement problems. The Preventive replacement strategy is widely applied in industrial areas and mostly applied on machine components problems. In Jainqiang [6] developed model based on Preventive replacement strategy and applied it for cutting tool problem of a CNC milling process. The main objective of the model [6] is to determine the optimal replacement intervals couple with the forecasting of tool replacement to minimize the production cost. In Bahrami [1] simplified the model to determine the optimum replacement time to minimize the downtime.

The preventive replacement is one of the effective strategies to reduce the probability of failure (reduce failure cost) and downtime in the deteriorate condition. In Jardine, A.K.S [7] stated that preventive replacement is the most appropriate

maintenance strategy for component which operates in the stage of wear-out life (CFR). However, the best time to carry out the Preventive replacement must be considered. If preventive replacement is applied too frequently, the cost of maintenance and system downtime will increase. While, if preventive replacement is applied occasionally, it will increase the downtime of sudden failures (breakdowns) plus downtime of maintenance. Hence, with compromise between these two replacement conditions (too frequently and occasionally) will result the optimum time of replacement (t_p), which minimize the total system cost of downtime due to maintenance downtime and sudden failure downtime. In this paper the failure time followed the weibull distribution and hence, the optimum replacement time can be determined.

2. The Constant-Interval Replacement Model

2.1 Notation

- $C(t_p)$ = The total expected cost/cycle time
- $F(t_p)$ = Probability that the part of the system would break in period $(0, t_p)$
- $f(t_p)$ = Probability density function
- C_p = Cost of preventive replacement
- C_f = Cost of failure replacement
- t_p = The optimum time which satisfies the $C(t_p)$
- α = Shape parameter
- θ = Scale parameter

In development of Constant-Interval Replacement Model, the decision criterion is defined by $C(t_p)$, "the total expected cost/cycle time" of replacing a part of the system in cycle period $(0, t_p)$. Bahrami [1] assumed that the expected number of failure occurring the cycle period $(0, t_p)$, is equal to the probability of occurrence a failure before time t_p , i.e. $F(t_p)$. Therefore, the final form of decision function, $C(t_p)$ can be expressed as below;

$$C(t_p) = \frac{c_p + F(t_p) * c_f}{t_p} \quad (1)$$

The best value, t_p can be obtained by standard differential calculus.

Setting $d/dt C(t_p) = 0$, knowing that, $c_f > 0$, which gives;

$$t_p = \frac{F(t_p) + \frac{c_p}{c_f}}{f(t_p)} \quad (2)$$

2.2 Special Case

The failure time follows weibull distribution with parameter α and θ . Then

(2) and (1) becomes

$$t_p = \frac{\left(1 - e^{-\left(\frac{t}{\theta}\right)^\alpha}\right) + \frac{c_p}{c_f}}{\frac{\alpha}{\theta} \left(\frac{t}{\theta}\right)^{\alpha-1} e^{-\left(\frac{t}{\theta}\right)^\alpha}} \quad (3)$$

$$C(t_p) = \frac{c_p + \left(1 - e^{-\left(\frac{t}{\theta}\right)^\alpha}\right) * c_f}{t_p} \quad (4)$$

Table 1: Optimum time and Expected cost

| t | t _p | C(t _p) | t | t _p | C(t _p) |
|----|----------------|--------------------|----|----------------|--------------------|
| 1 | 286.03 | 1.8077 | 16 | 89.82 | 6.5192 |
| 2 | 203.27 | 1.7540 | 17 | 88.92 | 6.6724 |
| 3 | 167.35 | 2.4670 | 18 | 88.22 | 6.8150 |
| 4 | 146.35 | 3.0240 | 19 | 87.68 | 6.9490 |
| 5 | 132.32 | 3.4801 | 20 | 87.27 | 7.0757 |
| 6 | 122.26 | 4.2292 | 21 | 87.08 | 7.1869 |
| 7 | 114.65 | 4.5482 | 22 | 86.95 | 7.2949 |
| 8 | 108.73 | 4.8390 | 23 | 86.82 | 7.4045 |
| 9 | 104.02 | 5.1060 | 24 | 86.88 | 7.4991 |
| 10 | 101.48 | 5.3530 | 25 | 87.02 | 7.5893 |
| 11 | 98.47 | 5.5820 | 26 | 87.24 | 7.7721 |
| 12 | 96.00 | 5.7957 | 27 | 87.54 | 7.8481 |
| 13 | 93.97 | 5.9955 | 28 | 87.91 | 7.9180 |
| 14 | 92.29 | 6.1825 | 29 | 88.34 | 7.8795 |
| 15 | 90.93 | 6.3563 | 30 | 88.83 | 7.9385 |

3. Conclusion

From the Table, the best value of, t_p (optimum time) is 86.88 ≈ 87 days. The time to perform the preventive replacements, t_p = 87 days satisfies the equation (1) that yields the minimum total cost/cycle time as C(t_p) = 7.4991 cost/cycle time. The methods and models that have been applied in this paper contribute to the maintenance decision making and can assist the maintenance manager to make better economic decision about the system maintenance. It provides the results essentially similar to those developed from the classical but more complicated model.

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