

Simulation of Waiting Line System Using Single-Line Multiple-Channel Models: A Case Study of NNPC Mega Stations in Owerri and Enugu State, Nigeria

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Abstract: *The research study's the waiting line system in NNPC Mega Station at Enugu and Owerri all are cities in Nigeria. The technique used for the analysis is the simulation of the single-line multiple channel technique. The used of the existing single-line multiple-channel models were absolved to analysis the data. From the result, it was observed that the number of servers needed to utilize the queuing systems in the two case study establishments were eight servers respectively. The results were also recommended to the case study establishments.*

Keywords: Arrival Rate, Service Rate, Waiting line, Probability, Servers, Waiting Time, System Capacity, System Utilization, Queuing System and Matlab Programme

1. Introduction to Basics of Queuing Theory

The theory of queues was initiated by the Danish mathematician A. K. Erlang, who in 1909 published "The theory of Probabilities and Telephone Conversation". He observed that a telephone system was generally characterized by either (1) Poisson input (the number of calls), exponential holding (service) time, and multiple channels (servers), or (2) Poisson input, constant holding time and a single channel. Erlang was also responsible in his later works for the notion of stationary equilibrium and for the first consideration of the optimization of a queuing system.

Applications of the theory to the telephony were soon appearing. In 1927, E. C. Molina published "Application of the Theory of Probability to Telephone Trunking Problems", and one year later Thornton Fry printed "Probability and its Engineering Uses" which expand much of Erlang's earlier work. In the early 1930's Felix Pollaczek did some further pioneering work on Poisson input, arbitrary output, and single and multiple channel problems. Other names working in the same field during that period included Kolmogorov and Khintchine in Russia, Crommelin in France and Palm in Sweden. The work in queuing theory picked up momentum rather slowly in its early days, but in 1950 started to accelerate and there have been a great deal of work in the area since then.

2. Objective of the Study

To simulate the existing single –line multiple channel waiting line models using Matlab programming.

3. Characteristics of Queuing System

The mechanism of the queuing process is very simple. Customers (not necessarily human customers) are arriving

for service, waiting for service if it is not immediate, and leaving the system as soon as they are served.

There are six basic characteristics of queuing processes which provide an adequate description of a queuing system: (1) arrival pattern of customers, (2) service pattern of servers, (3) number of service channels, (4) system capacity and (5) queue discipline.

In usual queuing systems the arrival pattern of customers is stochastic and it is thus necessary to know the probability distribution describing the time between successive customer arrivals (interarrival times). Also the arrival pattern can change with time so we differentiate between stationary and nonstationary arrival patterns. The same discussion applies to the service pattern of servers, a probability distribution is needed to describe the sequence of customer service times. Queue discipline refers to the manner in which customers are selected for service when a queue has formed. The most common discipline is first come, first served (FCFS), but there are many others like last come, first served (LCFS) which is applicable in many inventory systems as it is easier to reach the nearest item; randomly selecting for service (RSS) independent of the arrival time of the customer; and a variety of priority schemes, the customers with higher priority being served ahead of the lower priority customers regardless of the order in which they arrived to the system.

4. Types of Queuing System

There are four major types of queuing system and different combinations of the same can be adopted for complex networks. Lapin (1981) broadly categorized queuing system structures into the following.

1. Single-Server, Single-Phase System

This is a situation in which single queue of customers are to be served by a single service facility (server) one after the other. An example is bottles or cans of minerals or beer to be cocked in a production process. Diagrammatically it is depicted in figure 2.4.

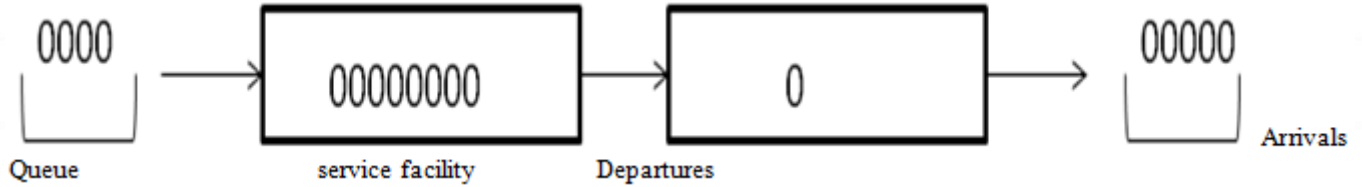


Figure 1: Single-server, Single phase System
Source: Adopted from Obamiro (2003)

2. Single-Server, Multiple-Phases System

In this situation, there's still a single queue but customers receive more than one kind of service before departing the queuing system as shown in figure 2.5. For example, in the university, students first arrive at the registration desk, get

the registration done and then wait in a queue for their forms to be signed, after signing; they join another queue for submission. Students have to join queue at each phase of the system.

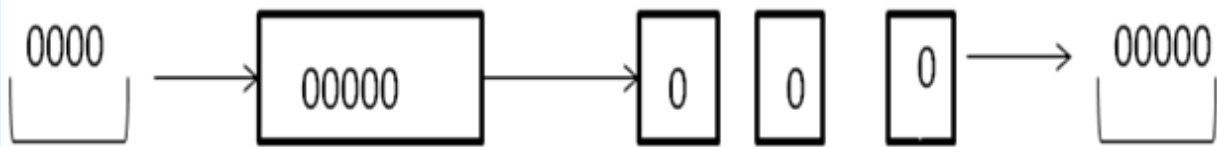


Figure 2: Single-server, multiple phases System
Source: Adopted from Obamiro (2003)

3. Multiple-Servers, Single-Phase System

This is a queuing system characterized by a situation whereby there is a more than one service facility (servers) providing identical service but drawn on a single waiting

line, Obamiro (2003). An example is a petroleum service station. As illustrated by figure 2.6.

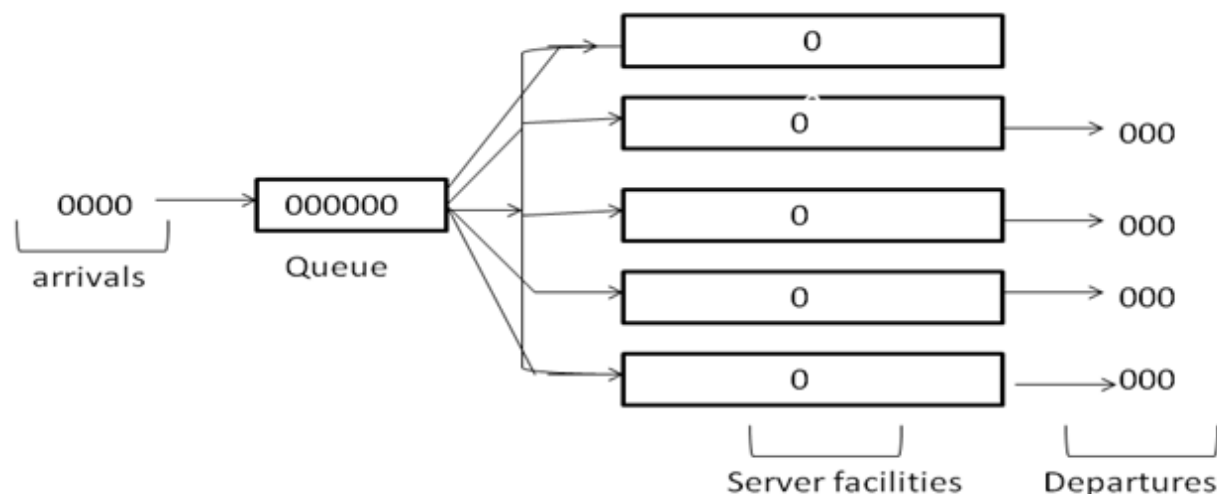


Figure 3: Multiple-servers, Single phase System
Source: Adopted from Obamiro (2003)

4 Multiple servers, Multiple-phases System:

According to Singh (2007), this type of system has numerous queues and a complex network of multiple phases of services involved as can be seen in figure 2.7. This type of service is typically seen in a hospital setting,

multi-specialty outpatient clinics, patient first form the queue for registration, and then he/she is triage for assessment, then for diagnostics, review, treatment, intervention or prescription and finally exits from the system or triage to different provider.

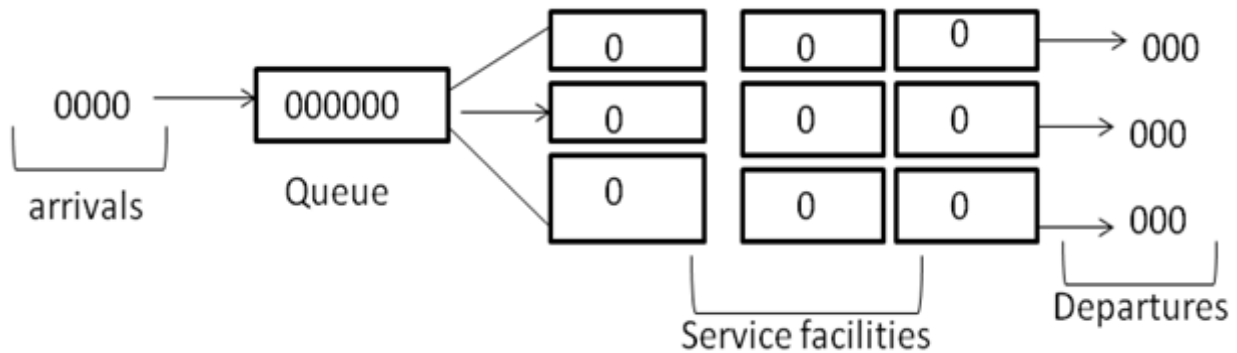


Figure 4: Multiple-servers, Multiple-phase System
Source: Adopted from Obamiro (2003)

5. Types of Queuing Models

Model A (M/M/1): Single-Channel Queuing Model with Poisson Arrivals and Exponential Service Times

The most common case of queuing problems involves the single-channel, or single server, waiting line. In this situation, arrivals form a single line to be serviced by a single station (see Figure 2.4). We assume that the following conditions exist in this type of system:

- a. Arrivals are served on a first-in, first-out (FIFO) basis, and every arrival waits to be served, regardless of the length of the line or queue.
- b. Arrivals are independent of preceding arrivals, but the average number of arrivals (arrival rate) does not change over time.
- c. Arrivals are described by a Poisson probability distribution and come from an infinite (or very, very large) population.
- d. Service times vary from one customer to the next and are independent of one another, but their average rate is known.
- e. Service times occur according to the negative exponential probability distribution.
- f. The service rate is faster than the arrival rate.

When these conditions are met, the series of equations shown can be developed. Prabhu (1987).

$$L_s = \frac{\lambda}{\mu - \lambda} \quad (1)$$

$$W_s = \frac{1}{\mu - \lambda} \quad (2)$$

$$L_Q = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad (3)$$

$$W_Q = \frac{\lambda}{\mu(\mu - \lambda)} \quad (4)$$

$$\rho = \frac{\lambda}{\mu} \quad (5)$$

$$P_0 = 1 - \frac{\lambda}{\mu} \quad (6)$$

Model B (M/M/S): Multiple-Channel Queuing Model

Now let's turn to a multiple-channel queuing system in which two or more servers or channels are available to handle arriving customers and all servers are assumed to perform at the same rate. In this model, we still assume that customers awaiting service form one single line and then proceed to the first available server. It is also assumed that arrivals follow a poisson distribution and service times are exponentially distributed. Service is first come, first-served and there is an infinite queue capacity. The Multichannel, single-phase waiting lines are found in many banks today and most especially in the petroleum service stations. (Refer to Figure 2.6 for a typical multichannel configuration). The queuing equations for Model B (which also has the technical name M/M/S) are shown below. These equations are obviously more complex than those used in the single-channel model; yet they are used in exactly the same fashion and provide the same type of information as the simpler model.

$$\rho = \frac{\lambda}{m(\mu)} \quad (7)$$

$$p_0 = \left[\sum_{n=0}^{M-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^M}{M!(1 - \frac{\lambda}{M\mu})} \right]^{-1} \quad (8)$$

$$L_Q = \frac{\lambda \mu (\frac{\lambda}{\mu})^M}{(M-1)!(M\mu - \lambda)^2} P_0 \quad (9)$$

$$L_s = L_Q + (\bar{\lambda} / \bar{\mu}) \quad (10)$$

$$W_a = \frac{1}{M\mu - \lambda} \quad (11)$$

$$W_Q = \frac{L_q}{\lambda} \tag{12}$$

$$W_s = \frac{L_s}{\lambda} \tag{13}$$

$$P_w = \frac{W_Q}{W_a} \tag{14}$$

Blanc (2011), Sztrik (2011) and Nain (2004)

Model C (M/D/1): Constant-Service-Time Model

Some service systems have constant, instead of exponentially distributed, service times. When customers or equipment are processed according to a fixed cycle, as in the case of an automatic car wash or an amusement park ride, constant service times are appropriate. Because constant rates are certain, the values for L_q , W_q , L_s , and W_s are always less than they would be in Model A, which has variable service rates. As a matter of fact, both the average queue length and the average waiting time in the queue are halved with Model C. Constant-service-model formulas are given below.

$$L_Q = \frac{\lambda^2}{2\mu(\mu-\lambda)} \tag{15}$$

$$W_Q = \frac{\lambda}{2\mu(\mu-\lambda)} \tag{16}$$

$$L_s = L_q + \frac{\lambda}{\mu} \tag{17}$$

$$W_s = W_q + \frac{1}{\mu} \tag{18}$$

Medhi, (2003); Hillier and Lieberman, (2005); Tutuncu and Newland, (2009).

Model D: Limited-Population Model

When there is a limited population of potential customers for a service facility, we must consider a different queuing model. This model would be used, for example, if we were considering equipment repairs in a factory that has 5 machines, if we were in charge of maintenance for a fleet of 10 commuter airplanes, or if we ran a hospital ward that has 20 beds. The limited-population model allows any number of repair people (servers) to be considered. This model differs from the three earlier queuing models because there is now a *dependent* relationship between the length of the queue and the arrival rate. Let's illustrate the extreme situation: If your factory had five machines and all were broken and awaiting repair, the arrival rate would drop to zero. In general, then, as the *waiting line* becomes longer in the limited population model, the *arrival rate* of customers or machines drops.

Below, are the queuing formulas for the limited-population model. Note that they employ a different notation than Models A, B, and C. To simplify what can become time-consuming calculations, finite queuing tables have been developed that determine D and F . D represents the probability that a machine needing repair will have to wait

in line. F is a waiting-time efficiency factor. D and F are needed to compute most of the other finite model formulas.

$$X = \frac{T}{T+U} \tag{19}$$

$$L = N(1 - F) \tag{20}$$

$$W = \frac{L(T+U)}{N-L} = \frac{T(1-F)}{XF} \tag{21}$$

$$J = NF(1 - X) \tag{22}$$

$$H = FNX \tag{23}$$

$$N = J + L + H \tag{24}$$

Stevenson (2005).

Performance Measures of a Queuing System

Hillier and Lieberman (2005) put forth the following performance parameters in a queuing system:

- a. System Utilization (ρ): System Utilization is the most important measure of a queuing system. It is the ratio of system capacity used to available capacity. It measures the average time the system is busy. System utilization of zero means that there is nobody in the system. On the other hand, a system utilization of 1 or more signifies that there is infinite number of people on the waiting line. This means that the available service facility cannot cope with the arriving demand. Thus something has to be done on the service facility, Egolom (2001).
- b. Mean Number in the system (L_s): Mean number in the system is the average number of system users (entities) in the system; it includes those in the queue and those being served by the server(s).
- c. Mean Number in Queue (L_q): Mean number in the queue is the average or expected number of system users in the queue (waiting line), waiting for their turn to be served.
- d. The average waiting time for an arrival not immediately served (W_a)
- e. Mean Time in System (W_s): Mean time in the system is the expected value or average waiting time an entity will spend in the queuing system. It includes the average time waiting for service to begin and the average service time.
- f. Mean Time in Queue (W_q): Mean time in the queue is the expected value or average time an entity will spend in the queue, waiting for service to begin.
- g. probability of zero customers in the system (P_0)
- h. Probability of waiting (p_w): This is the probability that an arrival will have to wait for its service to begin.

Methodology: The research method used is the simulation of an existing single-line multiple channel models using Matlab program. The application of the models will reveal the waiting-line problems in the sector and the essence of simulating these models is to have the models been coded with a Matlab program so that the future works in this area

can be easier for any future researcher. This if adopted will contribute its knowledge in queuing system analysis.

Queuing system analysis of NNPC mega petroleum station Owerri

Table 1: Shows the experimental observation of arrival and service rate of Customers at NNPC mega petroleum station Owerri.

S/N	Time	Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
		λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ
	AM												
1	9:00-9:15	33	32	40	39	34	33	34	29	31	30	38	37
2	9:15-9:30	36	35	33	29	37	36	38	38	35	34	31	30
3	9:30-9:45	34	33	34	33	35	34	35	34	33	32	37	36
4	9:45-10:00	34	34	35	32	33	32	33	33	35	35	32	31
5	10:00-10:15	32	31	35	36	36	35	37	36	32	31	36	35
6	10:15-10:30	37	36	36	35	34	33	33	32	37	36	35	34
7	10:30-10:45	33	32	38	37	33	32	36	37	33	33	39	38
8	10:45-11:00	36	35	30	31	35	34	32	31	38	36	31	29
	PM												
9	3:00-3:15	38	37	29	30	40	39	39	38	38	39	33	33
10	3:15-3:30	36	29	39	38	32	29	30	30	33	30	34	33
11	3:30-3:45	37	38	33	31	36	37	36	35	35	34	37	36
12	3:45-4:00	33	30	32	30	36	35	34	34	35	33	35	34
13	4:00-4:15	34	34	30	29	33	32	33	32	37	36	36	35
14	4:15-4:30	34	33	37	36	32	31	37	36	33	32	33	32
15	4:30-4:45	40	39	35	34	37	36	34	33	35	35	36	36
16	4:45-5:00	31	28	36	37	31	30	34	35	32	32	32	31

(Source: Data generated by the researcher (2013))

Queuing system analysis of NNPC mega petroleum station Enugu

Table 2: Shows the result of the experimental observation of arrival and service rate of Customers at NNPC mega petroleum station Enugu.

S/N	Time	Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
		λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ
	AM												
1	9:00-9:15	41	40	39	37	38	36	41	39	43	41	42	40
2	9:15-9:30	41	39	42	40	39	39	41	37	39	36	45	43
3	9:30-9:45	42	36	41	41	38	40	40	38	39	37	44	44
4	9:45-10:00	40	41	40	40	37	39	39	37	36	38	39	37
5	10:00-10:15	40	43	46	39	43	38	45	36	41	40	45	40
6	10:15-10:30	43	40	40	41	37	40	39	38	40	38	39	38
7	10:30-10:45	39	41	42	43	43	41	40	44	40	41	44	44
8	10:45-11:00	40	39	41	39	42	37	39	40	41	39	42	38
	PM												
9	3:00-3:15	45	41	46	40	47	38	44	41	42	42	45	41

10	3:15-3:30	39	40	38	41	39	39	36	42	40	41	36	38
11	3:30-3:45	44	43	42	41	43	39	40	42	46	44	42	40
12	3:45-4:00	43	39	40	39	38	37	42	40	41	38	38	37
13	4:00-4:15	38	40	39	40	37	38	38	38	39	38	40	39
14	4:15-4:30	46	42	47	37	45	35	42	33	44	40	44	39
15	4:30-4:45	42	40	43	37	41	35	41	35	40	39	41	36
16	4:45-5:00	43	40	41	43	39	41	45	45	40	35	42	43

(Source: Data generated by the researcher (2013))

```
%PERFORMANCE EVALUATION OF QUEUING
(NNPC MEGA PETROLEUM STATION OWERRI)
%SINGLE LINE MULTI-SERVER SYSTEM
(MATLAB SOLUTION)
%*****
*****
```

```
lamda=34.6563; % Average Arrival Rate
mu=5.6111; % Average Service Rate
M=(2:12);%Number of servers
n=(0:11);
p=lamda./(M.*mu); % System Utilization
```

```
for k=0:11
    approxvalue(k+1)=(lamda/mu).^k./factorial(k);
end
N = 12; % The number of elements in the series

result(1) =approxvalue (1); % Initializing the result
for i=2:12
    result (i-1)= sum(approxvalue(1:i)); % I didn't suppress
the result here so that the result can be displayed after each
computation

end
```

```
P0=(result(1,:)+((lamda./mu).^M(1,:))./(factorial(M(1,:)).*
(1-(lamda./(M(1,:).*mu))))).^-1;%Probability system is
empty
```

```
L_q=((lamda*mu).*((lamda/mu).^M(1,:).*P0(1,:))./(facto
rial(M(1,:)-1).*((M(1,:).*mu)-lamda).^2);% Average
number in line
```

```
Wa=1./(M.*mu-lamda); % Average waiting time
Wq=L_q./lamda;% Average Time in Line
P_w=Wq./Wa;% Probability that arrival must wait
Ws=Wq+1./mu;% Average time in System
L_s=Ws.*lamda;% Average Number in System
```

```
%GRAPHS OF THE VARIOUS VARIABLES
AGAINST THE NUMBER OF SERVERS
%*****
*****
```

```
figure(1)
subplot(2,2,1)
plot(P0,M)%Probability system is empty against Number
of servers
xlabel('Number of Servers (M)')
ylabel('Probability a System is empty')
```

```
subplot(2,2,2)
plot(L_q,M)%Average number in line against Number of
servers
xlabel('Number of Servers (M)')
ylabel('Average Number in Line')
```

```
subplot(2,2,3)
plot(Wa,M)%Average waiting time against The number of
servers
xlabel('Number of Servers (M)')
ylabel('Average Waiting Time')
```

```
subplot(2,2,4)
plot(Wq,M)%Average time stayed in line against The
number of servers
xlabel('Number of Servers (M)')
ylabel('Average Time in Line')
```

```
figure(2)
Subplot(3,1,1)
plot(Ws,M)%Average time in the system against The
number of servers
xlabel('Number of Servers (M)')
ylabel('Average Time in the System')
```

```
subplot(3,1,2)
plot(P_w,M)%Probability that arrival must wait against
The number of servers
xlabel('Number of Servers (M)')
ylabel('Probability that Arrival must wait')
```

```
subplot(3,1,3)
plot(L_s,M)% The number of servers against Average
number in system
xlabel('Number of Servers (M)')
ylabel('Average Number in System')
```

Simulation Results for NNPC Mega Petroleum Station Owerri

Performance Evaluation Result for NNPC Mega Petroleum Station Owerri

```
>> p
p =
3.0882 2.0588 1.5441 1.2353 1.0294 0.8823
0.7720 0.6863 0.6176 0.5615 0.5147
>> P0
P0 =
-0.5108 -0.0923 -0.0218 -0.0052 -0.0004 0.0012
0.0017 0.0019 0.0020 0.0021 0.0021
>> L_q
L_q =
-6.8999 -6.6540 -6.8868 -8.6965 -37.9241 5.0633
1.3534 0.4893 0.1908 0.0752 0.0291
>> Wa
Wa =
-0.0427 -0.0561 -0.0819 -0.1515 -1.0104 0.2164
0.0977 0.0631 0.0466 0.0369 0.0306
>> Wq
Wq =
-0.1991 -0.1920 -0.1987 -0.2509 -1.0943 0.1461
0.0391 0.0141 0.0055 0.0022 0.0008
>> Ws
Ws =
-0.0209 -0.0138 -0.0205 -0.0727 -0.9161 0.3243
0.2173 0.1923 0.1837 0.1804 0.1791
>> P_w
P_w =
4.6656 3.4220 2.4267 1.6564 1.0830 0.6752
0.3996 0.2237 0.1181 0.0587 0.0275
>> L_s
L_s =
-0.7235 -0.4776 -0.7104 -2.5201 -31.7477 11.2397
7.5298 6.6657 6.3672 6.2516 6.2055

%PERFORMANCE EVALUATION OF QUEUING
(NNPC MEGA PETROLEUM STATION ENUGU)
%SINGLE LINE MULTI-SERVER SYSTEM
(MATLAB SOLUTION)
%*****
lamda=41.1458; % Average Arrival Rate
mu=6.5677; % Average Service Rate
M=(2:12);%Number of servers
n=(0:11);
p=lamda./(M.*mu); % System Utilization

for k=0:11
    approxvalue(k+1)=(lamda/mu).^k./factorial(k);
end
```

```
N = 12; % The number of elements in the series

result(1) = approxvalue (1); % Initializing the result
for i=2:12
    result (i-1)= sum(approxvalue(1:i)); % I didn't suppress
the result here so that the result can be displayed after each
computation

end

P0=(result(1,:)+((lamda./mu).^M(1,:))./(factorial(M(1,:)).*
(1-(lamda./(M(1,:).*mu))))).^(-1);%Probability system is
empty

L_q=((lamda*mu).*((lamda/mu).^M(1,:)).*P0(1,:))./(facto
rial(M(1,:)-1).*((M(1,:).*mu)-lamda.^2));% Average
number in line

Wa=1./(M.*mu-lamda); % Average waiting time
Wq=L_q./lamda;% Average Time in Line
P_w=Wq./Wa;% Probability that arrival must wait
Ws=Wq+1./mu;% Average time in System
L_s=Ws.*lamda;% Average Number in System

%GRAPHS OF THE VARIOUS VARIABLES
AGAINST THE NUMBER OF SERVERS
%*****
*****
figure(1)
subplot(2,2,1)
plot(P0,M)%Probability system is empty against Number
of servers
xlabel('Number of Servers (M)')
ylabel('Probability a System is empty')
subplot(2,2,2)
plot(L_q,M)%Average number in line against Number of
servers
xlabel('Number of Servers (M)')
ylabel('Average Number in Line')
subplot(2,2,3)
plot(Wa,M)%Average waiting time against The number of
servers
xlabel('Number of Servers (M)')
ylabel('Average Waiting Time')
subplot(2,2,4)
plot(Wq,M)%Average time stayed in line against The
number of servers
xlabel('Number of Servers (M)')
ylabel('Average Time in Line')
figure(2)
Subplot(3,1,1)
plot(Ws,M)%Average time in the system against The
number of servers
xlabel('Number of Servers (M)')
ylabel('Average Time in the System')
subplot(3,1,2)
plot(P_w,M)%Probability that arrival must wait against
The number of servers
xlabel('Number of Servers (M)')
ylabel('Probability that Arrival must wait')
subplot(3,1,3)
plot(L_s,M)% The number of servers against Average
number in system
```

xlabel('Number of Servers (M)')
 ylabel('Average Number in System')

to any order establishment where single-line multiple-channel queuing system.

Simulation Results for NNPC Mega Petroleum Station Enugu

PERFORMANCE EVALUATION RESULT FOR NNPC MEGA PETROLEUM STATION ENUGU

```
>> p
p =
3.1324 2.0883 1.5662 1.2530 1.0441 0.8950
0.7831 0.6961 0.6265 0.5695 0.5221
>> P0
P0 =
-0.5160 -0.0929 -0.0220 -0.0054 -0.0006 0.0010
0.0016 0.0018 0.0019 0.0019 0.0019
>> L_q
L_q =
-6.9758 -6.7108 -6.8933 -8.4722 -26.6267 6.0259
1.5238 0.5463 0.2134 0.0846 0.0331
>> Wa
Wa =
-0.0357 -0.0466 -0.0672 -0.1204 -0.5748 0.2071
0.0878 0.0557 0.0408 0.0322 0.0265
>> Wq
Wq =
-0.1695 -0.1631 -0.1675 -0.2059 -0.6471 0.1465
0.0370 0.0133 0.0052 0.0021 0.0008
>> Ws
Ws =
-0.0173 -0.0108 -0.0153 -0.0536 -0.4949 0.2987
0.1893 0.1655 0.1574 0.1543 0.1531
>> P_w
P_w =
4.7488 3.4973 2.4921 1.7105 1.1257 0.7071
0.4220 0.2385 0.1272 0.0639 0.0303
>> L_s
L_s =
-0.7109 -0.4460 -0.6284 -2.2074 -20.3618 12.2908
7.7887 6.8111 6.4783 6.3495 6.2979
```

6. Discussion and Conclusion

The simulation of the results for NNPC Mega Station Enugu and Owerri respectively show their results for system utilization, average number in line, average number in the system, the probability that the system is empty, probability that the system must wait, average waiting time, average time in line, waiting time in the system. From the results it was observed that the most suitable results for the two case study facilities were the utilization of the eight (8) serves in the each of the following systems. This is based on the statement of Egolum (2001) which says that system utilization of 0.8 to 0.1 is the best.

In conclusion, the use of simulation technique will help both case studies to facilitate and to properly follow up there queuing system with easy. The queuing system simulation program has already been written to simulate any single-line multiple-channel queuing system. The simulation program and its technique have already been recommended to the case studies. It can also be applicable

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