household. In that context more than half of the total minority is at least connected with six other German minorities.

Based on our Ipsos Democracy Pulse (see Hoops, Glantz and Michael 2013), we have used a RDD-similar technique for creating random mobile and landline numbers in Germany. Since there was no reliable data available before we expected that there would be no differences between these two countries. The assumed distribution of the sum of landline and mobile phone numbers of each person can be seen in Figure 2.

We have used λ= 0.399 for gravity sampling (Hoops, Schnapp & Schaefer-Rolffs 2013). Moreover, along with distance to the Danish-German border and also considered the election results of the minority party, the density of minority oriented organizations, schools and companies as relevant attractiveness variables. We supposed that our survey will constitute a public interest so we could use citizens registers to select a sample. This registers would be complete, correct and contain no duplicates. For facility-based sampling we restricted ourselves to German schools in Denmark so only people with children that go to school can be included.

We used bootstrapping to approximate the theoretical values of the bias, the cardinality of the sample and the variance of the inclusion probabilities. These statistics are computed by selecting sub-samples of the size of 100 people from the complete sample by different sampling techniques and repeated it 10,000 times. Therefore, the inclusion probability of a person in the sample can be estimated by the number of selections divided through the number of repetitions. While the bias is calculated by the relative medium deviation between sample and target population in some variables (age and number children), the cardinality of the sample space is measured as the number of different cases that are selected at least once divided through the number of target populations.

### 3.2 Results

In Table 2 you can see that the cardinality of the sample space is very low for facility-based sampling whereas gravity sampling has a complete sample space under our assumption of no missing minority in the register.

**Table 2:** Measures to compare different methods

| *Method* | *Bias* | *Cardinality of the sample space* | *Variance of the inclusion probability* |
|---|---|---|---|
| FBS | 63.1% | 38.6% | 25553.6 |
| GS | 9.5% | 100% | 127.5 |
| RDD | 9.4% | 96% | 10719.8 |
| SS | 17.4% | 98.4% | 3540.9 |

In RDD the cardinality of sample space is lower than in the gravity simulation as only people who have a landline or mobile connection can be interviewed. Because 1% of the total target population can't be recommended by any minority, the cardinality of the sample space in the snowball sampling simulation should be 99%. However, the cardinality is about 98.4%, so there is nearly one percent that could be selected, but is not. The facility-based method has the lowest cardinality among all sampling techniques, since only 3,153 out of 7,913 target people have school-going children.

On the contrary, facility-based sampling has the highest variance of the inclusion probabilities as compared to the others (see Figure 2), because there are many people who are eligible but have no chance to be part of the sample. The top-left plot shows only a few data points. Most of the people are selected between 310 and 330 times.

As we can see, gravity sampling produces the lowest variance of the inclusion probability and the selection frequencies are concentrated from 100 to 160 times. In contrast to RDD it is no household sample and enables roughly constant inclusion probabilities in the sample. The simulation of RDD shows a slightly larger variance with selection frequencies, which are lying in the range of 130 to 200 times. The more landline and mobile telephone numbers a minority member has, the greater the probability of that person being selected in the sample space. So one minority has been randomly selected 862 times due to the possession of many telephone numbers in total.

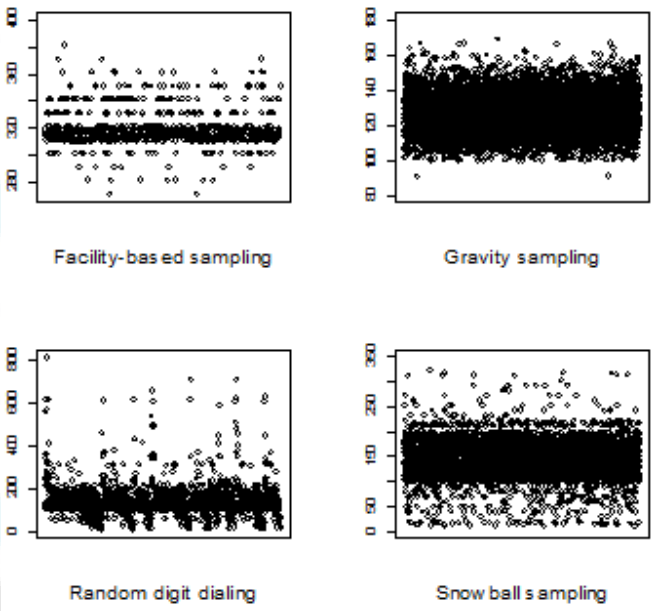


**Figure 2:** Plot of the inclusion frequencies of each technique

In snowball sampling the selection frequencies are concentrated between 100 to 200 times and a lower total variance of the inclusion probabilities is created. This is surely something surprising and calls our assumptions into question, because we expected a greater variance. The gravity sampling shows values from 91 to 169. Thus there is only less variation due to the expected frequency of about the average number of 126.37 (=(10,000*100)/7913) selections.

However, it is hardly surprising that all sampling methods create biased samples because of the very low sample size of 100. RDD and gravity samples show the lowest differences to the total population. So both methods are recommendable for representative studies. Furthermore, both biases should be asymptotically tending towards zero. Therefore, with

snowball and facility-based sampling there can only be samples produced which differ greatly from the mean of the population. Because we even compute the distortion across the central portion of children in the household, the samples in facility-based sampling are greatly biased since the researcher can only reach target people with children. Snowball sampling is biased because older people tend to have a greater network size and are more often selected.

## 4. Discussion and Conclusion

Because of low prevalence, traditional survey methods such as random digit dialing and citizens registry-office sampling are very expensive when carrying out surveys of hard-to-reach populations. Therefore, alternative methods such as gravity or snowball sampling with some limitations would be favorable and recommendable.

Our simulations have shown some advantages of gravity sampling. First, this method in combination with citizens registers has the largest cardinality of the sample space under the assumption of correct official data. Second, there is no significant distortion detected and inclusion probability variation is far less. Although we received positive results for gravity sampling, the computations in real studies might behave significantly differently. Therefore, the gravity sampling method should be strongly tested in practice.

It is, however, possible that we are overestimating the variance of the inclusion probabilities in the RDD simulation, because the inclusion probability does not increase proportionally to the amount of telephone numbers. Surely a person who has ten mobile numbers will not always possess ten mobile phones and will not always be equally reachable at each number. This should be taken into account for later simulations.

It is also possible to incorrectly estimate the network size of national minorities. If, for example, a single individual knows more than 16 other members and/or the proportion of the minority without a network is considerably larger, this could lead to an increase in the inclusion probability in random samples. Nonetheless, our conservative assumption should not influence the conclusions.

We must of course admit that the calculation of bias using age and number of children has been established somewhat subjectively. Especially since both variables are obviously correlated with each other. But there was no valid multivariate marginal distribution that we could have used for our simulations. It is also impossible to judge the value of the estimation of loading parameters lambda. So it only remains to be recommended that an appropriately practical method test should be conducted with a large sampling to calculation the deviations from official statistics.

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