



interest rate “i” to the rate of inflation “φ” and the real interest rate “r”. The real interest rate “r” is the interest rate after adjustment for inflation. It is the interest rate that lenders have to have to be willing to loan out their funds. The relation Fisher posted between these three rates is:

$$(1 + i) = (1 + r) (1 + \phi) = 1 + r + \phi + r\phi$$

This is equivalent to  $i = r + \phi (1 + r)$

Thus, according to this equation, if ‘φ’ increases by 1% the nominal interest rate increases by more than 1%. This means that if ‘r’ and ‘φ’ are known then ‘i’ can be determined. On the other hand, if ‘i’ and ‘φ’ are known then ‘r’ can be determined by the relationship is

$$1+r = (1+i) / (1+\phi) \text{ or } r = (i - \phi) / (1+\phi)$$

**3.2 Regression Model with Trigonometric function to accommodate cyclical fluctuation of Prices of Items/Inflation.**

In the current model it is assumed that inflation which plays vital role in calculation of real interest rate is assumed to be a regression model with trigonometric function to accommodate cyclical fluctuation of prices. Forecasting of inflation is carried out for 20 years based on the regression equation generated.

A model for a seasonal variation might include transcendental functions. The cycle of the model is as below . The model might be used to represent data for the four seasons of the year.

$$\phi = b_0 + b_1x + b_2 \sin ( 2t \pi /4 ) + b_2 \cos ( 2t \pi /4 )$$

Regression equation with trigonometric function is:

$$\phi = b_0 + b_1x + b_2 \sin ( 2t \pi /4 ) + b_2 \cos ( 2t \pi /4 )$$

Here ‘φ’ is inflation, ‘x’ is a variable and ‘t’ period, b<sub>0</sub>,b<sub>1</sub>,b<sub>2</sub> are the coefficients.

**Table 1: Quarterly data of inflation for two years**

Year (t)	0	1/4	1/2	3/4	1	5/4	3/2	7/4	2
φ	4.0	4.35	4.2	4.15	4.4	4.75	4.6	4.45	4.8

Following set of equations are used to obtain the values of coefficients of regression model

$$\phi = b_0 + b_1x + b_2 \sin ( 2t \pi /4 ) + b_2 \cos ( 2t \pi /4 ) (1)$$

$$\sum \phi = nb_0 + b_1 \sum x + b_2 \sum \sin ( T \pi /2 ) + b_2 \sum \cos ( 2t \pi /4 ) (2)$$

$$\sum (\phi x) = b_0 \sum x + b_1 \sum x^2 + b_2 \sum x. \sin ( 2 \pi t/4 ) + b_2 \sum x. \cos ( 2t \pi /4 ) (3)$$

$$\sum (\phi x^2) = b_0 \sum x^2 + b_1 \sum n^3 + b_2 \sum x^2. \sin ( 2 \pi t/4 ) + b_2 \sum x^2. \cos ( 2t \pi /4 ) (4)$$

When time origin is taken between middle of years the equation reduces to

$$\sum \phi = n b_0 + b_2 \sum \sin ( 2 \pi t /4 ) + b_2 \sum \cos ( 2 \pi t /4 ) (5)$$

$$\sum (\phi x) = b_1 \sum x^2 (6)$$

$$\sum (\phi x^2) = b_0 \sum x^2 + b_2 \sum x^2. \sin ( 2\pi t /4 ) + b_2 \sum x^2. \sin ( 2\pi /4 ) (7)$$

by solving above equations we get the values of regression coefficients

$$b_0 = 4.413, b_1 = -0.0817, b_2 = -0.022$$

The final regression equation is :  $F_n = 4.413 + 0.082x - 0.022\sin(2\pi t/4) - 0.022 \cos(2\pi t/4)$

**Table 2: Forecasted Inflation for 20 years**

x	t (Period in years)	φ
2	7.00	6.34
4		2
2	8.00	6.61
8		8
3	9.00	6.93
2		8
3	10.00	7.30
6		2
4	11.00	7.62
0		2
4	12.00	7.89
4		8
4	13.00	8.21
8		8
x	t (Period in years)	φ
-4	0.00	4.00
0	1.00	4.40
4	2.00	4.80
8	3.00	5.06
		2
1	4.00	5.33
2		8
1	5.00	5.65
6		8
2	6.00	6.02
0		2

x	t (Period in years)	φ
5	14.00	8.582
2		
5	15.00	8.902
6		
6	16.00	9.178
0		
6	17.00	9.498
4		
6	18.00	9.862
8		
7	19.00	10.18
2		2
7	20.00	10.45
6		8

#### 4.Replacement Problem with Linear Trend Running Cost

The replacement problems are concerned with the situation that arises when the efficiency of item decreases, failure or breakdown occurs. The decrease of efficiency or breakdown may be either gradual or sudden. The situation which demands the replacement of items are;

1. The old item has become inefficient or require expensive maintenance
2. The old item has failed due to accident or otherwise and does not work at all, or the old item is expected to fail shortly
3. A better design of equipment has been developed or due to obsolescence

The objective of replacement is to decide best policy to determine an age at which the replacement is most economical instead of continuing at increased maintenance costs. The problem of replacement is encountered in the case of both men and machines. It is possible to estimate the chances of failure of various ages. The fundamental objective of replacement is to direct the organization for maximizing its profit (or minimizing the cost). In the current paper It is assumed that running cost of the item follows linear trend with governing relation as:

$$R(n) = a_0 + a_1 n$$

'n' is time period,  $a_0, a_1$  are parameters or coefficients. The

model can be fitted to the data by using ordinary least square method so that

Following set of equations are used to obtain the values of coefficients of regression model

$$R = a_0 + a_1 t \quad (1)$$

$$\sum R = ma_0 + a_1 \sum t \quad (2)$$

$$\sum(R n) = a_0 \sum t + a_1 \sum t^2 \quad (3)$$

The following yearly maintenance cost (in rupees) of a machine is used to get trend equation of running cost. The machine is purchased at a total cost of  $C = \text{Rs.}4100$  and immediately after usage for first year its salvage value is reduced to  $\text{Rs.}1900$  and it remains the same in all subsequent periods.

**Table 3:** Running cost of Machine

Year (n)	1	2	3	4	5	6	7	8
R(n)	500	520	550	570	600	620	640	670

Year (n)	9	10	11	12	13	14	15
R(n)	690	720	740	760	790	810	840

by solving above equations we get the final regression equation as  $R_n = 668 + 35.78 (n)$  and using this trend, running cost are forecasted and used in replacement decision making.

**Table 4:** Calculation of average annual cost with money value (Forecasted running cost, inflation and real rate of interest)

1	2	3	4	5	6	7	8
Period	Inflation	Real interest rate	Present worth factor	Discount factor	Discount factor	Dividing discount factor	Maintenance Cost (Forecasted)
n	$\phi_n$	$r_n$	v	$v^n$	$v^{n-1}$	$\sum v^{n-1}$	$R_n$
1	3.4	3.773	0.964	0.964	1.000	1.000	703.780
2	3.8	3.375	0.967	0.936	0.967	1.967	739.560
3	4.04	3.167	0.969	0.911	0.940	2.907	775.340
4	4.37	2.911	0.972	0.892	0.918	3.824	811.120
5	4.69	2.691	0.974	0.876	0.899	4.724	846.900
6	5	2.500	0.976	0.862	0.884	5.608	882.680
7	5.33	2.318	0.977	0.852	0.872	6.479	918.460
8	5.65	2.158	0.979	0.843	0.861	7.340	954.240
9	6.03	1.987	0.981	0.838	0.854	8.195	990.020
10	6.29	1.881	0.982	0.830	0.846	9.040	1025.800
11	6.94	1.645	0.984	0.836	0.849	9.890	1061.580
<b>12</b>	<b>7.26</b>	<b>1.542</b>	<b>0.985</b>	<b>0.832</b>	<b>0.845</b>	<b>10.735</b>	<b>1097.360</b>
13	7.58	1.448	0.986	0.830	0.842	11.576	1133.140
14	7.91	1.357	0.987	0.828	0.839	12.416	1168.920

9	10	11	12	13
Maintenance Cost with money value $R_n v^{n-1}$	Cumulative Maintenance Cost with money value $\sum R_n v^{n-1}$	Salvage with money value $S_n v^n$	Total annual cost $TC = (C - S_n v^n + \sum R_n v^{n-1})$	Average annual cost $TC/\sum v^{n-1}$
703.780	703.780	1830.924	2972.856	2972.856
715.415	1419.195	1777.962	3741.232	1901.659
728.473	2147.668	1730.356	4517.312	1553.994
744.226	2891.893	1693.998	5297.895	1385.276
761.567	3653.460	1663.789	6089.671	1289.181
780.161	4433.620	1638.364	6895.256	1229.643
800.497	5234.118	1618.464	7715.653	1190.854
821.777	6055.895	1601.689	8554.206	1165.379
845.821	6901.716	1591.632	9410.084	1148.324
867.435	7769.151	1577.016	10292.135	1138.480
901.778	8670.929	1587.870	11183.059	1130.777
<b>927.318</b>	<b>9598.247</b>	<b>1581.197</b>	<b>12117.050</b>	<b>1128.768</b>
953.641	10551.888	1576.207	13075.681	1129.517
981.051	11532.939	1573.284	14059.655	1132.416

Table 5: Calculation of average annual cost without money value

Year n	Forecasted Maintenance cost $R_n$	Cumulative Maintenance Cost $\sum R_n$	Salvage value $S_n$	Loss due to resale (C-S)	Total cost $TC=(C+\sum R_n - S)$	Average cost per year $TC/n$
1	703.780	703.78	1900	2200	2903.78	2903.78
2	739.560	1443.34	1900	2200	3643.34	1821.67
3	775.340	2218.68	1900	2200	4418.68	1472.893
4	811.120	3029.8	1900	2200	5229.8	1307.45
5	846.900	3876.7	1900	2200	6076.7	1215.34
6	882.680	4759.38	1900	2200	6959.38	1159.897
7	918.460	5677.84	1900	2200	7877.84	1125.406
8	954.240	6632.08	1900	2200	8832.08	1104.01
9	990.020	7622.1	1900	2200	9822.1	1091.344
10	1025.800	8647.9	1900	2200	10847.9	1084.79
<b>11</b>	<b>1061.580</b>	<b>9709.48</b>	<b>1900</b>	<b>2200</b>	<b>11909.48</b>	<b>1082.68</b>
12	1097.360	10806.84	1900	2200	13006.84	1083.903

### 5. Results and Conclusion

When money value with inflation is considered, it is observed from the table 4, that the average annual cost of machine is decreasing gradually up to 12<sup>th</sup> year and from 13<sup>th</sup> year it is increasing. So it is advisable to replace the machine at the end of 12<sup>th</sup> year. But when money value is not considered the replacement period comes to 11<sup>th</sup> period (table 5). In general real time decision making depends on various uncontrollable parameters. In the current work of replacement of machine, only some macroeconomic parameters are considered with an assumption that the running cost has linear trend, but there is a lot of scope to develop a robust model considering different patterns of running cost, mileage of machine, technological changes, government policies (taxes etc.).

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