ISSN (Online): 2347-3878, Impact Factor (2014): 3.05

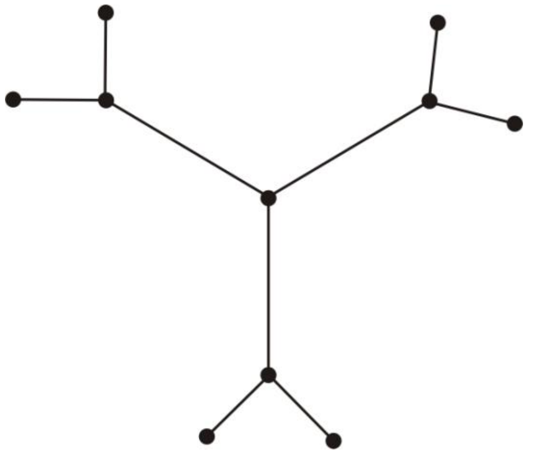

Figure 8: A Steiner tree in Petersen graph

We reduce 3 SAT to steiner problem in Petersen graphs. Let $x_1, x_2, \ldots, x_n$ be the variables. $c_1, c_2, \ldots, c_m$ the clauses in an arbitrary instance of 3SAT. Our aim is to construct a Petersen graph $G = (V, E)$ and a terminals set K, and a bound B such that Petersen graph contains steiner tree T for K at size at most B if and only if the given 3SAT instance is satisfiable.

Transforming 3SAT to Steiner problem in Peterson graph is constructed as follows. First we connect u and v by a variable path as shown in Figure 9.

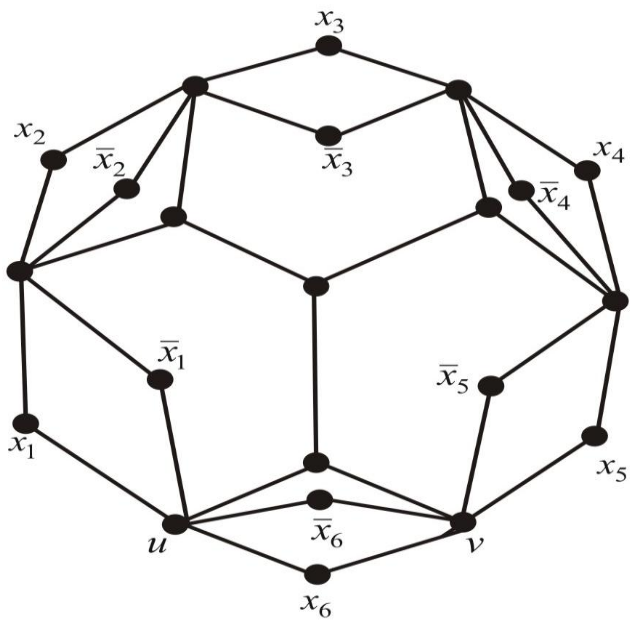

Figure 9: Transforming 3SAT to Steiner problem in Petersen graph

Then we create for every clause gadget consisting of a vertex C_i that is connected to the literals. contained in the clause C_i by paths of length $t = 2n+1$. As terminal set we choose $k = \{u, v\} \cup \{C_1 .. C_4\}$ and set $B = 2n + t.m.$

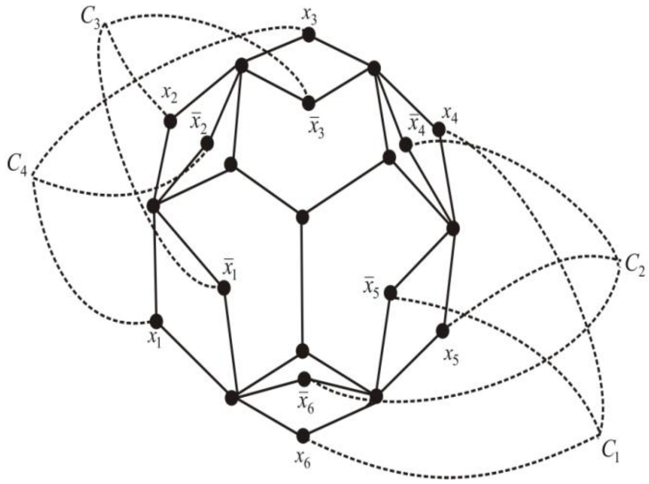

Figure 10: The clause gadget for the clause

$$C_1 = x_1 \vee \overline{x_2} \vee x_3.$$

The dashed lines indicated paths of length $t = 2n+1$ from C_i to the appropriate vertices on the variable path. Assume first that 3SAT for instance is satisfiable let $x_i \in P$ if x_i is said to be true in this assignment, and $\bar{x}_i \in P$ otherwise.To construct a Steiner tree for K we start with $u - v$ path *P* is reflecting a satisfying assignment.

x_i is true for all variables. Hence we arrive from our SAT problem with six variables and 4 clauses for a 3SAT problem.

$C_1 = (x_6 \vee \bar{x}_5 \vee x_4)$ $C_2 = (\bar{x}_6 \vee x_5 \vee \bar{x}_4)$

$C_3 = (\bar{x}_1 \vee x_5 \vee \bar{x}_3)$ $C_4 = (x \vee \bar{x}_2 \vee x_3)$

With six variables and five clauses.

Now we take $n = 6$; to form the clauses $\{C_1, C_2, C_3, C_4\}$ and the terminal set $K = \{u, v\} \cup \{C_1, C_2, C_3, C_4\}$ and $B = 2n + t.m$

$B = 2n + t.m$

$t = 2n + 1$

$n = 6 \Rightarrow t = 2(6) + 1 \quad = 13$

$m = 4 \Rightarrow B = 2n + t.m$

$= 2(6) + 13.4$

$= 12 + 52 = 64$

To construct a Steiner tree for K we starting with a $u - v$ path P reflecting a satisfying assignment.

Next observe that for every clause the vertex C_i can be connected to P by path of Length t.

In this way we obtain a Steiner tree for K of Length $2n + t.m = B.$

On the other hand, we assume now that T is a Steiner tree for K of Length at most B, Trivially for each clause to the vertex C_i has to be connected to the variable path.

Then $|E(T)| \geq (m+1).t > B,$

$\geq (4+1).13 > 64$

$\geq 5.13 > 64$
$\geq 65 > 64 \Rightarrow$ contradiction

This show that u and v can only be connected along the variable path, which requires at least 2n edges.

In this graphs $u - v$ path contains 24 edges and that each clause gadget is connected to this path using exactly t edges.

In this graph each clause gadget is connected to this path using exactly 12 edges. Thus $u - v$ path reflects a satisfying assignment.

In the figure 3, the Steiner tree contains 6 terminals.

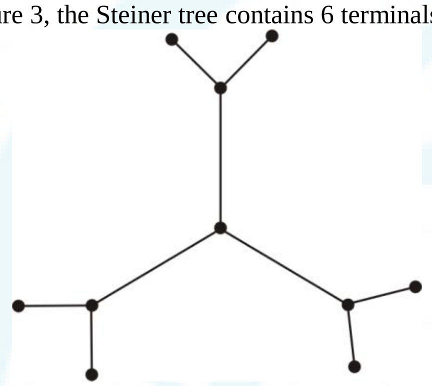

Figure 11: A Steiner minimum tree in Petersen graph

This implies that every full component of a Petersen graph contains at most four terminals.

Hence, Steiner tree in Petersen graph contains at most 4 terminals. This implies that R-Restricted Steiner Problem in Petersen graph is NP-Complete.

Result 1
Every $u - v$ Path of Steiner tree in Petersen graph is NP-Complete and every $u - v$ Path of Petersen graph contains exactly 2n edges.

Result 2
Transforming 3SAT to Steiner Problem in Petersen graph is NP-Complete.

Result 3
A Steiner tree of Petersen graph contains at most 4 terminals.

4. Conclusion

With Steiner tree in Petersen graph, we conclude that every $u - v$ Path contains exactly 2n edges, transforming 3SAT to Steiner problem in Petersen graph is NP-Complete and every full component of a Steiner tree in Petersen graph contains at most 4 terminals.

References

[1] G. Nirmala and C. Sujatha, Every u-v Path of NP – Complete Steiner graphs contains Exactly 2n-edges, International journal of scientific and research publication, Volume 3, Isssue 4, September 2014.

[2] G. Nirmala and D.R. Kirubaharan, Uses of Line graph, International Journal of Humanities and Sciences, PMU-Vol.2, 2011.

[3] G. Nirmala and M. Sheela, Fuzzy effective shortest spanning tree algorithm, International Journal of Scientific Transaction in Environment and Techno Vision, Volume I, 2012.

[4] G. Nirmala and C. Sujatha, Transforming 3SAT to Steiner Problem in Planar Graph which NP-Complete, Aryabhatta Journal of Mathematics and Informatics, Volume 6, No.2, December 2014.

[5] Han Jurgen Promel Angelika Steger 'The Steiner tree problem' page 42-58.

[6] Alessandra Santurari, Steiner Tree. NP-Completeness Proof, May 7, 2003.

[7] Michael R Gamey and David S. Johnson, Computers and Intractability. A Guide to the theory of NP-Completeness. W.H. Freeman and company, 1979.

[8] U.C. Berkeley – CS172, Automata, Computability and Complexity.

[9] Professor Luca Trevisan Notes on NP-Completeness, August 2009.

[10] Piete, Oloff de wet, Geometric Steiner Minimal Trees, University of South Africa, January 2008.

[11] Frank Harary, Graph Theory 1988, pp.32-40.

[12] S.P. Rajagopalan, R. Sattanathan, Graph theory, Chapter-5.

[13] R. Balakrishnan, K. Ranganathan, A Text Book of Graph theory, pp.152-180.

[14] G. Nirmala and C. Sujatha, "R-Restricted Steiner Problem is NP complete". International Journal of Science and Research, Volume 4, Issue 4, April 2015.