$$\text{Set } \frac{\partial J}{\partial k_2} = 0 \qquad (23)$$

Therefore

$$k_2 = \frac{-JRaBLa}{k_t} \pm \frac{\sqrt{(JRaBLa)^2 - (JLa + BRa + k_t k_e + k_t)(BRa + k_t k_e + k_t + JLa) + J^2 Ra^2 + 2JRaBLa + B^2 La^2}}{k_t} \qquad (24)$$

$$k_2 = 1.01499 \qquad (25)$$

$$J_{min} = 1.47 \qquad (26)$$

The system matrix H obtained for the compensated system is,

$$H = \begin{bmatrix} 0 & 1 \\ -2629.476 & -2833.7 \end{bmatrix} \qquad (27)$$

The feedback control signal is obtained as,

$$u = -x_1 - 1.015 x_2 \qquad (28)$$

This compensated system is considered to an optimal system which results in a minimum value for the performance index. The simulation of this compensated system is listed below and shown in figure 5 & figure 6. The BLDC drive system parameters are shown in Table 1

Table 1: BLDC Drive Parameters

SL. NO.	Parameter	Symbol	Unit	Value
1.	Stator Winding Resistance	Ra	Ω	1.4
2.	Stator Winding Inductance	La	H	0.0066
3.	Rotor inertia	J	Kg-m^2	0.00176
4.	Motor Viscous Friction Coefficient	B	Nm/rad/sec	0.0003888
5.	Torque Constant	K_t	Nm/Amp	0.03
6.	Velocity Constant	K_e	Volts/rad	0.0000181

4. Linear Quadratic Regulator (LQR)

This section deals with the design of a stable control system for BLDC drive based on quadratic performance indexes. The main advantage of using the quadratic optimal control scheme is that the system designed will be stable, except in the case where the system is not controllable. The matrix 'P' is determined from the solution of the matrix Riccatti equation. This optimal control is called the Linear Quadratic Regulator (LQR) [10], [11].

The optimal feedback gain matrix k can be obtained by solving the following Riccatti equation for a positive-definite matrix 'P'.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (29)$$

$$\text{Let } Q = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} (\mu \geq 0) \qquad (30)$$

$$\begin{bmatrix} 0 & -\frac{(\mathbf{B} R_a + k_t k_e)}{J L_a} \\ 1 & -\frac{(J R_a + B L_a)}{J L_a} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -\frac{(\mathbf{B} R_a + k_t k_e)}{J L_a} & -\frac{(J R_a + B L_a)}{J L_a} \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{k_t}{JLa} \end{bmatrix} [1] \begin{bmatrix} 0 & \frac{k_t}{JLa} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} = 0 \qquad (31)$$

Solving we obtain the following three equations,

$$\frac{P12^2 k_t^2}{J^2 L_a^2} + \frac{2(B R_a + k_t k_e)}{J L_a} - 1 = 0 \qquad (32)$$

$$P11 - \frac{P12(J R_a + B L_a)}{J L_a} - \frac{P22(B R_a + k_t k_e)}{J L_a} - \frac{P12\, P22\, k_t^2}{J^2 L_a^2} = 0 \qquad (33)$$

$$2 P_{11} - \frac{2 P_{22}(J R_a + B L_a)}{J L_a} - \frac{P22^2 k_t^2}{J^2 L_a^2} + \mu = 0 \qquad (34)$$

Solving these three equations we get,

$$P = \begin{bmatrix} -1.09\times10^{-3} + \sqrt{1+13298\mu} & 3.8\times10^{-4} \\ 3.8\times10^{-4} & -3.18\times10^{-5} + 3.35\times10^{-5}\sqrt{1+13298\mu} \end{bmatrix} \qquad (35)$$

The optimal feedback gain matrix is obtained as,

$$k = R^{-1} B^T P \qquad (36)$$

$$k = \begin{bmatrix} 0.981 & -0.08 + 0.0865\sqrt{1+13298\mu} \end{bmatrix} \qquad (37)$$

$$u = -kx = -0.981 x_1 - \left(-0.08 + 0.0865\sqrt{1+13298\mu}\right) x_2 \qquad (38)$$

Let assume μ =1, the control law u = 0.981$x_1 - 0.921 x_2$ This control signal yields an optimal result for any initial state under the given performance index. Figure (2) shows the block diagram for optimal control of the BLDC drive system.

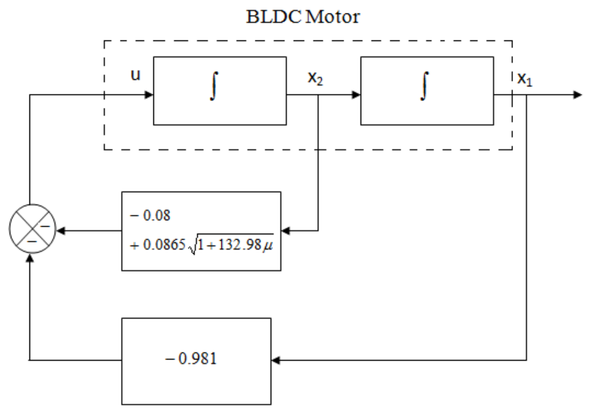

Figure 2: Optimal Control of the BLDC Drive System

5. Tuning of Q & R Matrix in LQR

In LQR the cost function which is to minimized is

$$J = \int_0^\infty (X^T Q X + U^T R U)\, dt \tag{39}$$

The two matrices Q and R are selected by the design engineer by trial and error method. Generally speaking, selecting a large value for Q requires the value of J to be small. On the other hand, selecting a large value for R, the control input u must be smaller to keep value of J small One should select value of Q to be positive semidefinite and R to be positive definite. This means that the scalar quantity $X^T Q X$ is always positive or zero at each time t. The Q & R matrix is tuned by trial & error method. The trial & method is done by MATLAB coding's. The best value of the Q & R matrix is calculated by checking the step response of the system.

The best value of

$Q=\begin{vmatrix} 0.41 & 0 \\ 0 & 0.0001 \end{vmatrix}$ & R=[1]

By tuning Q & R matrix the value of K_1 = 0.225 & K_2 = 0.0039,the control law

$$U = -0.0560\, x_1 - 0.0098 x_2 \tag{40}$$

6. Simulation Results and Discussions

MATLAB software package is used to determine the response of the system. Tuning of the Q & R matrix is done by separate coding. The regulation of speed and rate of change of speed is determined with and without tuning of Q & R matrix and the tracking of the motor is also determined. The Simulink model of the system shown in figure 3

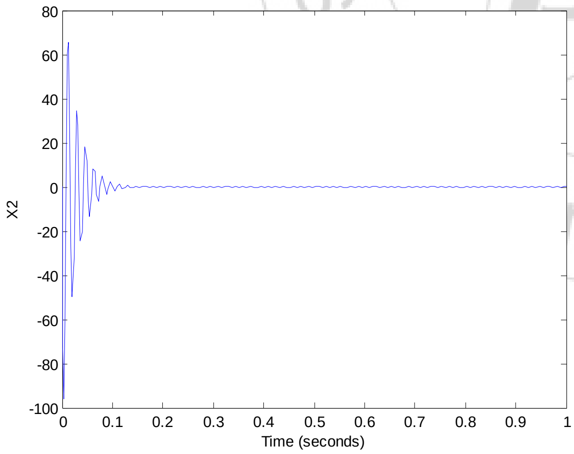

Figure 3: Regulation of rate of change of speed without tuning

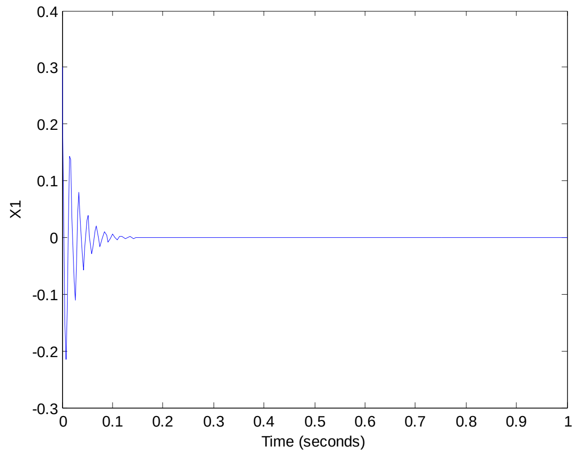

Figure 4: Regulation of speed without tuning

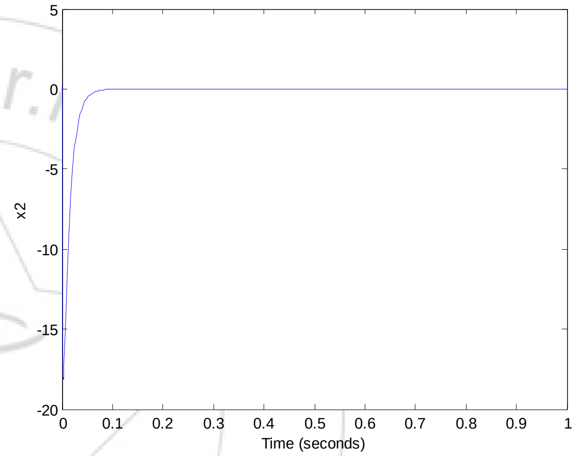

Figure 5: Regulation of rate of change of speed with tuning

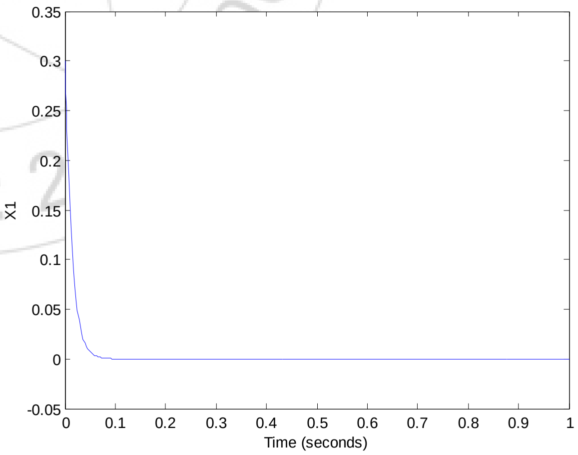

Figure 6: Regulation of speed with tuning

From figure 4 and figure 5 the system is regulated at 0.2 sec, but the system consists of large no of overshoot and undershoot. The system is not precisely regulated at this condition. From figure 5 and 6 the system is regulated at 0.1 sec without any oscillations. The system is completely controlled and the tracking of the motor is shown in figure 7.The motor is tracked at rated speed at 5.5 sec.

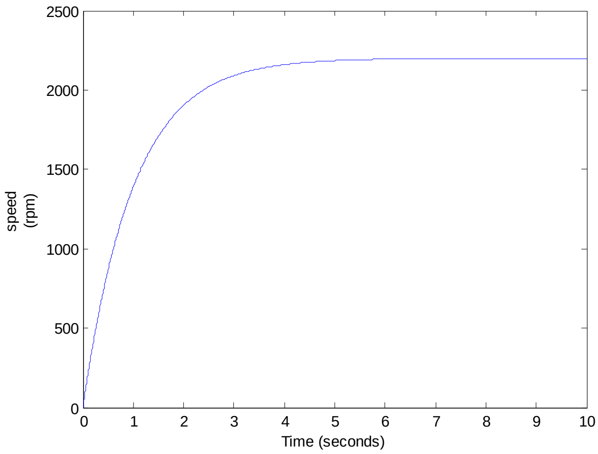

Figure 7: Tracking of BLDC motor at rated speed

7. Conclusion

In this paper a state variable feedback system was designed for BLDC drive system to achieve the desired system response. Also, an LQR system was designed for BLDC drive which results in a minimum value for the performance index. The LQR design provides an optimal state feedback control minimizes the quadratic state error and control effort

This optimal controlled BLDC drive system results in a minimum value for the performance index. Also, the control law given by equation (40) yields optimal result for any initial state under the given performance index. Both the transient and steady state response of the system is improved with LQR controller. This design based on the quadratic performance index yields a stable control system for the BLDC drive system

References

[1] P.Pillay and R.Krishnan, "Modelling, Simulation, and Analysis of Permanent Magnet Drives, Part I. The Permanent Magnet Synchronous Motor Drive" *IEEE Trans. Indus. Applications*, vol.25, No.2,pp 265-273,Mar/Apr. 1989.

[2] P.Pillay and R.Krishnan, "Modelling, Simulation, and Analysis of Permanent Magnet Drives, II. The BLDC Motor Drive" *IEEE Trans. Indus. Applications*, vol.25, No.2,pp 274-279,Mar/Apr. 1989.

[3] R.Krishnan," Electric Motor Drives, Modelling, Analysis and Control", PHI Learning Private Limited, New Delhi, 2009.

[4] Anand Sathyan, N.Milivojevic,Y-J Lee and M.Krishnamoorthy, "An FPGA-Based Novel Digital PWM Control Scheme for BLDC Motor Drives" *IEEE Trans. on Industrial Electronics*,Vol.35, pp 3040-3049 August 2009.

[5] Liviu Ioan Iepure, Ion Boldea "Hybrid I-f Starting and Observer-Based Sensorless Control of Single-Phase BLDC-PM Motor Drives*", Int. J Adv Manuf Technol*. 28: 942-949, May 2005. *IEEE transactions on industrial electronics, vol. 59, no. 9, september 2012*

[6] R. Venkitaraman,B. Ramaswami, "Thyristor Converter-Fed Synchronous Motor Drive", *Electric Machines and Electromechanics*, 6: 433-449, 1981.

[7] Tae-won chun, "Sensorless control of BLDC motor Drive for automotive fuel pump using a hysteresis comparator", *IEEE Trans. on Industrial Electronics*, Vol.55,No.9, pp3415-3425, Sep 2008.

[8] F.Rodigeuz, P.Desai and A.Emadi, "A Novel Digital Control Technique for Trapezoidal Brushless Dc Motor Drives", *in Proc. Electron, Technol. Conf., Chicago, IL*, Nov. 2004.

[9] F.Rodigeuz, and A.Emadi, "A Novel Digital Control Technique for Brushless Dc Motor Drives: Conduction-angle control", *in Proc. IEEE Int. Elect. Mach. Drives Conf.*, pp 308-314, May 2005.

[10] Katsuhiko Ogata, Modern Control Engineering (International Edition) Prentice- Hall of India, New Delhi 1988

[11] Richard C.Dorf, "Modern Control Systems", Addison Wesley Longman Inc., USA, 1998.

[12] R. Shanmugasundram, K. Muhammad Zakariahand,, N. Yadaiah "Implementation and Performance Analysis of Digital Controllers for Brushless DC Motor Drives", *IEEE Transactions On Mechatronics, Vol. 19, No. 1, February 2014*

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