



$$\delta_r = \frac{(\delta_{rL} + \delta_{rR})}{2} \quad (4) \quad \zeta_k' = f_k(x, \zeta_1, \dots, \zeta_k) + g_2(x, \zeta_1, \dots, \zeta_k)u \quad (11)$$

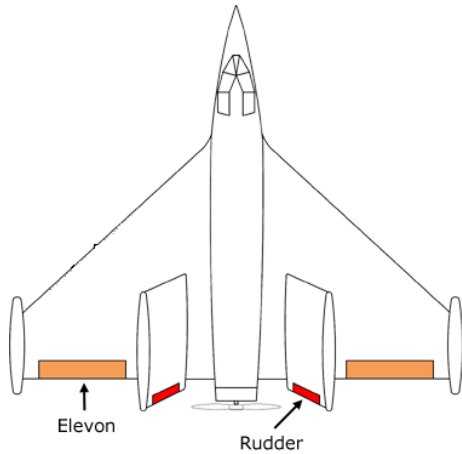


Figure 1: Control Surfaces

The non-linear set of equations of which describes the motion of the vehicle is as follows [6]:

$$\dot{p} = \frac{1}{I_{xx}}(L_{\beta} \sin \beta + L_p P + L_{\delta a} \sin \delta a + L_{\delta r} \sin \delta r + L_r r) \quad (5)$$

$$\dot{q} = \frac{1}{I_{yy}}(M_{\alpha} \sin \alpha + M_q q + M_{\delta a} \sin \delta a + M_{\delta r} \sin \delta r) \quad (6)$$

$$\dot{r} = \frac{1}{I_{zz}}(N_{\beta} \beta + N_r r + N_{\delta a} \delta a + N_{\delta r} \delta r + N_p p) \quad (7)$$

$$\dot{\alpha} = \frac{Z_{\alpha}}{V_T} \sin \alpha - g \frac{\sin \gamma}{V_T} \theta + \left(\frac{Z_{\alpha}}{V_T} + 1\right) q \quad (8)$$

$$\dot{\beta} = \frac{Y_{\delta} \beta}{V_T} + \frac{Y_{\delta} p}{V_T} + \left(\frac{Y_r}{V_T} - 1\right) r + \frac{g}{V_T} \phi \quad (9)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (10)$$

In the above set of equations p, q and r are the roll rate, pitch rate and yaw rate respectively. Here,  $\alpha$  is the angle of attack,  $\beta$  is the side slip angle, and  $\gamma$  is the flight path angle. The equations are represented in terms of aerodynamic forces and moments, where L is called the rolling moment, M the pitching moment and N is the yawing moment.  $I_{xx}, I_{yy}, I_{zz}$  are the moment of inertia in the x, y and z directions respectively.  $\phi$  is the roll angle, g is the acceleration due to gravity and  $V_T$  is the vehicle velocity. The coefficients Y and Z in the equations represents side force and downward force respectively. The dependence of the aerodynamic forces on the angle-of-attack and the side slip angle is crucial to stability and control.

### 3. Theory of Controller Design

#### 3.1 Basic Backstepping Technique

Backstepping designs by breaking down complex nonlinear systems into smaller subsystems, then designing control Lyapunov functions and virtual controls for these subsystems and finally integrating these individual controllers into the actual controller, by stepping back through the subsystems [5].

Consider a system of the form

$$\dot{x} = f(x) + g(x)\zeta_1$$

$$\dot{\zeta}_1 = f_1(x, \zeta_1) + g_1(x, \zeta_1)\zeta_2$$

$$\dot{\zeta}_2 = f_2(x, \zeta_1, \zeta_2) + g_2(x, \zeta_1, \zeta_2)\zeta_3$$

⋮

#### 3.2 Adaptive Backstepping Controller Design

Adaptive Backstepping Controllers are dynamic and more complex than the static controllers. What is achieved with this complexity is that, an Adaptive Backstepping Controller guarantees not only that the plant 'x,' remains bounded, but also regulation and tracking of a reference signal. In its basic form, the Adaptive Backstepping Control design employs overparametrization and this means that the dynamic part of the controller is not of minimal order. Consider

$$\begin{aligned} x_1' &= x_2 + \theta \phi(x_1) \\ x_2' &= u \end{aligned} \quad (12)$$

Where  $\theta$  is a known constant parameter and  $x_2$  as the first control input. Denote  $\theta_0$  as the estimated value for the parameter  $\theta$  and the estimation error  $\theta_e$  is given by

$$\theta_e = \theta - \theta_0 \quad (13)$$

Next the candidate Lyapunov function is selected as

$$v_2(x, \theta_e) = \frac{1}{2} x_1^2 + \frac{1}{2\gamma} \theta_e^2 \quad (14)$$

Where  $\gamma$  is the adaptation gain. With the control law.

$$x_2 = -k_1 - \theta \phi(x_1) = \alpha_1(x_1, \theta) \quad (15)$$

And the adaptation law

$$\dot{\theta}_0 = \gamma \phi(x_1) x_1 \quad (16)$$

The derivative of the candidate Lyapunov function becomes negative definite and is given by

$$\dot{v}_1 = -k_1 x_1^2 < 0 \quad (17)$$

In the eqn. (15)  $\alpha_1$  is called a stabilizing function for  $x_2$

The deviation of  $x_2$  from the stabilizing function is given by

$$Z = x_2 - \alpha_1(x_1, \theta) \quad (18)$$

Augmenting the Lyapunov function by adding the error Variable

$$v_2(x, Z, \theta_e) = v_1(x_1, \theta_e) + \frac{1}{2} Z^2 \quad (19)$$

By the proper selection of 'u,' the overall Lyapunov function '  $V_2$ ,' becomes negative definite which implies that as  $x_1$  tends to zero, then z also tends to zero asymptotically.

#### 4. Adaptive Backstepping Controller Design for Lateral Dynamics

The side slip angle  $\beta$  and the yaw rate r are used to completely define the lateral dynamics of the system. The equations of motion for the lateral dynamics are as follows

$$\begin{aligned} \dot{\beta} &= \frac{Y_{\beta}}{V_T} \sin \beta - x_2 \\ \dot{r} &= \frac{1}{I_z} [N_{\beta} \sin \beta + N_{\delta r} \sin \delta r] \end{aligned} \quad (20)$$

The state variables are selected as  $x_1 = \beta, x_2 = r$ . the control variable is selected as  $u = \delta_r$ . The system equation can now be expressed as

$$\begin{aligned} \dot{x}_1 &= \frac{Y_{\beta}}{V_T} \sin x_1 - x_2 \\ \dot{x}_2 &= \frac{1}{I_z} [N_{\beta} \sin x_1 + N_{\delta r} \sin u] \end{aligned} \quad (21)$$

In the case of actuator stuck

$$U = 2\delta r \tag{22}$$

The system dynamics now becomes

$$\begin{aligned} \dot{x}_1 &= \frac{Y\beta}{V_t} \sin x_1 - x_2 \\ \dot{x}_2 &= \frac{1}{I_z} [N_\beta \sin x_1 + N_{\delta r} \sin 2u] \end{aligned} \tag{23}$$

The control law is to be designed such that the system stabilizes for whatever be the initial conditions. For applying the Adaptive Backstepping Control design procedure, the system can now be expressed as

$$\begin{aligned} \dot{x}_1 &= \Phi_1 \sin x_1 - x_2 \\ \dot{x}_2 &= \Phi_2 \sin x_1 + \Phi_3 \sin 2u \end{aligned} \tag{24}$$

Where  $\Phi_1, \Phi_2, \Phi_3$  are the unknown parameters in the system.

The first error variable is defined as

$$e = x_1 - \theta_{sp} \tag{25}$$

Where  $\theta_{sp}$  is the desired set point using the lyapunov function

$$V_1 = \frac{1}{2} e^2 \tag{26}$$

And using the derivative of the Lyapunov function, the virtual control law can be formulated as

$$x_{2des} = \Phi_1 \sin x_1 + k_1 e - \theta_{sp}' \tag{27}$$

Where  $k_1 > 0$  and is a design parameter which guarantees  $\dot{V}_1 < 0$ . The second error variable  $\zeta$  is defined as

$$\zeta = x_2 - x_{2des} \tag{28}$$

By augmenting the Lyapunov function  $V_1$  with the error variable  $\zeta$  and the unknown parameters in the system, we get

$$V_2 = \frac{1}{2} e^2 + \frac{1}{2} \zeta^2 + \frac{1}{2\gamma_1} \Phi_{1e}^2 + \frac{1}{2\gamma_2} \Phi_{2e}^2 + \frac{1}{2\gamma_3} \Phi_{3e}^2 \tag{29}$$

Where  $\Phi_{1e}, \Phi_{2e}, \Phi_{3e}$  are the parameter estimation errors of  $\Phi_1, \Phi_2, \Phi_3$  where  $\Phi_{*e} = \Phi - \Phi_{*0}$  and \* stands for 1, 2, 3. The variables  $\Phi_{10}, \Phi_{20}, \Phi_{30}$  are the parameter estimates with  $\gamma_1, \gamma_2, \gamma_3$  are the adaptation gain constants. With the control law

$$u_{des} = \frac{1}{2\Phi_{30}} \sin^{-1}(-k_2 \zeta + \theta_{sp}'' + k_1 \dot{e} + \Phi_{10} \cos x_1 \sin x_1) \tag{30}$$

And the parameter update laws given by

$$\begin{aligned} \dot{\Phi}_{10} &= -\gamma_1 \zeta \cos x_1 \\ \dot{\Phi}_{20} &= -\gamma_2 \zeta \sin x_1 \\ \dot{\Phi}_{30} &= \gamma_3 \zeta \sin 2u \end{aligned} \tag{31}$$

The derivative of the augmented Lyapunov function becomes negative definite

$$\dot{V}_2 = -k_1 e^2 - k_2 \zeta^2 \leq 0 \tag{32}$$

Where  $k_1 > 0; k_2 > 0$ . Therefore by Laselles theorem, the system is globally asymptotically stable at the equilibrium point of the system.

## 5. Simulation Results and Discussion

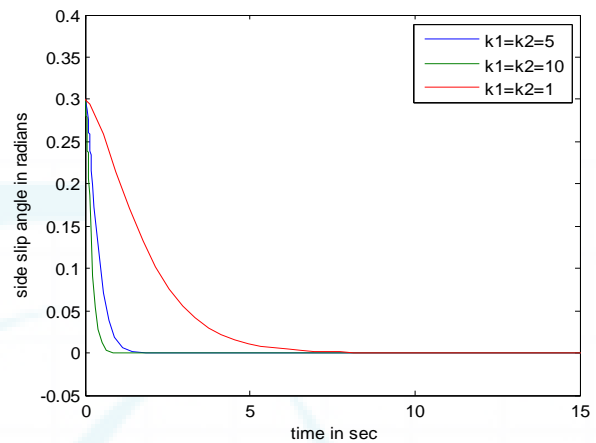


Figure 2: Side slip angle vs time

Fig. 2 shows the variation of side slip angle with time in seconds for an initial value of 17.18 degrees and different values of  $k_1$  and  $k_2$ . Fig. 3 shows the variation of yaw rate with respect to  $k_1$  and  $k_2$ .

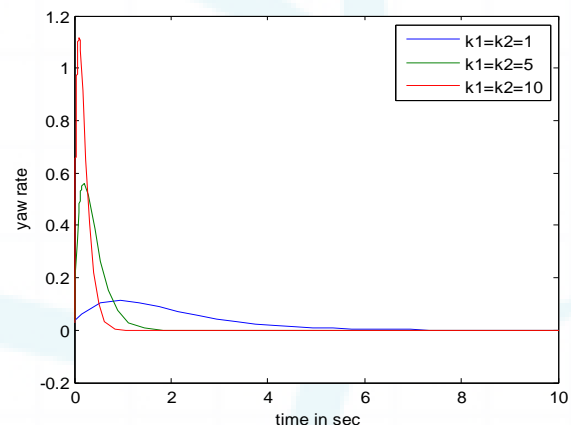


Figure 3: Yaw rate vs time

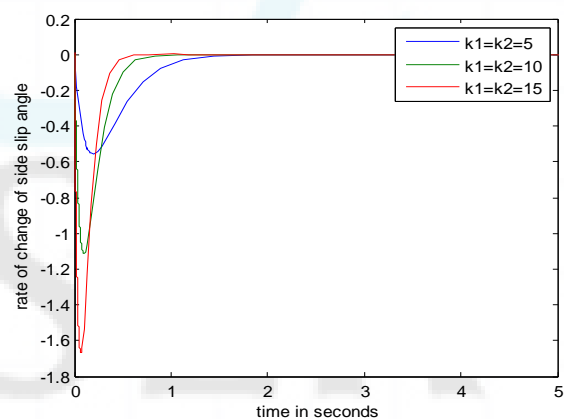
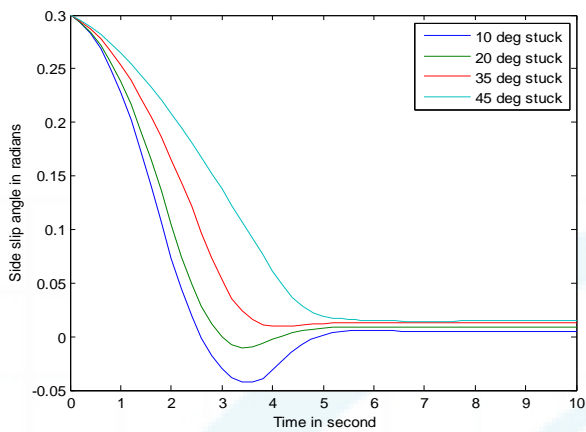


Figure 4: Rate of change of Side slip angle



**Figure 5:** Actuator stuck error tolerance for lateral motion

Fig. 4 shows the variation of rate of change of side slip angle for different values of  $k_1$  and  $k_2$ . Fig. 5 shows the fault tolerance of RLV for lateral motion at different values of actuator stuck. It is inferred that the system tolerates the fault within 5% upto an actuator stuck upto 45 degree.

## 6. Conclusion

In this paper a nonlinear adaptive control has been implemented on Reusable Launch Vehicle. Apart from the Backstepping design procedure in which only non-linearities had been taken care of, in the Adaptive Backstepping design uncertainties associated with the constant parameters of the system is also dealt with. Simulation results shows that the proposed controller compensates for fault due to actuator stuck within acceptable tolerance level. The relatively large estimation time and over parameterization are the two disadvantage of this control scheme.

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## Author Profile

**Mohammed Junaid R** was born in Kerala, India in 16/01/1992. He received B.Tech degree in Electrical and Electronics Engineering from TKM College of Engineering, Karicode, Kollam. in 2013. Currently, he is pursuing his M Tech degree in Industrial Instrumentation & Control from TKM College of Engineering, Karicode, Kollam. His research interests include Control Systems.