$$\left[\frac{0}{49},\frac{1}{49}\right]\cup\left[\frac{2}{49},\frac{3}{49}\right]\cup\left[\frac{4}{49},\frac{5}{49}\right]\cup\left[\frac{6}{49},\frac{7}{49}\right]\cup\left[\frac{14}{49},\frac{15}{49}\right]\cup\left[\frac{16}{49},\frac{17}{49}\right]\cup\left[\frac{18}{49},\frac{19}{49}\right]\cup\left[\frac{20}{49},\frac{21}{49}\right]\cup\left[\frac{28}{49},\frac{29}{49}\right]\cup\left[\frac{30}{49},\frac{31}{49}\right]\cup\left[\frac{32}{49},\frac{33}{49}\right]\cup\left[\frac{34}{49},\frac{35}{49}\right]\cup\left[\frac{42}{49},\frac{43}{49}\right]\cup\left[\frac{44}{49},\frac{45}{49}\right]\cup\left[\frac{46}{49},\frac{47}{49}\right]\cup\left[\frac{48}{49},\frac{49}{49}\right]$$

Repeating this process, we get the Cantor middle $3/7$ set.

4.1 Theorem:

Extraction of Cantor middle $\frac{3}{7}$ set from Farey sequence.

Proof:

Consider the interval $\left[\frac{k}{7^{n-1}},\frac{k+1}{7^{n-1}}\right]$, where n denotes the iteration.

Let $n=1$,

$$\left[\frac{7^{n-1}k}{7^{n-1}},\frac{7^{n-1}(k+1)}{7^{n-1}}\right]=[0,1]$$

In this iteration, Farey and Cantor are same intervals.

For $n=2$, the interval $\left[\frac{7^{n-1}k}{7^{n-1}},\frac{7^{n-1}(k+1)}{7^{n-1}}\right]$ can be written as

$$\left[\frac{7k}{7},\frac{7(k+1)}{7}\right]=\left\{\left[\frac{7k}{7},\frac{7k+1}{7}\right]\cup\left[\frac{7k+2}{7},\frac{7k+3}{7}\right]\cup\left[\frac{7k+4}{7},\frac{7k+5}{7}\right]\cup\left[\frac{7k+6}{7},\frac{7k+7}{7}\right]\right\}$$

The boundary points are non-reducible Farey fractions. The intervals $\left(\frac{7k+1}{7},\frac{7k+2}{7}\right),\left(\frac{7k+3}{7},\frac{7k+4}{7}\right),\left(\frac{7k+5}{7},\frac{7k+6}{7}\right)$ are removed from $\left[\frac{7k}{7},\frac{7(k+1)}{7}\right]$ to get Cantor set interval.

For $n\geq 3$, the successive removable fractions are identified from the set

$$\varphi_n=\left\{\left(\frac{7^{n-1}}{7^n},\frac{2.7^{n-1}}{7^n}\right),\left(\frac{3.7^{n-1}}{7^n},\frac{4.7^{n-1}}{7^n}\right),\left(\frac{5.7^{n-1}}{7^n},\frac{6.7^{n-1}}{7^n}\right)\right\}$$

Each closed interval in the $(n-1)$th iteration is multiplied and divided by 7 and written as union of four closed intervals as below. The partitioned intervals for $n=3$ is given below for illustration

$$\left[\frac{7^2k}{7^{n-1}},\frac{7^2k+7}{7^{n-1}}\right]=\left\{\left[\frac{7^2k}{7^{n-1}},\frac{7^2k+1}{7^{n-1}}\right]\cup\left[\frac{7^2k+2}{7^{n-1}},\frac{7^2k+3}{7^{n-1}}\right]\cup\left[\frac{7^2k+4}{7^{n-1}},\frac{7^2k+5}{7^{n-1}}\right]\cup\left[\frac{7^2k+6}{7^{n-1}},\frac{7^2k+7}{7^{n-1}}\right]\right\}$$

$$\left[\frac{7^2k+14}{7^{n-1}},\frac{7^2k+21}{7^{n-1}}\right]=\left\{\left[\frac{7^2k+14}{7^{n-1}},\frac{7^2k+15}{7^{n-1}}\right]\cup\left[\frac{7^2k+16}{7^{n-1}},\frac{7^2k+17}{7^{n-1}}\right]\cup\left[\frac{7^2k+18}{7^{n-1}},\frac{7^2k+19}{7^{n-1}}\right]\cup\left[\frac{7^2k+20}{7^{n-1}},\frac{7^2k+21}{7^{n-1}}\right]\right\}$$

$$\left[\frac{7^2k+28}{7^{n-1}},\frac{7^2k+35}{7^{n-1}}\right]=\left\{\left[\frac{7^2k+28}{7^{n-1}},\frac{7^2k+29}{7^{n-1}}\right]\cup\left[\frac{7^2k+30}{7^{n-1}},\frac{7^2k+31}{7^{n-1}}\right]\cup\left[\frac{7^2k+32}{7^{n-1}},\frac{7^2k+33}{7^{n-1}}\right]\cup\left[\frac{7^2k+34}{7^{n-1}},\frac{7^2k+35}{7^{n-1}}\right]\right\}$$

$$\left[\frac{7^2k+42}{7^{n-1}},\frac{7^2k+49}{7^{n-1}}\right]=\left\{\left[\frac{7^2k+42}{7^{n-1}},\frac{7^2k+43}{7^{n-1}}\right]\cup\left[\frac{7^2k+44}{7^{n-1}},\frac{7^2k+45}{7^{n-1}}\right]\cup\left[\frac{7^2k+46}{7^{n-1}},\frac{7^2k+47}{7^{n-1}}\right]\cup\left[\frac{7^2k+48}{7^{n-1}},\frac{7^2k+49}{7^{n-1}}\right]\right\}$$

The remaining removable sets of intervals ψ_n, $(n=3)$ are depicted as follows.

$$\left(\frac{7^2k+1}{7^{n-1}},\frac{7^2k+2}{7^{n-1}}\right),\left(\frac{7^2k+3}{7^{n-1}},\frac{7^2k+4}{7^{n-1}}\right),\left(\frac{7^2k+5}{7^{n-1}},\frac{7^2k+6}{7^{n-1}}\right),$$
$$\left(\frac{7^2k+15}{7^{n-1}},\frac{7^2k+16}{7^{n-1}}\right),\left(\frac{7^2k+17}{7^{n-1}},\frac{7^2k+18}{7^{n-1}}\right),\left(\frac{7^2k+19}{7^{n-1}},\frac{7^2k+20}{7^{n-1}}\right),$$
$$\left(\frac{7^2k+29}{7^{n-1}},\frac{7^2k+30}{7^{n-1}}\right),\left(\frac{7^2k+31}{7^{n-1}},\frac{7^2k+32}{7^{n-1}}\right),\left(\frac{7^2k+33}{7^{n-1}},\frac{7^2k+34}{7^{n-1}}\right),$$
$$\left(\frac{7^2k+43}{7^{n-1}},\frac{7^2k+44}{7^{n-1}}\right),\left(\frac{7^2k+45}{7^{n-1}},\frac{7^2k+46}{7^{n-1}}\right),\left(\frac{7^2k+47}{7^{n-1}},\frac{7^2k+48}{7^{n-1}}\right).$$

The removal of $\varphi_n\cup\psi_n$ from $\widetilde{F_N}$ will result in Cantor Middle $\frac{3}{7}$ set

5. Conclusion

It is observed that the identifications of removable sets for Cantor middle $\frac{2}{5}$, $\frac{3}{7}$ sets from $\widetilde{F_{5^n}}$ and $\widetilde{F_{7^n}}$ differs slightly in approach the other Cantor middle sets from Farey fractions can be studied and if possible we generalized.

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