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The Uniqueness of Equilibria of a Complex Recurrent Neural Network

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Abstract: ***Here we have presented signal propagation delays in complex recurrent neural network. The type of connection possible is generally referred to as the architecture of the neural network. Whenever the neural network makes a mistake, some weigh and some thresholds have to be changed to companion for this error.***

Keywords: recurrent neural network, signal propagation delays, biological system, the binary stream, short term memory (STM) equation

1. Introduction

The amount of time it takes for the head of the single to travel from the sender to the receiver it. i.e. signal propagation delays are being represent everywhere in biological system. When timing falling behind is of the property of relative size or extends as the typical time scale of a particular process.

Many of the current leaders in the field began to publish their work during the 1970's. The early work of Tevuo Kohonen [7] dealt with associative memory nets. Grossberg [8] work which was very mathematical and very biological is widely known. Reilly and cooper [10] used a "Hybrid networks" with multiple layers. Each layer using a different problem solving strategy.

Also in 1982, there was a joint USA. Japan announced a new fifth generation effort on conference on neural networks. In 1985, parker's [12] work come to the attention on the parallel distributed processing group led by psychologists David Rumel Hart of California at San Diego and James Mc Clell and of Carnerger, refine and publish it.

The binary stream was initiated by the classical Mc Culloch and Pitts [1] model of thresholds logic systems that describe how the activities or the short term memory (STM) traces. Caianioello [2] used a binary STM equation that is influenced by activities at multiple times in the past.

Rosenblatt [3] used on STM equation that evolves in continuous time, whose activities can spontaneously de cay. Widrow [4] drew inspiration from the brain to introduce the gradient descent Adeline adaptive pattern recognition pattern. Anderson [5] initially described his institutions about neural pattern recognition using a spatial cross correlation function. Kohonen [6] made a transition from linear algebra concepts such as the moore-pen rose pseudo inverse to more biological motivated studies that are summarized in his book Kohonen, [9], [11].

Here we take this system

$$z'(t) = -z(t) + g(z(t-\tau)) \quad (1)$$

Here we see that this system described by the set of equation (1) may have more than one equilibria but there are conditions for which such a system will have a single equilibrium point. Indeed, system (1) can be written as

$$z'(t) = -Ez(t) + Ag\big(z(t-\tau_*)\big) \quad (2)$$

Where E is the *nxn* identity matrix, $z = (z_1, z_2, \dots, z_n)^T$

$$g[z(t-\tau_*)] = \big[g_1\big(z_1(t-\tau_1)\big), g_2\big(z_2(t-\tau_*)\big), \dots, g_n\big(z_n(t-\tau_n)\big)\big]^T$$

and

$$A = (a_{ij})_{nXn} = \begin{bmatrix} 0\ 1\ 0 \ \dots\dots\dots\dots 0\ 0 \\ 0\ 0\ 1 \ \dots\dots\dots\dots 0\ 0 \\ \dots\dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots \\ \omega_1\ \omega_2\ \omega_3\ \dots\dots \omega_{n-1}\ 0 \end{bmatrix}$$

Definition (1): Denote B the set of all solutions of system (1), the norm

$\| z \| = sup_{t \in R}$ $|z(t)|$, then $(B, \|.\|)$ construct a Banach space.

Lemma (1): The solution of system (1) is globally bounded.

Proof: Since the activation functions $z_i(t-\tau_i)$ are bounded. Then in system (1) there exists M_i (i = 1,2,…. , n) such that $z_i(t) \le M_i$ (i = 1,2,…. , n). Let $M = \max\{M_1, M_2, \dots, M_n\}$ then $|z_i(t)| \le M$. Hence z (t) is globally bounded.

Lemma (2): Suppose that $\omega_i \neq 0$ and there exists a set of $\eta_i \in (0,1]$, i = (1,2,……, n). Such that $0 < \omega_1\eta_1 < \omega_2\eta_2 < \cdots < \omega_n\eta_n < 1$ holds. Then there is a unique equilibrium point for that system (1).

Proof: An equilibrium point z* is a constant solution of system (1) i.e. it is satisfies that the time delays do not affect the number of equlibria. Indeed if a set k = = {z: z ϵ B and z satisfies -Ez + Ag (z) = 0}. Then k is a closed compact subset of B. For any z ϵ k., define a map

$H(z) = (H_1(z), H_2(z), \dots\dots H_n(z))^T = -Ez + Ag(z)$

$H_i(z) = -z_{ai} + z_{\beta i}(z_{i+1})\ ,\ (i = 1,2, \dots.. (n-1))$

$H_n(z) = -z_n + \omega_1 g_1(z_1) + \omega_2 g_2(z_2) + \cdots + \omega_{n-1} g_{n-1}(z_{n-1})$

We will so that H is a contradiction mapping on k, system (1) has a unique equilibrium point for any z ϵ k,

Since $-Ez + Ag(z) = 0$

Then,

$z_1 = g_1(z_2)$

$z_2 = g_2(z_3)$

$z_3 = g_3(z_4)$

....

....

$z_{n-1} = g_{n-1}(z_n)$

$z_n = \omega_1 g_1(z_1) + \omega_2 g_2(z_2) + \cdots + \omega_{n-1} g_{n-1}(z_{n-1})$ (3)

For any $z_\alpha, z_\beta \in k$ with $z_\alpha \neq z_\beta$,

We get

$H(z_\alpha) - H(z_\beta) = -E[z_\alpha - z_\beta] + A[g(z_\alpha) - g(z_\beta)]$ (4)

Or

$H_1(z_\alpha) - H_1(z_\beta) = -[z_{\alpha 1} - z_{\beta 1}] + [g(z_{\alpha 2}) - g(z_{\beta 2})]$

$H_2(z_\alpha) - H_2(z_\beta) = -[z_{\alpha 2} - z_{\beta 2}] + [g(z_{\alpha 3}) - g(z_{\beta 3})]$

....

....

$H_{n-1}(z_\alpha) - H_{n-1}(z_\beta) = -[z_{\alpha(n-1)} - z_{\beta(n-1)}] + [g(z_{\alpha n}) - g(z_{\beta n})]$

$H_n(z_\alpha) - H_n(z_\beta) = -[z_{\alpha n} - z_{\beta n}] + \omega_1[g(z_{\alpha 1}) - g(z_{\beta 1})] + \cdots + \omega_{n-1}[g(z_{\alpha(n-1)}) - g(z_{\beta(n-1)})]$ (5)

Note that *g (z)* is a monotonically increasing function $|g(z)| \leq 1$ and its derivative w.r.t. "z" Satisfies

$\frac{d}{dz} g(z) = g'(z)$,

then $g'(0) = 1$ and $g(z) < 1 \quad (z \neq 0)$

Let $g_k(z)$ denote the k[th] iteration of $g(z)$

i.e.

$g_k(z) = g\left(g(g \dots\dots g(z))\right)$

Especially,

$g_1(z) = g(z)$

$g_2(z) = g(g(z))$

$g_3(z) = g\left(g(g(z))\right)$

Because,

$g'_{k+1} = g'_k(z)[1 - g'_{k+1}(z)]$

$= g'_k(z)[1 - g^2_{k+1}(z)][1 - g^2_k(z)] \dots [1 - g^2(z)] \leq 1$

So we have,

$$g_{k+1}(z) - g_k(z) \leq 1 \qquad (6)$$

For example,

$g_2(z) = g(g(z)) < g(z) = g_1(z)$

It can be shown that for every integer *k*>1, $g_k(z)$ is odd, strictly increasing and bounded from (4.3), we get

$$-[z_{\alpha n} - z_{\beta n}] + \omega_1[g(z_{\alpha 1}) - g(z_{\beta 1})] + \omega_2[g(z_{\alpha 2}) - g(z_{\beta 2})] + \cdots + \omega_{n-1}[g(z_{\alpha(n-1)}) - g(z_{1\beta(n-1)})] = -[z_{\alpha n} - z_{\beta n}] + \omega_1[g_n(z_{\alpha n}) - g_n(z_{\beta n})] + \omega_2[g_{n-1}(z_{\alpha n}) - g_{n-1}(z_{\beta n})] + \dots + \omega_{n-1}[g(z_{\alpha n}) - g(z_{\beta n})] \qquad (7)$$

From the mean value theorem, we have

$g(z_\alpha) - g(z_\beta) = g'(\mu)(z_\alpha - z_\beta)$,

where lies between z_α and z_β , from (4.7) there exists $\mu_i, (i = 1,2, \dots, n)$ such that

$-[z_{\alpha n} - z_{\beta n}] + \omega_1 [g_n(z_{\alpha n}) - g_n(z_{\beta n})] + \omega_2 [g_{n-1}(z_{\alpha n}) - g_{n-1}(z_{\beta n})] + \dots\dots.. + \omega_{n-1} [g(z_{\alpha n}) - g(z_{\beta n})]$

$= -[z_{\alpha n} - z_{\beta n}] + \omega_1 g'_n(\mu_1)[z_{\alpha n} - z_{\beta n}] + \omega_2 g'_{n-1} g(\mu_2)[z_{\alpha n} - z_{\beta n}] + \dots\dots.. + \omega_{n-1} g'_2 \mu_{n-1} [z_{\alpha n} - z_{\beta n}]$

$= -[z_{\alpha n} - z_{\beta n}] + \omega_1 \eta_1 [z_{\alpha n} - z_{\beta n}] + \omega_2 \eta_2 [z_{\alpha n} - z_{\beta n}] + \dots + \omega_{n-1} \eta_{n-1} [z_{\alpha n} - z_{\beta n}]$

$= -[z_{\alpha n} - z_{\beta n}] + (\omega_1 \eta_1 + \omega_2 \eta_2 + \cdots + \omega_{n-1} \eta_{n-1})[z_{\alpha n} - z_{\beta n}]$ (8)

Where $\eta_i g_{n+1-i}(\mu_i) \leq 1$ (i=1,2,......,n-1)

So from $0 \leq \omega_1 \eta_1 + \omega_2 \eta_2 + \cdots + \omega_{n-1} \eta_{n-1} < 1$

$\exists\ a \emptyset_n\ (0 < \emptyset < 1)$ Such that

$\| H_n(z_\alpha) - H_n(z_\alpha) \| =$

$\| -[z_\alpha - z_\beta] + \omega_1 [g(z_{\alpha 1}) - g(z_{\beta 1})] + \cdots + \omega_{n-1}[g(z_{\alpha(n-1)}) - g(z_{\beta(n-1)})] \|$

$= \| -[z_{\alpha n} - z_{\beta n}] + ((\omega_1 \eta_1 + \omega_2 \eta_2 + \cdots + \omega_{n-1} \eta_{n-1})[z_{\alpha n} - z_{\beta n}]$

$\leq \emptyset_n \parallel [\ z_{\alpha n} - z_{\beta n}] \parallel$

$\leq \emptyset_n \parallel [z_{\alpha n} - z_{\beta n}] \parallel$ (9)

This implies that H_n *(z)* is a contraction mapping on k, similarly we have

$g(z_{\alpha n}) - g(z_{\alpha n}) = g'(\xi)[z_{\alpha n} - z_{\beta n}]$

$= g'(\xi)\{\omega_1[g(z_{\alpha 1}) - g(z_{\beta 1})] + \omega_2[g(z_{\alpha 2}) - g(z_{\beta 2})] + \cdots$

$+ \omega_{n-1}[g(z_{\alpha(n-1)}) - g(z_{\beta(n-1)})]\}$

$= g'(\xi)\{\omega_1 g'_{n-1}(\mu'_1)[z_{\alpha(n-1)} - z_{\beta(n-1)}]$

$+ \omega_2 g'_{n-2}(\mu'_2)[z_{\alpha(n-1)} - z_{\beta(n-1)}] + \cdots$

$+ \omega_{n-1} g'_1(\mu'_{n-1})[z_{\alpha(n-1)} - z_{\beta(n-1)}]\}$

$= g'(\xi)\{\omega_1 \eta'_1 + \omega_2 \eta'_2 + \cdots$

$+ \omega_{n-1}\eta'_{n-1}\}\left[[z_{\alpha(n-1)} - z_{\beta(n-1)}]\right]$ (10)

Where $\eta'_i = g'_{n-1}(\mu'_i) \leq 1 \ (i = 1,2, \dots \dots n-1)$

So far

$H_n(z_\alpha) - H_n(z_\beta) = -[z_{\alpha(n-1)} - z_{\beta(n-1)}] + g(z_{\alpha n}) - g(z_{\beta n})$

There exists a θ_{n-1} $(0 < \theta_{n-1} < 1)$ such that

$\parallel H_n(z_\alpha) - H_n(z_\alpha) \parallel =$

$\parallel -[z_{\alpha(n-1)} - z_{\beta(n-1)}] + g'(\xi)\{(\omega_1\eta'_1 + \omega_2\eta'_2 + \cdots + \omega_{n-1}\eta'_{n-1})[z_{\alpha(n-1)} - z_{\beta(n-1)}]\} \parallel$

$\leq \theta_{n-1} \parallel z_{\alpha(n-1)} - z_{\beta(n-1)} \parallel$ (11)

This implies that $H_n(z)$ is also a contraction mapping on k. Similarly one can show that H_n (i = 1,2,… …….., n-2) are contraction mapping on k. Therefore H is a contraction mapping on k from the contraction mapping principle there is a unique eqillibrium point for system (4.1). Note that g (0)=0, so the unique equilibrium point is exactly at (0, 0, ……..,0)^T

Lemma: Suppose that ω_i and there exists a set of $\eta_i > 1, (i = 1,2, \dots, n-1)$ satisfying $\eta_1 > \eta_2 > \cdots > \eta_{n-1}$. Such that

$\frac{\eta_2}{\eta_1}\omega_1 + \frac{\eta_3}{\eta_2}\omega_2 + \frac{\eta_4}{\eta_3}\omega_3 + \cdots + \frac{\eta_{n-1}}{\eta_{n-2}}\omega_{n-2} + \frac{\eta_1}{\eta_{n-1}} | \omega_{n-1} | < 1$

holds. Then system (1) has a unique equilibrium point.

Proof: Let (u, v) represents the inner product of the vectors $u, v \in k$

$sgn(u) = (sgn(u_1), sgn(u_2), \dots \dots, sgn(u_n))^T$, for real u, Sgn (u) defined by,

$$sgn(u) = \begin{cases} 1 & if \quad u > 0 \\ 0 & if \quad u = 0 \\ -1 & if \quad u < 0 \end{cases}$$

Then for $z_\alpha, z_\beta \in k$ and $z_\alpha \neq z_\beta$ $(i = 1,2, \dots, n)$,

We have,

$(g(z_{\alpha i}) - g(z_{\beta i}))\, sgn\,(z_{\alpha i} - z_{\beta i})$

$= | g(z_{\alpha i}) - g(z_{\beta i}) |$

$= g'(\varsigma_i) | z_{\alpha i} - z_{\beta i} |$

Where $0 < g'(\varsigma_i) < 1$

Where $0 < g'(\varsigma_i) < 1$. Since $\eta_1 < \eta_1 < \cdots < \eta_{n-1}$,

One can let

$g'(\varsigma_i) = \frac{\eta_{i+1}}{\eta_i} \quad (i = 1,2, \dots, n-2)$

Noting that $g'(\varsigma_{n-1})\omega_{n-1} < \frac{\eta_i}{\eta_{n-1}} |\omega_{n-1}|$.

Then

$\sum_{i=1}^{n-1} g'(\varsigma_i)\,\omega_i < \frac{\eta_2}{\eta_1}\omega_1 + \frac{\eta_3}{\eta_2}\omega_2 + \cdots + \frac{\eta_{n-1}}{\eta_{n-2}}\omega_{n-2} + \frac{\eta_1}{\eta_{n-1}} | \omega_{n-1} | < 1$

Therefore there exists $0 < \varepsilon < 1$ such that

$(H(z_\alpha) - H(z_\beta) sgn\,(z_\alpha - z_\beta))$

$= -\sum_{i=1}^{n}(z_{\alpha i} - z_{\beta i}) + \sum_{i=2}^{n-1}(g(z_{\alpha i}) - g(z_{\beta i})) + \sum_{i=1}^{n-1}\omega_i[g(z_{\alpha i}) - g(z_{\beta i})]\, sgn(z_\alpha - z_\beta)$

$= -\sum_{i=1}^{n} | (z_{\alpha i} - z_{\beta i}) | + \sum_{i=2}^{n-1} | (g(z_{\alpha i}) - g(z_{\beta i})) | \sum_{i=1}^{n-1}\omega_i[g(z_{\alpha i}) - g(z_{\beta i})]$

$\leq -\sum_{i=1}^{n} | (z_{\alpha i} - z_{\beta i}) | + \sum_{i=1}^{n} g'(\varsigma_i) | (z_{\alpha i} - z_{\beta i}) | + \sum_{i=1}^{n}\omega_i g'(\varsigma_i) | (z_{\alpha i} - z_{\beta i}) |$ (12)

$\leq \varepsilon \parallel z_{\alpha i} - z_{\beta i} \parallel$

This implies that $H(z_\alpha) \neq H(z_\beta)$ Whenever $z_\alpha \neq z_\beta$. So H is one to one and (0, 0, 0, ….0)^T is a unique equilibrium point.

2. Conclusion

Here we have investigated a Complex Recurrent Neural Network with time delays. By using fixed point theorem we have shown that the Complex Neural Network has a unique equilibrium point.

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