International Journal of Scientific Engineering and Research (IJSER)
ISSN (Online): 2347-3878
Index Copernicus Value (2015): 62.86 | Impact Factor (2015): 3.791

Prime Difference Speed Sequence Graceful Graphs

S. Uma Maheswari[1], K. Indirani[2]

[1]Department of Mathematics, CMS College of Science & Commerce, Coimbatore, India

[2]Head & Associate Professor, Dept. of Maths, Nirmala College for Women, Coimbatore, India

Abstract: ***The idea of prime difference speed sequence graphs are imposed on Fibonacci sequence. It is tested on various graphs like path, cycles, K_m, $K_{1,n}$, gear graph, $D(T_n)$, fan graph, star graph, wheel graph etc. and the graceful labeling of those graphs are obtained. The strongly prime difference speed sequence graphs are also obtained.***

Keywords: graceful labeling, difference speed sequence labeling, speed sequence graphs, prime difference speed sequence graphs, etc.

AMS Subject Classification: 05C78

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected graph G(V, E) with 'p' vertices and 'q' edges. A detailed survey of graph labeling can be found in the dynamic survey of labeling by J.A. Gallian. In this paper we introduce a new labeling called prime difference speed sequence labeling.

Definition 1.1
Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f: V(G) \rightarrow \{1, 2, \dots, p\}$ is called a prime labeling if for each edge $e = uv$, $gcd\{f(u), f(v)\} = 1$.A graph which admits a prime labeling is called a Prime graph.

Definition 1.2
In a graph G with q edges, if f is an injection from the vertices of G to the set $\{0, 1, 2,\dots,F_q\}$, where F_q is the q^{th} Fibonacci number of the Fibonacci sequence $F_1 = 1$, $F_2 = 2$, $F_3 = 3$, $F_4 = 5$,… and if each edge uv is assigned the label $|f(u)\text{-}f(v)|$, then we call the graph as a Fibonacci graceful graph.

Definition 1.3
A (p.q) graph G(V,E) is said to be a difference speed sequence graceful graph if there exists a bijection f: V(G) → $\{0, 1, 2, \dots q^2\}$ such that the induced mapping f: E(G) →$\{\Delta_i(x)\}$/ i=1, 2, 3, …n} defined by f(uv) = |f(u) – f(v) |is a bijection. Here $\Delta_m(x) = (\Delta_m x_k) = |(x_k - x_{k+m})|$ and (x) is the Fibonacci sequence. The function f is called a difference speed sequence graceful graph.

Definition 1.4
A (p, q) graph $G(V, E)$ is said to be a prime difference speed sequence graceful graph if there exists a bijection f : $V(G) \rightarrow \{\Delta_i(x) \,/\, i = 1,2,3 \dots\dots n\}$ and for each edge $e = uv, gcd\{f(u), f(v)\} = 1$.Here$\Delta_i(x) = (\Delta_i x_k) = |x_{k+i} - x_k|$ and (x_k) is any sequence. A graph which admits Prime difference speed sequence labeling is called a Prime difference speed sequence graph.

Definition 1.5
Let u and v be a two distinct vertices of graph G. A new graph $G_{u,v}$ is constructed by identifying (fusing) two vertices u and v by a single new vertex x such that every edge which was incident with either u or v in G is now incident with $x$in $G_{u,v}$.

Definition 1.6
A Gear graph is a graph obtained from Wheel graph, with a vertex added between each pair of adjacent vertices of an outer cycle.

Definition 1.7
A triangular snake T_n is obtained from a path $u_1, u_2, \dots\dots\dots\dots, u_n$by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n - 1$

Definition 1.8
A Double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path.

2. Prime Difference Sequence Labeling of Some Graphs by Fibonacci Numbers

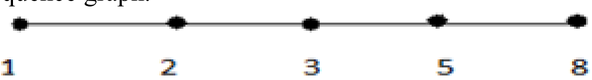
Theorem 2.1
The Path P_n is a Prime difference speed sequence graph

Proof:
Let $V(P_n) = \{u_1, u_2, \dots\dots\dots\dots, u_n\}$where $u_i = (\Delta_i x_k)$ where (x_k) is a Fibonacci sequence

When we label the vertices for the path using difference speed sequence, we see that the consecutive adjacent vertices having labels in such a manner that $gcd\{f(u), f(v)\} = 1$.

Here $f(u_1) = (x_1 - x_3), f(u_2) = (x_2 - x_4), f(u_3) = (x_3 - x_5), \dots, f(u_i) = (x_i - x_{i+2})$, such that it satisfies the condition $gcd\{f(u), f(v)\} = 1$.

Therefore P_n is a Fibonacci Prime difference speed sequence graph.



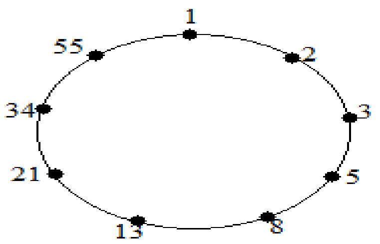
Theorem 2.2
The cycle $C_n$is a Prime difference speed sequence graph.

Proof:

Let $V(C_n) = \{u_1, u_2, \ldots\ldots\ldots\ldots, u_n\}$

The same labeling pattern is followed as in the path for both the cases (i) when n is odd and (ii) when n is even.
Let $u_1, u_2, \ldots\ldots\ldots\ldots, u_n$ be the n vertices.

The vertices are labeled such that $f(u_1) = (x_1 - x_3), f(u_2) = (x_2 - x_4), f(u_3) = (x_3 - x_5), \ldots, f(u_i) = (x_i - x_{i+2})$, such that it satisfies $gcd\{f(u), f(v)\} = 1$.



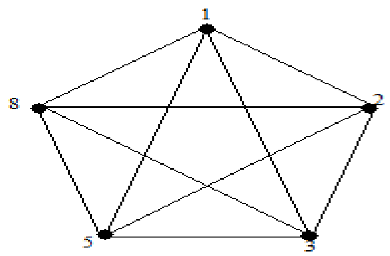
Theorem 2.3
The complete graph K_n is not a Prime difference speed sequence graph for n>4

Proof:
Let $V(K_n) = \{u_1, u_2, \ldots\ldots\ldots\ldots, u_n\}$

$$|E(G)| = \frac{n(n-1)}{2}$$

When we assign labels the condition $gcd\{f(u), f(v)\} = 1$ is satisfied for $n = 1 \; to \; 4$

When n>4, for atleast any one of the edges does not satisfy the required result. Therefore K_n, n>4 is not a prime difference speed sequence graph.



Theorem 2.4
The graph obtained by identifying any two vertices of $K_{1,n}$ is a Prime difference speed sequence graph.

Proof:

For $n = 1, 2$ the result is obvious.
When $n \geq 3$.
Let v_0 be the apex vertex and $v_1, v_2, v_3, \ldots, v_n$ be the consecutive pendant vertices of $K_{1,n}$.
By the nature of $K_{1,n}$ two of the vertices can be identified in the following two ways.

Case 1: The apex vertex v_0 is identified with any of the pendant vertices (say v_1).
Let the new vertex be u_0 and the resultant graph be G.

Then deg $(v_i) = 1$, for $i = 2, 3, \ldots, n$ and
deg $(u_0) = n + 1$ as there is a loop incident at u_0.

Define $f: V(G) \rightarrow \{\Delta_i(x)/ \, i=1, 2, 3, \ldots n\}$ as $f(v_i) = i$ for $i = 2, 3, \ldots, n$ and $f(u_0) = 1$.

Clearly f is an injection and $gcd\{f(u), f(v)\} = 1$for every pair of adjacent vertices u and v of G.

Hence G is a Prime difference speed sequence graph.

Case 2: Any two of the pendant vertices (say v_{n-1} and v_n) are identified.
Let the new vertex be u_{n-1} and the resultant graph be G.

So, in G, deg $(v_i) = 1$, for $i = 1, 2, \ldots, n$-2,
deg $(u_{n-1}) = 2$ and
deg $(v_0) = n$.

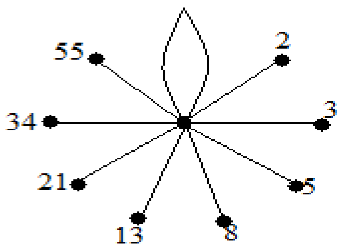
Define $f: V(G) \rightarrow \{\Delta_i(x)/ \, i=1, 2, 3, \ldots n\}$ as
$f(v_i) = i + 1$ for i = 0,1,2…,n-2 and
$f(u_{n-1}) = n$.

Obviously f is an injection and $gcd\{f(u), f(v)\} = 1$ for every pair of adjacent vertices u and v of G.

Hence G is a Prime difference speed sequence graph.

Example 2.5
The Prime difference speed sequence labeling of the graph obtained by identifying the apex vertex with a pendant vertex of $K_{1,9}$

Since the self-loop is counted twice,
The $\deg(u_0)= n(e)+2=$ number of edges incident at $u_0+2=8+2=10$

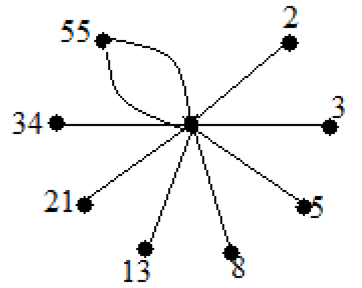


Example 2.6 The Prime difference speed sequence graph labeling obtained by identifying the apex vertex with two pendant vertices with a pendant vertex of $K_{1,9}$

Suppose we fix $f(v_i) = (x_i - x_{i+2})$, for all i=0,1,…,n.

Then the vertices $v_n$and v_{n-1} are identified.

In all the pendant vertices $\deg(v_1) = 1$ for all i=2 to n-2, $\deg(u_{n-1})=2$, $\deg(v_0)=9$.



Theorem 2.7
The graph obtained by identifying two vertices with label 1 and p of prime difference speed sequence graph is also a prime difference speed sequence graph if p is a prime and G is a prime labeling of G.

Proof:
Assume that f is a prime labeling of G.
Assign a as the label for the vertex v_a for $a = 1\ to\ p$.
Then the new vertex of the graph G' will be u_1 which is obtained by identifying v_1 and v_p of G.
We define $f_1: \{u_1, v_2, v_3, \dots, v_{p-1}\} \to \{1, 2, \dots, p-1\}$ as

$$f_1(x) = \begin{cases} f(v_a) & if\ x = v_a, a = 2, \dots, p-1 \\ 1 & \text{if } x = u_1 \end{cases}$$

Then $f_1(x) = \begin{cases} a & if\ x = v_a, a = 2, \dots, p-1 \\ 1 & \text{if } x = u_1 \end{cases}$

Clearly f is an injection.

We claim that $gcd\{f(u), f(v)\} = 1$ for any arbitrary edge $e = uv$ of G'. Then the following cases arises. To prove our claim, the following cases are to be considered.

Case 1
If $u = u_1$, then
$gcd\{f_1(u), f_1(f_1(v))\} = gcd\{f_1(u_1), f_1(v)\}$
$= \text{gcd}\{1, f_1(v)\}$
$= 1$

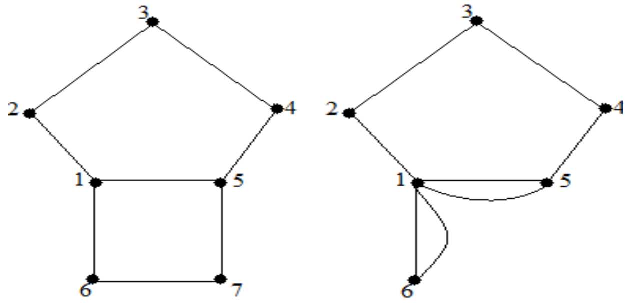
Case 2
If $u \neq u_1$, and $v = u_1$, then $\text{gcd}\{f_1(u), f_1(f_1(v))\} = \text{gcd}\{f_1(u), f_1(u_1))\}$
$= \text{gcd}\{f_1(u), 1\}$
$= 1$

Case 3
If $u \neq u_1$, and $v \neq u_1$, then $u = v_a : v = v_b$ for some $a, b = 2, 3, \dots, p-1$ with $a \neq b$
then $gcd\{f_1(u), f_1(f_1(v))\} = gcd\{f_1(v_a), f_1(v_b)\}$
$= \text{gcd}\{f(v_a), f(v_b)\}$
$= 1$
as v_a and v_b are adjacent vertices in the prime graph G.
Hence f_1 admits a prime difference speed sequence labeling for G'.

Therefore G' is a prime difference speed sequence graph.

Example 2.8

In the following figures, the prime labeling of a graph G of order 6 and the prime labeling for the graph G' obtained by identifying the vertices of G with label 1 and 7 are shown.

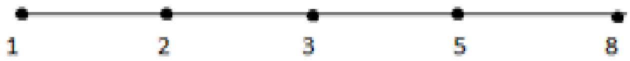


Theorem 2.9
The graph obtained by identifying any two vertices of P_n is a Prime difference speed sequence graph.

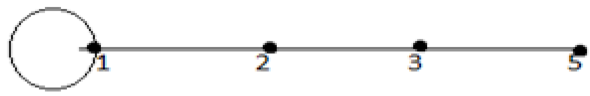
Proof:
Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of P_n.
Let u be the new vertex of a graph G obtained by identifying two distinct vertices v_a and v_b of P_n.

Then G is nothing but a cycle or loop with at most two paths attached at u. Such a graph is a Prime difference speed sequence graph.

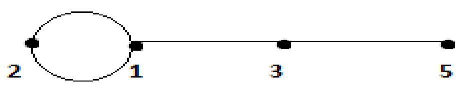
Example 2.10
The various graphs obtained by identifying any two vertices of path P_n are as follows.



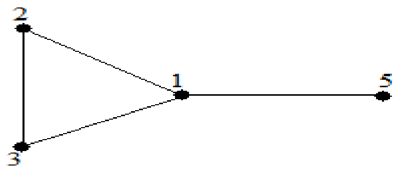
A prime difference sequence labeling of P_5



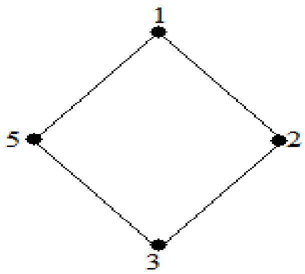
A prime difference sequence labeling of the graph obtained by identifying v_1 and v_2 of P_5



A prime difference sequence labeling of the graph obtained by identifying v_1 and v_3 of P_5



A prime difference sequence labeling of the graph obtained by identifying v_1 and v_4 of P_5



A prime difference sequence labeling of the graph obtained by identifying v_1 and v_5 of P_5

Theorem 2.11
A Gear graph G_r, $r \geq 3$ is a Prime difference speed sequence graph.

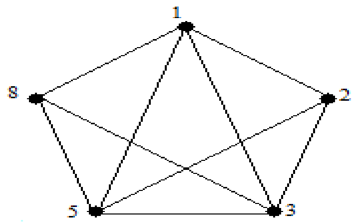
Proof:
A Gear graph G_r, $r \geq 3$ has $2r + 1$ vertices and $3r$ edges.
Step 1: Assign the central vertex as label 1
Step2: Remaining vertices in the outer cycle can be labeled as 2,3,5, ...upto the n^{th} vertex.
Let v_0 be the apex vertex $v_1, v_2, v_3, \ldots, v_{2r+1}$ be the rim vertices.
Let
$f(u_0) = (x_1 - x_3), f(v_2) = (x_2 - x_4), \ldots \ldots \ldots.., f(v_r) = (x_{2r+1} - x_{2r+3})$
so that $gcd\{f(u), f(v)\} = 1$ satisfying the prime difference speed sequence graph.

3. Strongly Prime Difference Speed Sequence Graphs

Definition 3.1
A graph G is said to be a strongly prime difference speed sequence graph if for any vertex v of G there exists a prime difference speed sequence labeling f satisfying $f(v) = 1$

Theorem 3.2
The complete graph K_n is not Strongly Prime difference speed sequence graph for n>4

Proof:
It is obvious that K_1 and K_2 are Strongly Prime difference speed sequence graph
Any vertex of K_3 and K_4 can be assigned the labels easily so that it satisfies the strongly prime difference speed sequence graph.
Let $v_1, v_2, \ldots \ldots \ldots \ldots, v_n$ be the n vertices of a graph.
Let n>4.
Fix any vertex as v_1 and labeling the vertices consecutively, we see that atleast any one of the edge does not satisfy $gcd\{f(u), f(v)\} = 1$.
Therefore $n > 4$ is not a Strongly Prime difference speed sequence graph.



Theorem 3.3
Every path is a Strongly Prime difference speed sequence graph.

Proof:
Let $v_1, v_2, \ldots \ldots \ldots \ldots, v_n$ be the consecutive vertices of P_n.
If v_a is any arbitrary vertex of P_n,then the following two cases arises:

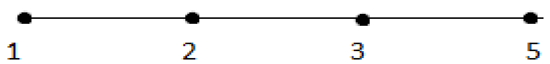
Case 1:
If v_a is either of the pendant vertices $(say\ v_a = v_1)$ then the function $f: V(P_n) \rightarrow \{\Delta_i(x)/\ i = 1, 2, 3, \ldots n\}$ defined by $f(v_i) = i, for\ all\ i = 1, 2, 3, \ldots n\}$is a prime difference speed sequence labeling with $f(v_a) = f(v_1) = 1$.

Case 2:
If v_a is not a pendant vertex then $a = j$ for some $j \in \{2,3, \ldots, n-1\}$ then the function $f: V(P_n) \rightarrow \{\Delta_i(x)/\ i = 1, 2, 3, \ldots n\}$is a prime difference speed sequence labeling with $f(v_a) = f(v_j) = 1$.

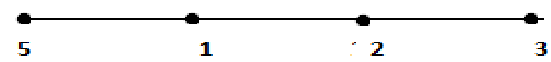
Thus P_n is a strongly Prime difference speed sequence graph.

Example 3.4

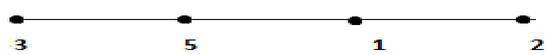
By assigning label 1 to any arbitrary vertex of pathP_4, we get different strongly prime difference speed sequence graphs.



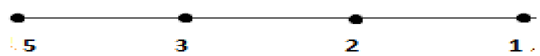
A strongly prime difference sequence labeling of P_4 having v_1 as label 1



A strongly prime difference sequence labeling of P_4 having v_2 as label 1



A strongly prime difference sequence labeling of P_4 having v_3 as label 1



A strongly prime difference sequence labeling of P_4 having v_4 as label 1

References

[1] Binod Chandra Tripathy and AyhanEsi, A new type of difference sequence space, Int. J of Sci and Tech, 1 (1), 2006, 11-14.
[2] Indirani K, Rate sequence spaces, thesis submitted to Mother Teresa Women's University, December 2009.
[3] KathiresanK.M., and AmuthaS., Fibonacci graceful graphs, ArsCombin., 97(2010), 41- 50
[4] Tout A., Dabbouey A.N. and Howalla K. Prime Labeling of graphs, National Academy Science Letters, 11(1982), 365-368