

Removal of Image Blurring and Salt Pepper Noise Using Variation Models

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Abstract: For the past recent decades, image denoising has been analyzed in many fields such as computer vision, statistical signal and image processing. It facilitates an appropriate base for the analysis of natural image models and signal separation algorithms. Moreover, it also turns into an essential part to digital image acquiring systems to improve qualities of image. These two directions are vital and will be examined in this work. Noise and Blurring of images are two degrading factors and when image is corrupted with both blurring and mixed noises de-noising and de-blurring of image is very difficult. In this paper, Gauss-Total Variation model (G-TV model) and Gaussian Mixture-Total Variation Model (GM-TV Model) are discussed and results are presented and it is shown that blurring of image is completely removed using G-TV model; however, image corrupted with blurring and mixed noise can be recovered with GM-TV model and using DCT runtime can be reduced significantly.

Keywords: G-TV, GM-TV, Blurring and Noise

1. Introduction

The field of processing of digital image alludes to the use of computer algorithms to extract helpful data from digital images [1-4]. The whole procedure of image processing may be divided into three prime stages:

- 1) Image acquisition: conversion of 3D visual information into 2D digital form that is perfect for storing, processing and transmission purpose.
- 2) Processing: enhancing quality of image by means of enhancement, restoration, etc.
- 3) Analysis: drawing out image features; quantifying shapes and recognition.

In image processing, input is an image scene while output is a corresponding digital image at first stage. On the other side both input and output are digital images at second stage of processing, where the output is an enhanced form of the input. In the last stage, input is still a digital image but the output is description of the contents.

Noise

Noise of image is the irregular changes of color or brightness information in images created by the sensor and circuitry of a scanner or digital camera [5-13]. Image noise is taken as an unwanted by-product of image taken.

Gaussian Blur

Gaussian Blur is that pixel weights are not same – as per curve of a bell-shaped, they go down to edges from kernel centre. The result of Gaussian Blur is a filter that combines a specific pixel quantity incrementally, that follows a bell-shaped curve. In the center, the blurring is thick but it feathers at the edge [12-13].

2. Various Image Restoration Filters

Noise affects a lot of images and it is the variation in data that could not be explained. Analysis of image is generally simple if we are able to remove this noise. In the similar

way as filters are used in chemistry to remove unwanted particles from fluids by making them to pass through a layer of sand or charcoal; signal processing engineers have increased the extent of the word filter to incorporate functions which accentuate elements of interest in data.

With the application of this comprehensive definition, these filters may be implemented to focus edges that is, border of objects or parts of objects in images. Filters provide guidance to visual interpretation of images. Not limiting to this, it can be used as a precursor to following digital processing, like segmentation [12-17].

Restoration of image or Denoising is the method of getting the original image from the corrupted image provided the information of the degrading factors as demonstrated in Figure 2.11. It is used for the purpose of pulling out the noise from the degraded image. In this process of removing, it does not influence and maintains the edges and other details.

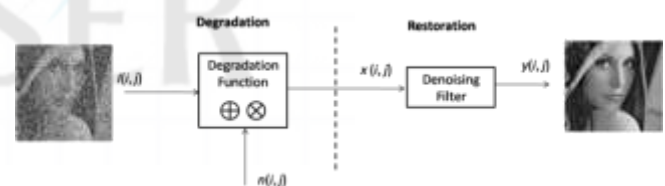


Figure 1: Image Degradation and Restoration Process

The image degradation and restoration process is illustrated in the above figure 2. $O(i, j)$ is an input object $n(i, j)$ is degrading term (may consist noise, blurring or both) so $x(i, j)$ is

$$x(i, j) = O(i, j) + n(i, j) \quad (1)$$

or

$$x(i, j) = O(i, j) \times n(i, j) \quad (2)$$

$$y(i, j) = L[x(i, j)] \quad (3)$$

L is filter operator.

Various procedures have been put forward in the recent past years for image filtering. Among all of them, most preferred method is linear filtering.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Where $x(t)$ is input image, $h(t)$ is filter impulse function and $y(t)$ is the output image.

3. Gauss-Total Variation model (G-TV Model)

A new interpretation of the ROF model is developed in this section that based on statistical approaches. In the following, we consider that the noise intensity $n(x)$ or $(k*f)(x)-g(x)$ is a random variable and all these random variables are not dependent and identically-distributed (i.i.d.) as a Gaussian distribution $N(0, \sigma^2)$, i.e.,

$$g(x) = (k * f)(x) + n(x) \tag{4}$$

$$p((k * f)(x) - g(x) / \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{|(k * f)(x) - g(x)|^2}{2\sigma^2}\right\} \tag{5}$$

$$L(f, \sigma^2) = \prod_{x \in \Omega} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{|(k * f)(x) - g(x)|^2}{2\sigma^2}\right\} \tag{6}$$

Minimizing log-likelihood function

$$E_1(f, \sigma^2) = \frac{1}{2} \int_{\Omega} \left\{ \frac{|(k * f)(x) - g(x)|^2}{\sigma^2} \right\} dx + \frac{1}{2} \int_{\Omega} \ln(\sigma^2) dx \tag{7}$$

Where, σ^2 is unknown constant. Minimizing the above equation is equivalent to minimizing the residual

$$\frac{1}{2} \|k * f - g\|_{L^2}^2 \tag{8}$$

The minimization problem defined above is ill-posed; hence we incorporate a regularization term and gets the following cost functional [17-18]

$$E(f, \sigma^2) = E_1(f, \sigma^2) + \lambda J(f) \tag{9}$$

Considering TV regularization term as

$$J(f) = \int_{\Omega} \sqrt{|\nabla f|^2} dx \tag{10}$$

$$E_1(f, \sigma^2) = \frac{1}{2} \int_{\Omega} \left\{ \frac{|(k * f)(x) - g(x)|^2}{\sigma^2} \right\} dx + \frac{1}{2} \int_{\Omega} \ln(\sigma^2) dx + \lambda \int_{\Omega} |\nabla f| dx \tag{11}$$

Algorithm

The flow diagram of the Gauss and Gaussian Mixture model is given below:

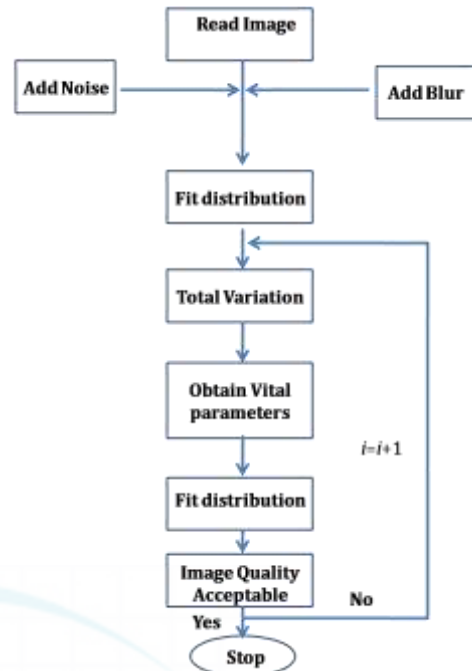


Figure 2: Flow diagram

4. Results



Figure 3: Original Lena image

In the experiment, figure 3 shows the original image of Lena. Gaussian Blur has corrupted this image with mean 25, and variance as 1, 5 and 7 respectively and resultant images are illustrated in Figure 4(a), 4(b) and 4(c) respectively.

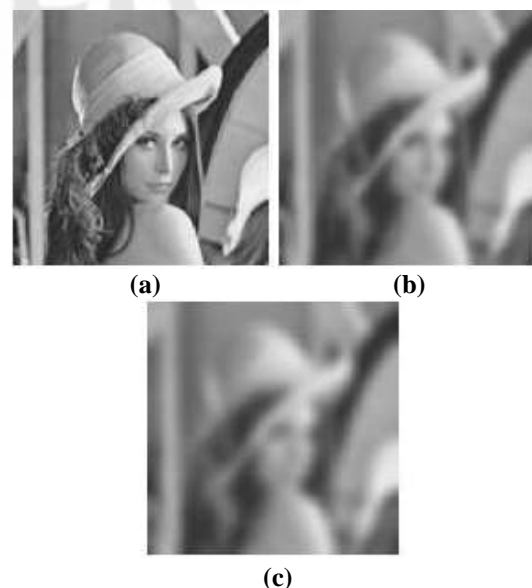


Figure 4: Blurred Lena image

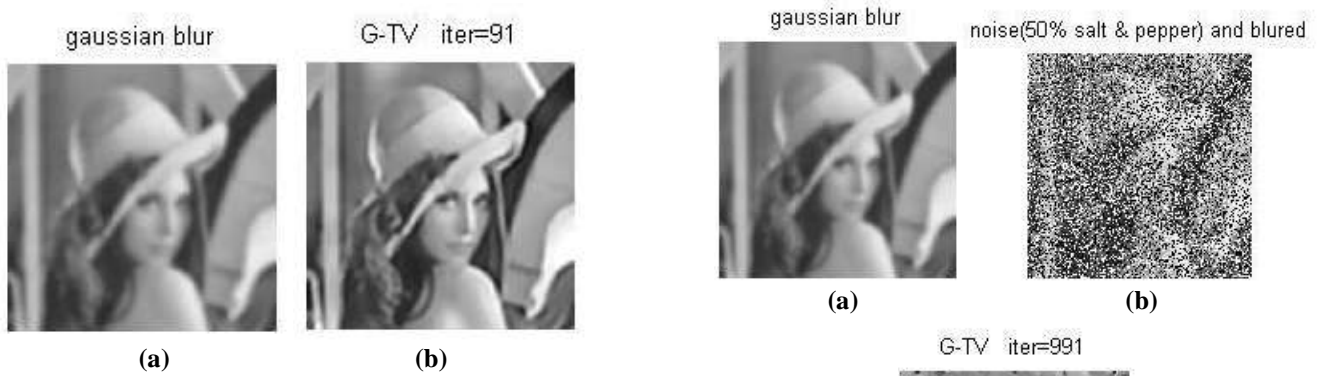


Figure 5: (a) Blurred and (b) Recovered Lena image

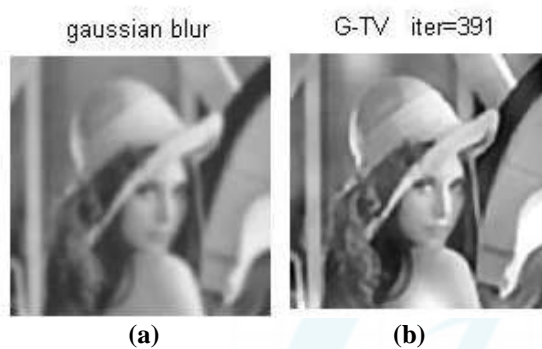


Figure 6: (a) Blurred and (b) Recovered Lena image

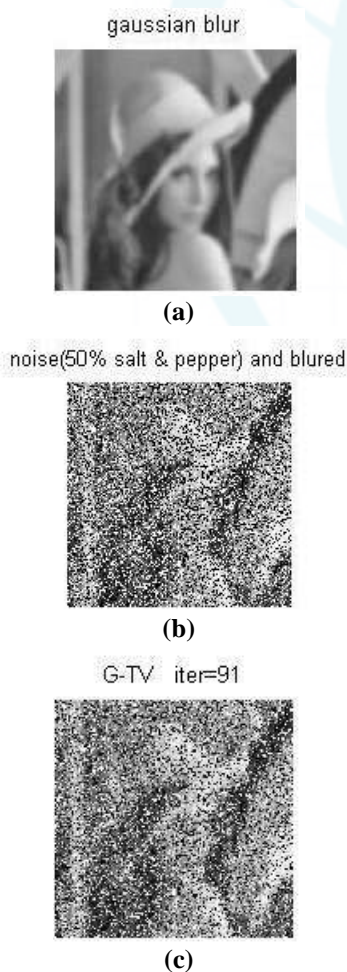


Figure 7: (a) Blurred and (b) Blurred and Noisy image (c) recovered image with G-TV model with 91 iterations

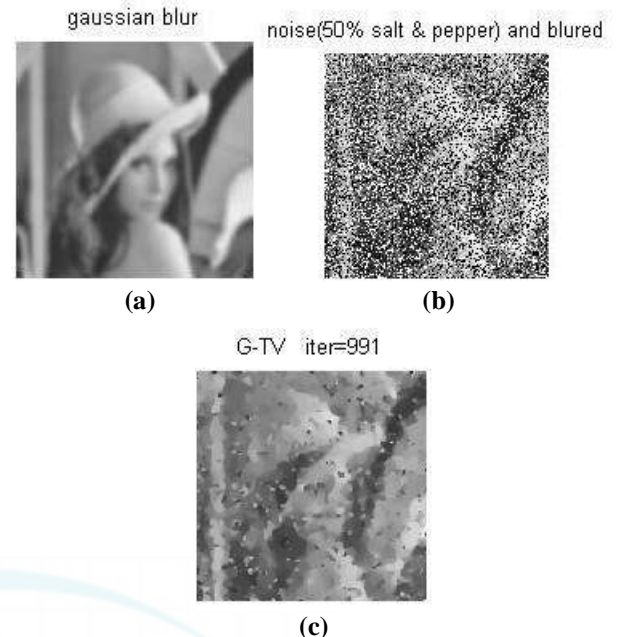


Figure 8: (a) Blurred and (b) Blurred and Noisy image (c) recovered image with G-TV model with 991 iterations

In figure 4, Lena image is corrupted with Gaussian blur with mean 25 and variance 3 and image is free from any other noise. The simulation was run for 600 iterations, and after 91 iterations significant improvement was found in the blurred image (Fig 5(b)).

In figure 6, Lena image is corrupted with Gaussian blur with mean 25 and variance 5 and image is free from any other noise. The simulation was run for 600 iterations, and after 391 iterations significant improvement was found in the blurred image (Fig 6(b)). But improvement is much lesser in comparison to fig 5(b).

In figure 7, Lena image is corrupted with Gaussian blur with mean 25 and variance 3 and image is corrupted with salt and pepper noise (50%). The simulation was run for 200 iterations, and after 91 iterations no significant improvement was found in the blurred image (Fig 8(b)). In figure 8(c) results are obtained after 991 iterations and still improvement is very less. However, recovered image is much better in comparison to 91 iterations.

5. Gaussian Mixture-Total Variation Model

G-TV model presented in previous chapter is very useful in the reconstruction of images with uniform distributed noise and blur without making any change in the regularization parameter λ directly. Though, it still is not quite effective when blur and mixed noise has contaminated the image. Hence in this section we propose a new model to address this issue.

Assume at each point $x \in \Omega$, the intensity of noise $n(x)$ or $(k * f)(x) - g(x)$ is a random variable and all the random variables $\{n(x) | x \in \Omega\}$ are independent and identically-distributed with the following probability density function [18]:

$$p(n(x)|z) = \sum_{i=1}^q a_i p_i(n(x)|\mu_i, \sigma_i^2) \tag{12}$$

where each P_i is a Gaussian density function with mean μ_i and variance σ_i^2 , and the parameter set $z = \{a_1, \dots, a_q, \mu_1, \dots, \mu_q, \sigma_1^2, \dots, \sigma_q^2\}$ is chosen such that

$$\sum_{i=1}^q a_i = 1 \tag{13}$$

In other words, the probability density function (PDF) is a mixture of q individual Gaussian components with different ratios.

$$E_l(f, z) = \int_{\Omega} -\ln \sum_{i=1}^q \frac{a_i}{\sigma_i} \exp \left\{ -\frac{|(k * f)(x) - g(x) - \mu_i|^2}{2\sigma_i^2} \right\} dx$$

and

$$J(f) = J_{\beta}(f) = \int_{\Omega} \sqrt{|\nabla f|^2} dx \tag{14}$$

The above is to be minimized under the constraints

$$\sum_{i=1}^q a_i = 1$$

Again use flow diagram for GM-TV model.



Figure 9: Results for G-TV and GM-TV model (only Blur)

In figure 9, only Gaussian blurring is considered with mean 25 and variance 1. Thus due to the lesser variance blurring is less. In above figure Gaussian blur and blur image is same as no noise is added. The recovered image using G-TV and GM-TV is shown and image is well recovered in 141 iterations.

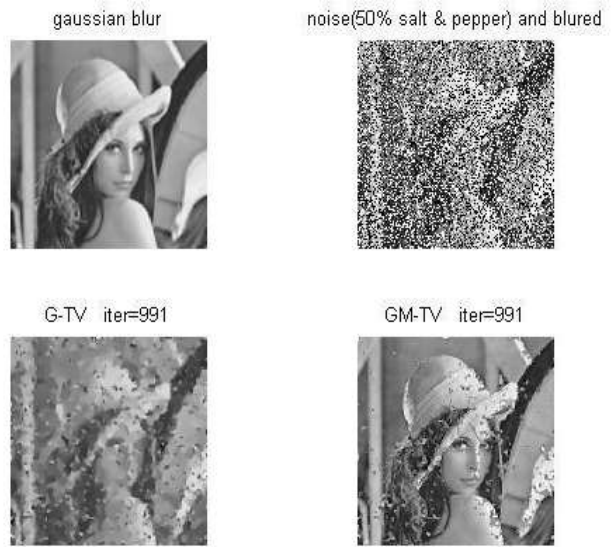


Figure 10: Results for G-TV and GM-TV model (Blur and Noise)

In figure 10, only Gaussian blurring is considered with mean 25 and variance 7. Thus due to the lesser variance blurring is less. In above figure Gaussian blur is similar to original Lena image however as noise is introduced, the blurred and noised image is not clearly visible. The recovered image using G-TV and GM-TV is shown, and using GM-TV model image recovery is better in comparison to G-TV model.

DCT and Size Reduction

Considering DCT of block size 8x8, the size of an image can be reduced considerably as shown in below table 1, while the quality of image remains same (SSIM=1).

Table 1: File size reduction using DCT

Number of considered DFT components	Percentage Reduction	File Size
.....	65.0 KB
20	68.75	9.05 KB
15	76.56	8.22 KB
10	84.38	6.95 KB

Due to this size reduction the run time complexity gets reduced and G-TV and GM-TV algorithms becomes faster.

Table 2: RUN Time complexity reduction

File Size	Time
65.0 KB	63 seconds
9.05 KB	59 seconds
8.22 KB	57 seconds
6.95 KB	51 seconds

Therefore, it can be concluded that using DCT and selecting only a few co-efficient size of the image can be reduced significantly thus run time of G-TV and GM-TV reduced considerably.

6. Conclusions

In this work, two methods are detailed which are capable of removing blurring and noises in digital images. On the basis of obtained results, following conclusions can be made: The G-TV model is quite effective in reconstructing images with blur and uniform distributed noise without changing the regularization parameter λ directly. However, it still could not work well when the image is contaminated with blur and mixed noise. GM-TV model is quite effective in reducing both blurring and noise. Using DCT size of the images can be reduced significantly thus in turn reduces runtime complexity.

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