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Fuzzy Critical Path Method Using Octagonal Fuzzy Numbers

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Abstract: ***In this paper a modified ranking approach to determine the critical path in fuzzy project network, where the duration of each activity time is represented by an octagonal fuzzy numbers. In octagonal fuzzy numbers is given to access the critical path from the initial node to terminal node by using ranking of fuzzy numbers.***

Keywords: Fuzzy set, Fuzzy numbers, Octagonal fuzzy numbers, critical path method, ranking function

1. Introduction

Critical path method (CPM) has been demonstrated to be a useful tool in the planning and control of complicated projects in management and engineering. The critical path method worked out at the beginning of the 1960's with the help of the critical path, the decision maker can adopt a better strategy of optimizing the time and the available resource to ensure the earliest completion and the quality of the project.

Zadeh[1] introduced an alternative way to deal with imprecise data to employ the concept of fuzziness. Liang and Han [2] developed an algorithm which is presented to perform critical path analysis in a fuzzy environment. M.H. Oladeinde and C.A.: Oladeinde [3] consider the effectiveness of the decision makers risk. Dubois et al [4] presented a project network defined as a set of activities that must be performed according to procedure constraints starting which must start after completion of other specified activities. Jain [5] proposed the concept of fuzzy numbers. Channas and Radosinski [6] analyzed the use of fuzzy numbers. Ravisanker et al [7] used Trapezoidal fuzzy numbers to rank the set of fuzzy numbers in a fuzzy project work. The project duration equals the length of the longest path through the project duration.

This paper presents another approach, which has not been proposed in the literature so far. We use a new ranking formula for octagonal fuzzy numbers and apply to the expected duration for each activity in the fuzzy project network to find the critical path.

2. Preliminaries

2.1. Fuzzy Set.

Let X be a set. A fuzzy set A on X is defined to be a function $A : X \to [0, 1]$ or μ_A: $X \to [0, 1]$. Equivalently, A fuzzy set A is defined to be the class of objects having the following representation:$A = \{(x, \mu A(x), x \in X)\}$ where μ_A: $X \to [0, 1]$ is a function called the membership function of A.

2.2. Fuzzy Number

The fuzzy number A is a fuzzy set whose membership function satisfies the following conditions:

(1) $\mu_A(x)$ is piecewise continuous.
(2) A fuzzy set A of the universe of discourse X is convex.
(3) A fuzzy set of the universe of discourse X is called a normal fuzzy set if $x_i \in X$ exists.

2.3. Trapezoidal Fuzzy Number.

A fuzzy number with membership function in the form

$$\mu_A(x)=\begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ 1, b \le x \le c \\ \frac{d-x}{d-c}, c \le x \le d \\ 0, othwise \end{cases} \quad (1)$$

is called a trapezoidal fuzzy number $A = (a, b, c, d)$.

2.4. Triangular Fuzzy Number.

A fuzzy number with membership function in the form

$$\mu_A(x)=\begin{cases} \frac{x-a}{b-a}\ a \le x \le b \\ \frac{x-c}{b-c}, b \le x \le c \\ 0, othwise \end{cases} \quad (2)$$

is called a triangular fuzzy number A = (a, b, c)

2.5 Octagonal Fuzzy Number.

Here, we present a method for ranking fuzzy numbers. This method calculates the ranking scores of the octagonal fuzzy number. Fuzzy numbers (A, B, C) are an octagonal fuzzy number denoted by $(A, B, C)=(a, b, c, d, e, f, g, h)$ where (a, b, c, d, e, f, g, h) are real numbers and its membership functions $\mu_A(x)$, $\mu_B(x)$, and $\mu_C(x)$ are given below:

$$\mu_A(x)=\begin{cases}\frac{x-a}{d-a}, a \le x \le d\\ 1, b \le x \le c\\ \frac{x-h}{e-h}, e \le x \le h\\ 0, othwise\end{cases} \quad \mu_B(x)=\begin{cases}\frac{x-a}{c-a}, a \le x \le c\\ 0.8, c \le x \le f\\ \frac{x-h}{f-h}, f \le x \le h\\ 0, othwise\end{cases}$$

$$\mu_C(x)=\begin{cases}\frac{x-a}{b-a}, a \le x \le b\\ 0.6, b \le x \le g\\ \frac{x-g}{g-h}, g \le x \le h\\ 0, othwise\end{cases} \qquad (3)$$

3. Arithmetic operation on octagonal fuzzy numbers

Let A_1 and A_2 be two octagonal fuzzy numbers parameterized by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$, respectively.

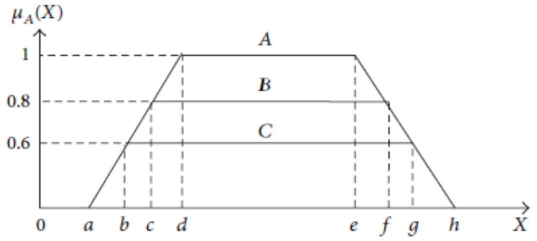


Figure 1: Graphical representation of an octagonal fuzzy number

The simplified fuzzy number arithmetic operations between the fuzzy numbers A_1 and A_2 are as follows:

Fuzzy numbers addition: ⊕ : $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \oplus (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8)$

Fuzzy numbers subtraction: ⊖ : $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \ominus (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) = (\max(0, a_1 - b_8), \max(0, a_2 - b_7), \max(0, a_3 - b_6), \max(0, a_4 - b_5), \max(0, a_5 - b_4), \max(0, a_6 - b_3), \max(0, a_7 - b_2), \max(0, a_8 - b_1))$.

Ranking of octagonal fuzzy numbers

The parametric methods of comparing fuzzy numbers, especially in fuzzy decision making theory are more efficient than non-parametric methods. Cheng's centroid point method [9], Chu and Tsao's method [8], Abbasbandy and Assady's [10] sign-distance method was all non-parametric and was applicable only for normal fuzzy numbers. The non-parametric methods for comparing fuzzy numbers have some drawbacks in practice.

Definition 4.1: A measure of fuzzy number Ãω is a function Mα: $\mathbf{R}\omega(I) \rightarrow \mathbf{R}^+$ which assigns a non-negative real number Mα(Ãω) that expresses the measure of Ãω.

$$M_\alpha^{Oct}(\tilde{A}\omega) = \frac{1}{2}\int_\alpha^k \big(l_1(r) + l_2(r)\big)dr + \frac{1}{2}\int_k^\omega \big(s_1(t) + s2tdt\ where\ 0<\alpha<1$$

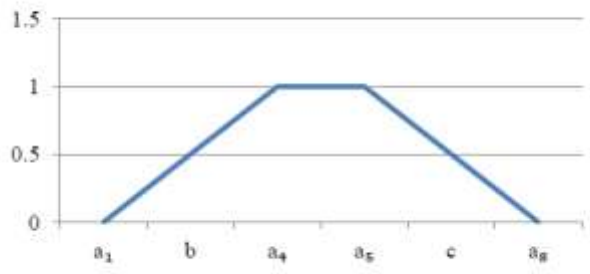
Definition 4.2: The measure of an octagonal fuzzy number is obtained by the average of the two fuzzy side areas, left side area and right side area, from membership function to α axis.

Definition 4.3: Let Ã be a normal octagonal fuzzy number. The value $M_0^{Oct}(\tilde{A})$, called the measure of Ã is calculated as follows:

$$M_\alpha^{Oct}(\tilde{A}\omega) = \frac{1}{2}\int_0^k \big(l_1(r) + l_2(r)\big)dr + \frac{1}{2}\int_k^1 \big(s_1(t) + s2tdt\ where\ 0<k<1$$

$$= \frac{1}{4}\,[(a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1-k)] \qquad (4)$$

Remark 4.1: Consider the trapezoidal number $(a_1, b, a_4, a_5, c, a_8)$ which is got from the above octagonal number by equating $a_2 = a_3 = b$, $a_6 = a_7 = c$, for k = 0.5 and ω = 1



The measure of the normal fuzzy trapezoidal number is given by

$$M_0^{tra}(\tilde{A}) = \frac{1}{8}\,(a_1 + 2b + a_4 + a_5 + 2c + a_8) \quad (5)$$

Remark 4.2: If k=0.5, $M_0^{Oct}(\tilde{A}) = \frac{1}{8}(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8)$ (6)

When a_2 coincides with a_3 and a_6 coincides with a_7 it reduces to trapezoidal fuzzy number, which is given by Equation (5).

Remark 3.3:

If $a_1 + a_2 + a_7 + a_8 = a_3 + a_4 + a_5 + a_6$ (7)

then the measure of an octagonal number is the same for any value of k ($0<k<1$).

4. Fuzzy Critical Path Algorithm

Step 1: Construct network diagram according to Fulkerson rule
Step 2: Calculate Earliest starting time according to forward pass calculation
i.e., $E_j = \text{Max}_i \{E_i + \widetilde{D}_{ij}\}$, i = no of preceding nodes
Step 3: Calculate Earliest finishing time EFT = EST + NT

Step 4: Calculate Latest Starting time according to backward pass calculation.

i.e., $L_i = \text{Min}_j \{L_i - \tilde{D}_{ij}\}$, j = number of succeeding nodes.

Step 5: Calculate the Latest Finishing time LFT = LST – NT

Step 6: Calculate Total Floating time TFT = LFT – EFT

Step 7: If TFT = 0 those activities are called critical activities.

5. Description of the method

Octagonal fuzzy numbers are converted in to expected (normal time) time by ranking method. These expected time treated as the time between the nodes and fuzzy critical path is calculated by using conventional method mentioned above in equation (6).

Numerical Example:

Suppose that there is a project network with the set of fuzzy events $\tilde{V}$ = {1, 2, 3, 4, 5, 6, 7, 8} the fuzzy activity time for each activity is shown in table 1(all duration are in hours)

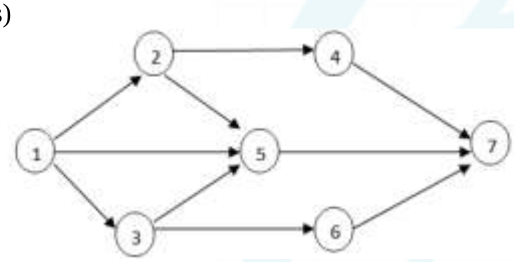


Figure 1: Fuzzy Project network

Table 1: Activity duration of each activity in a fuzzy project network

Activity	Fuzzy activity time
1-2	(2, 3, 4, 5, 6, 7, 7)
1-3	(4, 4, 5, 6, 7, 7, 8, 9)
1-5	(3, 4, 4, 5, 6, 6, 7, 8)
2-4	(5, 5, 6, 7, 7, 9, 10)
2-5	(6, 7, 7, 8, 9, 10, 11, 12)
3-5	(8, 9, 10, 11, 12, 14, 15, 16)
3-6	(6, 7, 8, 9, 10, 11, 12, 13)
4-7	(10, 12, 14, 15, 15, 16, 18, 20)
5-7	(12, 12, 14, 15, 15, 16, 18, 20)
6-7	(13, 13, 15, 16, 17, 18, 19, 20)

From the above example using ranking method (Equation 6) find the duration of each activity.

Table 2: Expected duration in a fuzzy project network

Activity	Fuzzy activity time	Expected Duration
1-2	(2, 3, 4, 5, 6, 7, 7)	4.875
1-3	(4, 4, 5, 6, 7, 7, 8, 9)	6.25
1-5	(3, 4, 4, 5, 6, 6, 7, 8)	5.375
2-4	(5, 5, 6, 7, 7, 9, 10)	7.25
2-5	(6, 7, 7, 8, 9, 10, 11, 12)	8.75
3-5	(8, 9, 10, 11, 12, 14, 15, 16)	11.875
3-6	(6, 7, 8, 9, 10, 11, 12, 13)	9.5
4-7	(10, 12, 14, 15, 15, 16, 18, 20)	15
5-7	(12, 12, 14, 15, 15, 16, 18, 20)	15.5
6-7	(13, 13, 15, 16, 17, 18, 19, 20)	16.375

The revised form of activity and duration as follows

Activity	Duration
1-2	4.88
1-3	6.25
1-5	5.38
2-4	7.25
2-5	8.75
3-5	11.88
3-6	9.5
4-7	15
5-7	15.5
6-7	16.38

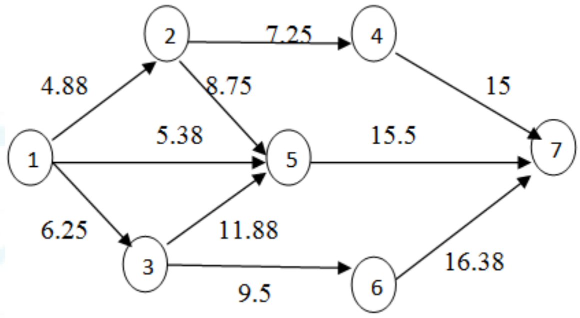


Figure: Project Network – II

Now to apply algorithm for identifying the critical path. The result shows that critical path for fuzzy project network is 1-3-5-7

6. Conclusion

This paper develops a simple ranking approach introduced to identify the fuzzy critical path. We used Octagonal fuzzy numbers to represent the activity duration in the project network, to find the critical path. The proposed approach is more effective in determining the activity criticalities and finding the critical path.

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