

$\tau_1\tau_2$ - \tilde{g} -Closed sets in Bitopological Spaces

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Abstract: *The aim of this paper is to introduce $\tau_1\tau_2$ - \tilde{g} -Closed sets, $\tau_1\tau_2$ - \tilde{g} -Open sets in Bitopological Spaces and to study about their properties.*

Keywords: $\tau_1\tau_2$ - \tilde{g} -Closed sets, $\tau_1\tau_2$ - \tilde{g} -Open sets

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1. Introduction

Levine [8] introduced semi open sets in 1963 and also Levine [7] defined generalized closed sets in 1970. AblEl-Monsef et al.[2] introduced β -open sets. Veerakumar [11] introduced $\#$ -g-closed sets in topological spaces. O.Ravi [10] and S.Ganesan [10] declared \tilde{g} -closed sets in topological spaces. Kelley [6] initiated the study of bitopological spaces in 1963. A nonempty set X equipped with two topologies τ_1 and τ_2 is called a bitopological space and is denoted by (X, τ_1, τ_2) . Since then several topologists generalized many of the results in topological spaces to bitopological spaces. Fukutake [3] introduced generalized closed sets in bitopological spaces. Fukutake [4] defined semi open sets in bitopological spaces.

2. Preliminaries

Definition 2.1: A subset A of a bitopological space (X, τ_1, τ_2) is called a

1. $\tau_1\tau_2$ -semi open [2] if $A \subset \tau_2 cl(\tau_1 int(A))$ and it is called $\tau_1\tau_2$ -semi closed [2] if $\tau_2 int(\tau_1 cl(A)) \subset A$
2. $\tau_1\tau_2$ -pre open[5] if $A \subset \tau_2 int(\tau_1 cl(A))$ and $\tau_1\tau_2$ -preclosed [5] if $\tau_2 cl(\tau_1 int(A)) \subset A$
3. $\tau_1\tau_2$ - α -open [9] if $A \subset \tau_1 int(\tau_2 cl(\tau_1 int(A)))$.

Definition 2.2: A subset A of a bitopological space (X, τ_1, τ_2) is called a

1. $\tau_1\tau_2$ -g-closed [3]($\tau_1\tau_2$ -generalized closed) if $\tau_2 cl(A) \subset U$, whenever $A \subset U$, U is τ_1 -open.
2. $\tau_1\tau_2$ -sg-closed [4]($\tau_1\tau_2$ -semi generalized closed) if $\tau_2 scl(A) \subset U$, whenever $A \subset U$, U is τ_1 -semi open.
3. $\tau_1\tau_2$ -gs-closed [5] ($\tau_1\tau_2$ -generalized semi closed) if $\tau_2 scl(A) \subset U$, whenever $A \subset U$, U is τ_1 -open.
4. $\tau_1\tau_2$ - α g-closed [9] ($\tau_1\tau_2$ - α -generalized closed) if $\tau_2 \alpha cl(A) \subset U$, whenever $A \subset U$, U is τ_1 -open.
5. $\tau_1\tau_2$ - $g\alpha$ -closed [9] ($\tau_1\tau_2$ -generalized α -closed) if $\tau_2 \alpha cl(A) \subset U$, whenever $A \subset U$, U is τ_1 - α -open.
6. $\tau_1\tau_2$ - \hat{g} -closed[5] if $\tau_2 cl(A) \subset U$, whenever $A \subset U$, U is τ_1 - semi open.

3. $\tau_1\tau_2$ - \tilde{g} -Closed sets

Definition 3.1

A subset A of (X, τ_1, τ_2) is called a $\tau_1\tau_2$ - \tilde{g} -closed if $\tau_2 cl(A) \subset U$, whenever $A \subset U$ and U is τ_1 -sg-open in (X, τ_1) .

Example 3.2

Let

$$X = \{a, b, c\}; \tau_1 = \{X, \phi, \{a\}\}; \tau_2 = \{X, \phi, \{a\}, \{a, b\}\}.$$

Then $X, \phi, \{c\}, \{b, c\}$ are $\tau_1\tau_2$ - \tilde{g} -Closed sets.

Theorem 3.3

Every τ_2 -closed set is $\tau_1\tau_2$ - \tilde{g} -closed sets.

Proof:

Let A be τ_2 -closed. Then $\tau_2 cl(A) = A \Rightarrow \tau_2 cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -sg-open. $\Rightarrow A$ is $\tau_1\tau_2$ - \tilde{g} -closed sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.4

$$\text{Let } X = \{a, b, c, d\}; \tau_1 = \{X, \phi, \{a, b\}\}; \tau_2 = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}.$$

$\tau_1\tau_2$ - \tilde{g} -closed sets are

$$X, \phi, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}.$$

Therefore $\{d\}, \{b, d\}$ are $\tau_1\tau_2$ - \tilde{g} -closed set but they are not τ_2 -closed.

Theorem 3.5

Every $\tau_1\tau_2$ - \tilde{g} -closed set is $\tau_1\tau_2$ -g-closed.

Proof:

Let $A \subseteq U$, U is τ_1 -open. Then U is τ_1 -sg-open. $\Rightarrow \tau_2 cl(A) \subseteq U$.[$\because A$ is $\tau_1\tau_2$ - \tilde{g} -closed] $\Rightarrow \tau_2 cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open. $\Rightarrow A$ is $\tau_1\tau_2$ -g-closed sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6

$$\text{Let } X = \{a, b, c, d\}; \tau_1 = \{X, \phi, \{a, b\}\}; \tau_2 = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$$

$\tau_1\tau_2$ - \tilde{g} -closed sets are

$X, \phi, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}$

$\tau_1\tau_2$ - g -closed sets are

$X, \phi, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}.$

Here $\{c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}$ is $\tau_1\tau_2$ - g -closed set but they are not $\tau_1\tau_2$ - \tilde{g} -closed set.

Theorem 3.7

Every $\tau_1\tau_2$ - \tilde{g} -closed set is $\tau_1\tau_2$ - \hat{g} -closed.

Proof:

Let $A \subseteq U$, U is τ_1 -semi open. Then U is τ_1 - sg -open.

Since A is $\tau_1\tau_2$ - \tilde{g} -closed, $\tau_2 - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -semi open. $\Rightarrow \tau_2 - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -semi open. $\Rightarrow A$ is $\tau_1\tau_2$ - \hat{g} -closed sets.

Theorem 3.8

Every $\tau_1\tau_2$ - \tilde{g} -closed set is $\tau_1\tau_2$ - gs -closed.

Proof:

Let $A \subseteq U$, U is τ_1 -open. U is τ_1 -open $\Rightarrow U$ is τ_1 - sg -open. $\Rightarrow \tau_2 - cl(A) \subseteq U$ (By assumption) But $\tau_2 - scl(A) \subseteq \tau_2 - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open. $\Rightarrow \tau_2 - scl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open. $\Rightarrow A$ is $\tau_1\tau_2$ - gs -closed.

Theorem 3.9

Every $\tau_1\tau_2$ - \tilde{g} -closed set is $\tau_1\tau_2$ - αg -closed.

Proof:

Let $A \subseteq U$, U is τ_1 -open. U is τ_1 -open $\Rightarrow U$ is τ_1 - sg -open. $\Rightarrow \tau_2 - cl(A) \subseteq U$ (By assumption) But $\tau_2 - \alpha cl(A) \subseteq \tau_2 - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open. $\Rightarrow \tau_2 - \alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open. $\Rightarrow A$ is $\tau_1\tau_2$ - αg -closed.

Theorem 3.10

Every $\tau_1\tau_2$ - \tilde{g} -closed set is $\tau_1\tau_2$ - $g\alpha$ -closed.

Proof:

Let $A \subseteq U$, U is τ_1 - α -open. U is τ_1 - α -open $\Rightarrow U$ is τ_1 - sg -open. $\Rightarrow \tau_2 - cl(A) \subseteq U$ (By assumption) But $\tau_2 - \alpha cl(A) \subseteq \tau_2 - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 - α -open. $\Rightarrow \tau_2 - \alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 - α -open. $\Rightarrow A$ is $\tau_1\tau_2$ - $g\alpha$ -closed.

Theorem 3.11

Union of two $\tau_1\tau_2$ - \tilde{g} -closed sets is $\tau_1\tau_2$ - \tilde{g} -closed.

Proof:

Assume that A and B are $\tau_1\tau_2$ - \tilde{g} -closed sets. Let $A \cup B \subseteq U$ where U is τ_1 - sg -open. Then $A \subseteq U$ and $B \subseteq U$. $\Rightarrow \tau_2 - cl(A) \subseteq U$ and $\tau_2 - cl(B) \subseteq U \Rightarrow \tau_2 - cl(A) \cup \tau_2 - cl(B) \subseteq U$. But $\tau_2 - cl(A \cup B) = \tau_2 - cl(A) \cup \tau_2 - cl(B) \subseteq U$. Therefore $\tau_2 - cl(A \cup B) \subseteq U$ whenever $A \cup B \subseteq U$ and U is τ_1 - sg -open. Therefore $A \cup B$ is $\tau_1\tau_2$ - \tilde{g} -closed.

Remark 3.12

The intersection of two $\tau_1\tau_2$ - \tilde{g} -closed sets need not be $\tau_1\tau_2$ - \tilde{g} -closed.

This can be seen from the following example.

Example 3.13

Let $X = \{a, b, c\}$; $\tau_1 = \{X, \phi, \{b, c\}\}$; $\tau_2 = \{X, \phi, \{a\}\}$

$\tau_1\tau_2$ - \tilde{g} -closed sets $X, \phi, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}.$

$A = \{a, b\}$ and $B = \{b, c\}$

$A \cap B = \{b\}$ is not $\tau_1\tau_2$ - \tilde{g} -closed set.

Theorem 3.14

Let A be $\tau_1\tau_2$ - \tilde{g} -closed and $A \subset B \subset \tau_2 - cl(A)$ then B is $\tau_1\tau_2$ - \tilde{g} -closed.

Proof:

Let $B \subseteq U$ where U is τ_1 - sg -open. Then $A \subset B \subset U \Rightarrow \tau_2 - cl(A) \subseteq U$.

Given $B \subset \tau_2 - cl(A)$ and $\tau_2 - cl(B)$ is the smallest closed set containing B .

$\therefore B \subset \tau_2 - cl(B) \subset \tau_2 - cl(A) \subset U \Rightarrow \tau_2 - cl(B) \subset U \Rightarrow B$ is $\tau_1\tau_2$ - \tilde{g} -closed.

Theorem 3.15

If A is $\tau_1\tau_2$ - \tilde{g} -closed then $\tau_2 - cl(A) - A$ does not contains any non-empty τ_1 - sg -closed set.

Proof:

Suppose $\tau_2 - cl(A) - A$ contains a not empty τ_1 - sg -closed set F .

Then $F \subset \tau_2 - cl(A) - A \Rightarrow F \subset \tau_2 - cl(A)$ but $F \not\subset A \Rightarrow F \subset A^c \Rightarrow A \subset F^c$ where F^c is τ_1 - sg -open. $\Rightarrow \tau_2 - cl(A) \subset F^c \Rightarrow F \subset (\tau_2 - cl(A))^c$.

We have $F \subset \tau_2 - cl(A) \cap (\tau_2 - cl(A))^c = \phi \therefore \tau_2 - cl(A) - A$ does not contains any non-empty τ_1 - sg -closed set.

Theorem 3.16

Let A be $\tau_1\tau_2$ - \tilde{g} -closed. Then A is τ_2 -closed iff $\tau_2 - cl(A) - A$ is τ_1 - sg -closed set.

Proof:

Suppose that A is $\tau_1\tau_2$ - \tilde{g} -closed and τ_2 -closed. Then $\tau_2 - cl(A) = A$.

$\Rightarrow \tau_2 - cl(A) - A = \phi$. which is τ_1 - sg -closed set.

Conversely, assume that A is A be $\tau_1\tau_2$ - \tilde{g} -closed and $\tau_2 - cl(A) - A$ is τ_1 - sg -closed set. Since A is $\tau_1\tau_2$ - \tilde{g} -closed, $\tau_2 - cl(A) - A$ does not contains any non-empty τ_1 - sg -closed set. $\Rightarrow \tau_2 - cl(A) - A = \phi \Rightarrow \tau_2 - cl(A) = A \Rightarrow A$ is τ_2 -closed.

Theorem 3.17

If A is $\tau_1\tau_2$ - \tilde{g} -closed and $A \subset B \subset \tau_2 - cl(A)$ then $\tau_2 - cl(B) - B$ contains no non-empty τ_1 - sg -closed set.

Proof:

Let A be $\tau_1\tau_2$ - \tilde{g} -closed and $A \subset B \subset \tau_2 - cl(A)$. By theorem 3.14, B is $\tau_1\tau_2$ - \tilde{g} -closed. Since B is $\tau_1\tau_2$ - \tilde{g} -closed, then by theorem 2.15 $\tau_2 - cl(B) - B$ contains no non-empty τ_1 - sg -closed set. Hence $\tau_2 - cl(B) - B$ contains no non-empty τ_1 - sg -closed set.

Theorem 3.18

For each $x \in X$, the singleton $\{x\}$ is either τ_1 -sg-closed set or its complement $\{x\}^c$ is $\tau_1\tau_2$ - \tilde{g} -closed.

Proof:

Suppose $\{x\}$ is not τ_1 -sg-closed, then $\{x\}^c$ will not be τ_1 -sg-open. $\Rightarrow X$ is the only τ_1 -sg-open set containing $\{x\}^c \Rightarrow \tau_2 - cl\{x\}^c \subset X \Rightarrow \{x\}^c$ is $\tau_1\tau_2$ - \tilde{g} -closed. $\Rightarrow \{x\}$ is $\tau_1\tau_2$ - \tilde{g} -open.

Theorem 3.19

Arbitrary union of $\tau_1\tau_2$ - \tilde{g} -closed sets $\{A_i, i \in I\}$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ - \tilde{g} -closed if the family $\{A_i, i \in I\}$ is locally finite on X .

Proof:

Let $\{A_i/i \in I\}$ be locally finite on X and each A_i is $\tau_1\tau_2$ - \tilde{g} -closed in X .

To prove: $\cup A_i$ is $\tau_1\tau_2$ - \tilde{g} -closed.

Let $\cup A_i \subset U$ where U is τ_1 -sg-open. $\Rightarrow A_i \subset U$, for every $i \in I$. $\Rightarrow \tau_2 - cl(A_i) \subset U$ for every $i \in I$. $\Rightarrow \cup \tau_2 - cl A_i \subset U$. Since A_i s locally finite, $\tau_2 - cl(\cup A_i) = \cup \tau_2 - cl(A_i) \Rightarrow \tau_2 - cl(\cup A_i) \subset U$ whenever $\cup A_i \subset U$ where U is τ_1 -sg-open. $\Rightarrow \cup A_i$ is $\tau_1\tau_2$ - \tilde{g} -closed.

4. $\tau_1\tau_2$ - \tilde{g} -open sets**Definition 4.1**

A subset A of bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ - \tilde{g} -open iff $X - A$ is $\tau_1\tau_2$ - \tilde{g} -closed.

Example 4.2

In example 3.2, $\phi, X, \{a\}, \{a, b\}$ are $\tau_1\tau_2$ - \tilde{g} -open sets in X .

Theorem 4.3

A set A is $\tau_1\tau_2$ - \tilde{g} -open iff $F \subseteq \tau_2 - int(A)$ where F is τ_1 -sg-closed and $F \subseteq A$.

Proof:

Suppose A is $\tau_1\tau_2$ - \tilde{g} -open. Then A^c is $\tau_1\tau_2$ - \tilde{g} -closed. Suppose that F is τ_1 -sg-closed and $F \subseteq A$. Then F^c is τ_1 -sg-open and $A^c \subseteq F^c$. Therefore $\tau_2 - cl(A^c) \subseteq F^c$ (since A^c is $\tau_1\tau_2$ - \tilde{g} -closed) $\Rightarrow X - \tau_2 int(A) \subset F^c$ [since $cl(X - A) = X - int(A)$]. Hence $F \subseteq \tau_2 - int(A)$.

Conversely, suppose that $F \subseteq \tau_2 - int(A)$ where F is τ_1 -sg-closed and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is τ_1 -sg-open. Take $U = F^c$. Since $F \subseteq \tau_2 - int(A)$, we have

$[\tau_2 - int(A)]^c \subseteq F^c = U \Rightarrow \tau_2 - cl(A^c) \subseteq U$ [since $cl(A^c) = (int A)^c$]. Therefore A^c is $\tau_1\tau_2$ - \tilde{g} -closed. Thus A is $\tau_1\tau_2$ - \tilde{g} -open.

Remark 4.4

Every τ_1 -open set is $\tau_1\tau_2$ - \tilde{g} -open but the converse is not true in general as can be seen from the following two examples.

Example 4.5

In example 3.4,

$\tau_1\tau_2$ - \tilde{g} -open sets $\phi, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \{a, b, c\}$. $\{a, c\}$ is $\tau_1\tau_2$ - \tilde{g} -open in X but not τ_1 -open in X .

Theorem 4.6

If A and B are $\tau_1\tau_2$ - \tilde{g} -open sets in bitopological space (X, τ_1, τ_2) then their intersection is a $\tau_1\tau_2$ - \tilde{g} -open set.

Proof:

If A and B are $\tau_1\tau_2$ - \tilde{g} -open sets, then A^c and B^c are $\tau_1\tau_2$ - \tilde{g} -closed sets. $A^c \cup B^c$ is $\tau_1\tau_2$ - \tilde{g} -closed sets (by theorem 3.11). $(A \cap B)^c$ is $\tau_1\tau_2$ - \tilde{g} -closed. $\Rightarrow A \cap B$ is $\tau_1\tau_2$ - \tilde{g} -open set.

Remark 4.7

The union of two $\tau_1\tau_2$ - \tilde{g} -open sets is need not be $\tau_1\tau_2$ - \tilde{g} -opens set in X .

This can be seen from the following example.

Example 4.8

Let $X = \{a, b, c, d\}$; $\tau_1 = \{X, \phi, \{a, b\}\}$; $\tau_2 = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$

$\tau_1\tau_2$ - \tilde{g} -closed set are

$X, \phi, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$.

$\tau_1\tau_2$ - \tilde{g} -open set are

$X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}$.

Here $\{a\}$ and $\{b\}$ are $\tau_1\tau_2$ - \tilde{g} -open sets but they union is not $\tau_1\tau_2$ - \tilde{g} -open set.

Theorem 4.9

The arbitrary intersection of $\tau_1\tau_2$ - \tilde{g} -open sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ - \tilde{g} -open if the family $\{A_i^c, i \in I\}$ is locally finite in (X, τ_1) .

Proof:

Let $\{A_i^c, i \in I\}$ is locally finite in (X, τ_1) and A_i is $\tau_1\tau_2$ - \tilde{g} -open in X for each $i \in I$. Then A_i^c is $\tau_1\tau_2$ - \tilde{g} -closed in X for each $i \in I$. By theorem 3.19, we have $\cup(A_i^c)$ is $\tau_1\tau_2$ - \tilde{g} -closed in X . Consequently, let $(\cap A_i)^c$ is $\tau_1\tau_2$ - \tilde{g} -closed in X . therefore $\cap A_i$ is $\tau_1\tau_2$ - \tilde{g} -open in X .

Theorem 4.10

If $\tau_2 - int(A) \subset B \subset A$ and A is $\tau_1\tau_2$ - \tilde{g} -open in X then B is also $\tau_1\tau_2$ - \tilde{g} -open in X .

Proof:

Suppose $\tau_2 - int(A) \subset B \subset A$ and A is $\tau_1\tau_2$ - \tilde{g} -open. Then $A^c \subset B^c \subset X - \tau_2 - int(A) = \tau_2 - cl(X - A) = \tau_2 - cl(A^c)$. Since A^c is $\tau_1\tau_2$ - \tilde{g} -closed set, by theorem 3.14, B^c is $\tau_1\tau_2$ - \tilde{g} -closed set. $\Rightarrow B$ is $\tau_1\tau_2$ - \tilde{g} -open in X .

Theorem 4.11

If a set A is $\tau_1\tau_2$ - \tilde{g} -closed in X , then $\tau_2 - cl(A) - A$ is $\tau_1\tau_2$ - \tilde{g} -open set.

Proof:

Suppose that A is $\tau_1\tau_2$ - \tilde{g} -closed in X . To prove $\tau_2 - cl(A) - A$ is $\tau_1\tau_2$ - \tilde{g} -open set. Let F be τ_1 -sg-closed set and $F \subseteq \tau_2 - cl(A) - A$. Since A is $\tau_1\tau_2$ - \tilde{g} -closed set in X , we have $\tau_2 - cl(A) - A$ contains no non-empty τ_1 -sg-closed set. Since $F \subseteq \tau_2 - cl(A) - A$, we have

$F = \phi \subseteq \tau_2 - int(\tau_2 - cl(A) - A)$.

Therefore $\tau_2 - cl(A) - A$ is $\tau_1\tau_2$ - \tilde{g} -open set.

Theorem 4.12

If a set A is $\tau_1\tau_2$ - \tilde{g} -open in a bitopological space (X, τ_1, τ_2) then $G = X$ whenever G is τ_1 -sg-open satisfies $\tau_2 - \text{int}(A) \cup A^c \subseteq G$.

Proof:

Suppose that A is $\tau_1\tau_2$ - \tilde{g} -open in a bitopological space (X, τ_1, τ_2) and G is τ_1 -sg-open and $\tau_2 - \text{int}(A) \cup A^c \subseteq G$. Then $G^c \subseteq [\tau_2 - \text{int}(A) \cup A^c]^c = \tau_2 - \text{cl}(A^c) - A^c$. Since G is τ_1 -sg-open, we have G^c is τ_1 -sg-closed. Since A is $\tau_1\tau_2$ - \tilde{g} -open, we have A^c is $\tau_1\tau_2$ - \tilde{g} -closed. Therefore $\tau_2 - \text{cl}(A^c) - A^c$ contains no non-empty τ_1 -sg-closed set in X . (by theorem 3.17). Consequently $G^c = \phi$. Hence $G = X$.

Example 4.13

Let $X = \{a, b, c, d\}$; $\tau_1 = \{X, \phi, \{a, b\}\}$; $\tau_2 = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$

$\tau_1\tau_2$ - \tilde{g} -open set are

$X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}$.

$A = \{c, d\}$; $A^c = \{a, b\}$

$\tau_2 - \text{int}(A) = \tau_2 - \text{int}(\{c, d\}) = \{c\}$

$\tau_2 - \text{int}(A) \cup A^c = \{c\} \cup \{a, b\} = \{a, b, c\} \subseteq X$.

X is τ_1 -sg-open but $A = \{c, d\}$ is not $\tau_1\tau_2$ - \tilde{g} -open.

Theorem 4.14

The intersection of $\tau_1\tau_2$ - \tilde{g} -open set and τ_2 -open set is always $\tau_1\tau_2$ - \tilde{g} -open set.

Proof:

Suppose that A is $\tau_1\tau_2$ - \tilde{g} -open set and B is τ_2 -open set. Since B is τ_2 -open, we have B^c is τ_2 -closed. "Every τ_2 -closed set is $\tau_1\tau_2$ - \tilde{g} -closed". Therefore B^c is $\tau_1\tau_2$ - \tilde{g} -closed. $\Rightarrow B$ is $\tau_1\tau_2$ - \tilde{g} -open. By theorem 4.6, $A \cap B$ is $\tau_1\tau_2$ - \tilde{g} -open set.

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