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A Practical Approach on Revenue Equivalence Theorem

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Abstract: ***The emergence of internet has made auctions very popular where people can request for bids via the internet from a wide range of buyers and for a large number of commodities. We aim to study the equivalences of the revenue generated for the seller by the four basic auction types - English auction, Dutch auction, First price sealed-bid and Second price sealed-bid auctions - as stated by the Revenue Equivalence Theorem. We empirically verify the results of the theorem through various experiments. Since all the benchmark conditions rarely hold together in real world, some auctions may yield better revenue than some others. We, therefore, analyse the auctions by violating some of the benchmark assumptions so as to simulate the real world scenario.***

Keywords: Revenue equivalence theorem, Types of auctions, RET, Experiments on Revenue equivalence theorem, Practical approach on Revenue equivalence theorem

1. Introduction

Auctions are mechanisms for buying or selling of goods where participants submit bids and the allocation and pricing of goods are based on specific rules set by the mechanism. They have been used as means of ex- change of goods for most of the history. Earliest evidence of using auction date back to 500 B.C. in the Babylon civilization. In the recent times, auctions are of great significance as the primary means of allocating natural resources and procurement of goods and ser- vices. Auctions conducted for the allocation of mobile spectrum, coal fields and other national resources involve huge amount of money and is of national importance. Therefore, it becomes essential to study about various types, their properties and their suitability under different environments.

In this report, we focus on the selling of a single in- divisible item based on the four basic auction types - English, Dutch, First-price sealed bid and Second- price sealed bid auctions. English auction is the most common type of auction. It is also known as open ascending price auction, where the seller puts an indivisible item for sale with a low reserve price. The seller then keeps on increasing the price until exactly one buyer remains interested in buying. The item is then sold to that buyer with the same price. The auction format used in the Indian Premier League for buying players is a very good example for English auction.

Dutch auction is another type of auction where the auctioneer puts up an item for sale with a reason- ably high initial price, usually known as asking price. If no one buys the item for that price then the auctioneer will keep on decreasing the asking price until somebody buys the item or a predetermined minimum price is reached. Here the buyer who bids first wins the auction and will pay an amount equal to his bid, which is the asking price of the auctioneer to which he agreed to buy the item.

First Price sealed bid auction is a kind of auction in which the seller will advertise the product details and invite interested buyers to submit their bids in a sealed fashion. After the interested buyers submit their sealed bids all the bids will be compared and the highest bidder will get the item awarded. The winner will pay an amount equal to his bid amount.

Second Price sealed bid auction is a kind of auction very similar to First Price sealed bid auction where the seller will advertise the product details and invite interested buyers to submit their sealed bids. Later the highest bidder will get the item awarded and he have to pay only an amount equivalent to the second highest bid. Here the bidders are always going to pay an amount less than his bid for the item.

The rest of the paper is organized as follows. Section 3 briefly discusses the revenue equivalence the- orem and proofs of various equivalences. In Section 4, we describe the results of empirical investigation conducted through simulation and real world experiments. Section 5 contains some conclusions we could observe and scope for future work.

2. Relevant Work

The paper by McAfee and McMillan[1] provides an excellent literature on various auction types and their equivalences. The paper discusses various benchmark conditions under which the theorem is valid in an intuitive fashion. The book by Vijay Krishna[2] on Auction Theory is a very good reference on the theoretical aspects of the theorem including the proofs of various equivalences. Other useful references include the Lecture notes on Revenue Equivalence Theorem by Prof. Y Narahari[3] and experiments conducted on the revenue equivalence theorem by David LuckingReiley[5]

3. Revenue Equivalence Theorem

The Revenue Equivalence theorem states that under the benchmark model, all the four basic auction for- mats yield the same average revenue to the seller. The benchmark model assumptions are

1) The bidders are risk neutral.
2) The independent-private-values assumption applies.
3) The bidders are symmetric.
4) Payment is a function of bids alone. This result was first shown by Vickery(1961).

Proof of Revenue Equivalence Theorem

The revenue equivalence between Dutch-First price auctions and English-Second price auctions is intuitive and quite straight forward. If we can also show equivalence in expected revenue between first price and second price auctions, the revenue equivalence theorem will be proved. In the remaining part of this section, we provide the proof of revenue equivalence of first price and second price auctions. This proof is based on the one presented in the Lecture notes of Prof. Narahari[3]. The Revenue Equivalence Theo- rem is proved based on the following theorem.

Theorem 1.1 Assume an auction with:

1) n risk-neutral bidders.
2) Bidder valuations lies in real interval $[\theta_1,\theta_2]$ with $\theta_1 < \theta_2$.
3) Bidder valuations are drawn from $[\theta_1,\theta_2]$ with a strictly positive density $\varphi_i(.) > 0$. Let $\varphi_i(.)$ be the cumulative distribution function.
4) Independent bidders.

Now consider a given pair of Bayesian Nash Equilibrium of two different auctions procedures that satisfy:

1) For every bidder i, for each possible realization of (θ_1, ..., θ_n), bidder i has an identical probability of getting the good in the two auctions.
2) Every bidder i has the same expected pay off in the two auctions when his valuation for the object is at its lowest possible level.

Then the two auctions generate the same expected revenue to the seller. We now show that both the first price auction and the second price auction satisfy the conditions of the theorem on revenue equivalence of two auctions. In both the auctions, the bidder with the highest valuation wins the auction. Bidders' valuations are drawn from $[\theta_1,\theta_2]$ and a bidder with valuation at the lower limit of the interval has a payoff of zero in both the auctions. Hence theorem can be applied to the equilibrium of the two auctions.

4. Experiments

The risk attitude of bidders is modelled as risk-averse, risk-neutral or risk-affine. A person who is risk-averse tend to be reluctant to play a strategy with an uncertain pay off over a strategy with more certain, but possibly lower pay off. A player is risk-averse if and only if his utility function is concave[4]. A risk-affine player behaves opposite to a risk-averse player and a risk-neutral player has equal probability of choosing the lower and higher uncertain situations. The utility function of a risk-affine player is convex while that of a risk neutral player is a straight line.

The valuations of bidders is modelled either by independent-private-values model or common-value model[1]. In independent-private-values model, each bidder observes his valuation of the object for sale and is known only to the bidder. Further, the valuations of every bidder is statistically independent of others. The auctioning of an antique where the bidders buy for their private use and not for resale is an example scenario for this model. In common-value model, each bidder is uncertain about his own valuation, probably arising out of the asymmetric nature of information available to different bidders. The valuations are affiliated and could change when the bidder comes to know about the valuations of others. The auctioning of an antique where the bidders buy for resale is an example scenario for this model.

Bidders are symmetric if they choose valuation from the same probability distribution. Experimentations were carried out in two phases. In the first phase, all the four auctions namely the English Auction, Dutch Auction, First Price Sealed Bid Auction and the Second Price Sealed bid Auctions were modelled using MATLAB. These auction models were simulated and the expected revenue was estimated. In the second phase of experiment, a practical auction environment was created in ebay.com with an iTunes Gift card as an item for sale.

Modelling of Simulation Experiments

Modelling English Auction

The English Auction was modelled as follows:

- A single indivisible item was assumed for the auction
- The auctioneer will put a reserve price. The item will not be sold if there is no interested buyer for this price
- Bidders with valuations greater than the reserve price will go to the next round during which the reserve price is incremented by a fixed increment
- The auction will stop at the point where the second last bidder drops out
- To minimize numerical boundary issues the fixed increment is made as small as possible.
- Valuations are drawn from a Normal Distribution N (μ, σ) with μ = reserve price + 10 and σ = 5.0
- Hundred experiments each with 100 players with different valuation distributions were carried out and the expected value is found out

Modelling Dutch Auction

The Dutch Auction was modelled as follows:

- A single indivisible item was assumed for the auction
- The auctioneer will put a reserve price which will be much higher than the true valuation of the item. This is understood as the seller will like to maximize his/her revenue
- If there are no interested buyers then the reserve price is reduced by a fixed amount
- The auction will stop at the point one bidder be- comes interested to buy the item
- To minimize numerical boundary issues the fixed decrement is made as small as possible.
- Valuations are drawn from a Normal Distribution N (μ, σ) with μ = reserveprice − 10 and σ = 5.0
- Hundred experiments each with 100 players with different valuation distributions were carried out and the expected value is found out

Modelling First Price Sealed Bid Auction

The first price sealed bid auction was modelled as follows:

- A single indivisible item was assumed for the auction
- The bids are derived from a Uniform Distribution, given by $b_i = ((N-1)/N)*V_i$ under risk neutrality

- The bidding is done only once
- The item is awarded to bidder who has the highest bid
- One hundred experiments are carried out and the expected value is found out.

Modelling Second Price Sealed Bid Auction

The second price sealed bid auction was modelled as follows:

- A single indivisible item was assumed for the auction.
- Valuations are drawn from a Normal Distribution N(μ,σ) with μ = 110 and σ = 5.0
- Since its a weakly dominant strategy to bid one's valuation in a second price auction, the bids are assumed to be equal to each players' valuation
- The item is awarded to bidder who has the highest bid, but he will only pay the second highest bid
- One hundred experiments are carried out and the expected value is found out

5. Simulation

The four models of auctions were simulated under the benchmark conditions. The results were collaborating with the theoretical versions which suggests that all the four form of auctions yields the same expected revenue to the seller. A comparison of the revenue between first price sealed bid and second price sealed bid auction is shown in Figure 1. As evident from the figure, the revenue obtained from both the First Price Sealed Bid Auction and Second Price Sealed Bid Auction are similar. The First price sealed bid auction is equivalent to the Dutch Auction and the Second price sealed bid auction is similar to English Auction. So a similar comparison can be made in this regard. The average revenue from all the four Auction models were found to be same under the benchmark assumptions.

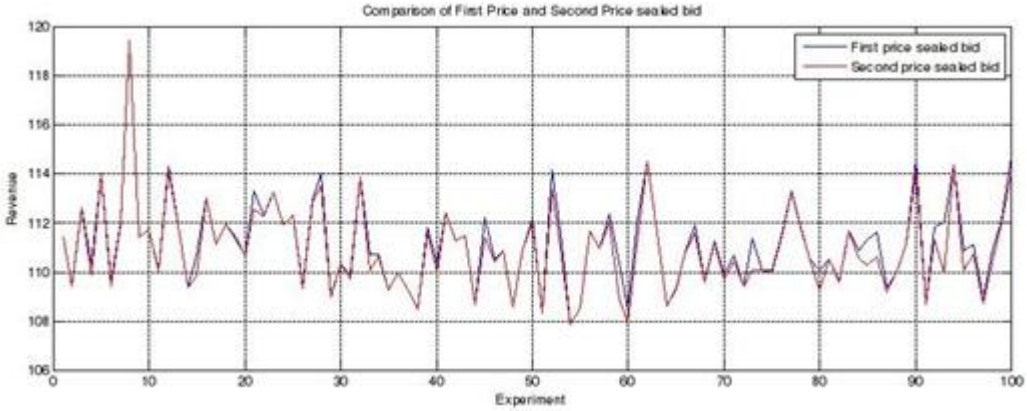

Figure 1

The behaviour of the celebrated Revenue Equiva- lence Theorem when one or more of the benchmark conditions are violated is analysed and studied in the later part of the experiment. The conditions violated are the risk neutral and Independent Private Values.

Risk Averse Bidders

When the bidders become Risk Averse they will bid more so that the probability of winning the item be- comes high. Thus under such conditions the First Price and the Dutch auctions will fetch more revenue than English or Second Price Auction. The risk averseness is modelled in First Price Sealed bid auction only as it is not applicable in a weakly dominant environment of Second Price Sealed Bid Auction. The bidding is modelled as $b_i = ((N-1)/(N-1+r))V_i$ where r isthe Risk Aversion Coefficient and it takes value between 0 and 1[6] . The result is shown in Figure 2 where the First Price Sealed Bid Auction with Risk Averse bidders is compared with Risk Neutral bidders and Second Price Sealed Bid Auction.

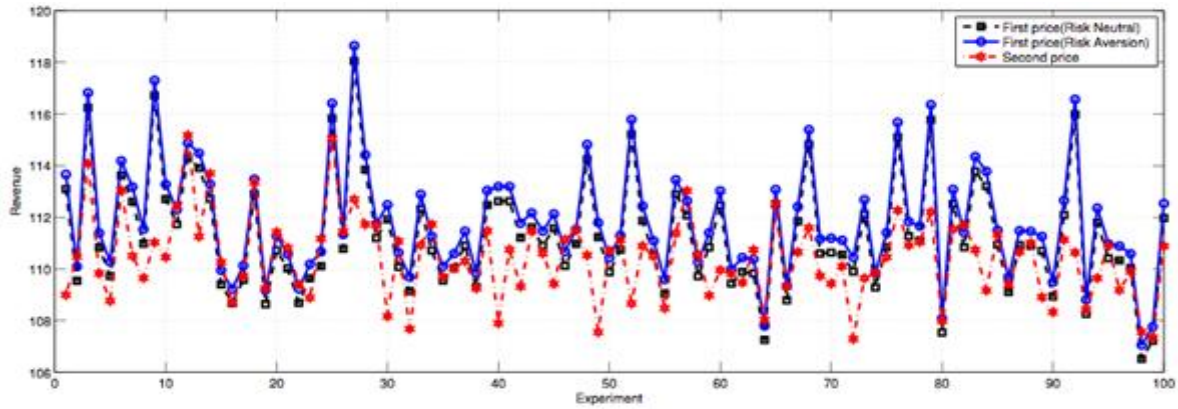

Figure 2

Benchmark Model

Affiliated Valuations

In the case of Affiliated Valuations, bidders will in- crease their valuations thinking that their counter- parts perceive the value of the item to be high. This is modelled by increasing the valuations of all losers and decreasing the valuation of the winner. A bidder who had lost in Auction 'i' will perceive that his/her valuation was much below the winning bid and in Auction 'i+1' he/she will increase the valuations.by (Δ/2).where Δ = winnervaluation − loservaluation.

The winner will reduce his valuation by a small amount to increase his utility. Since to implement this logic the valuations of the bidders need to be known English Auction was chosen for the analysis.

The result of the experiment is shown in Figure 3. Since the valuations are going up due to dependence in valuations among the bidders the average revenue obtained is much higher than that obtained in Second Price Sealed Bid Auction which has weakly dominant strategy equilibrium.

Risk Aversion

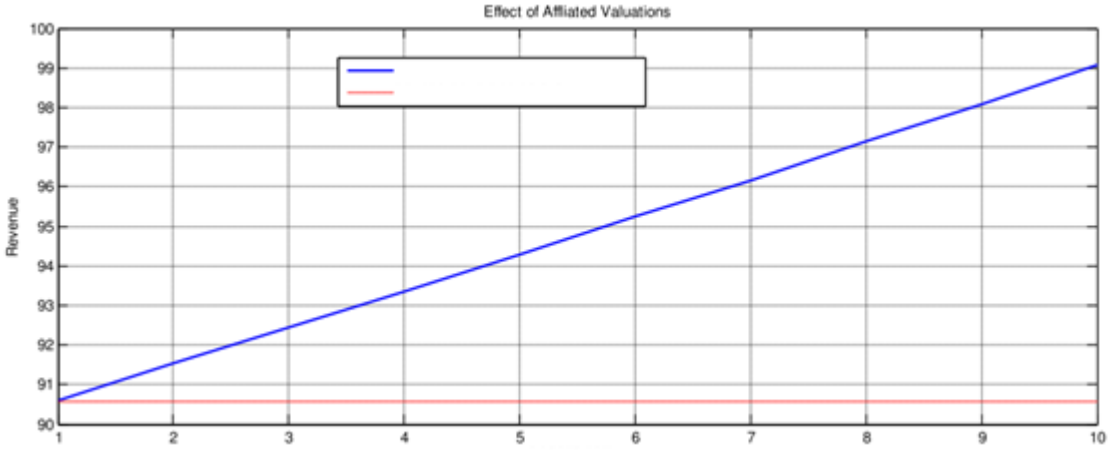

Figure 3

Summary of Simulation Experiments

Table 1 summarizes the results from the simulation experiments on Revenue Equivalence Theorem.

Table 1: Summary of Simulation Experiments	
Conditions	Auctions
Independent Private Value + Risk Neutral	D = F = S = E
Independent Private Value + Risk Averse	D = F < S = E
Affiliated Private Value + Risk Neutral	D = F < E

6. Real World Experiments

As part of the real world experimentation we auctioned same kind of product by using three basic auction formats such as English auction, First price sealed bid auction and Second price sealed bid auction. These auctions are conducted in the internet market where the interested buyers can participate in the auction. We announced each auction one after the other in order to avoid the bidders waiting for the next auction. If the buyers know the fact that the same item is coming for auction in the next week or so, they are likely to skip the current one and wait for the next one. So we conducted English auction first and we announced the details of the other auctions to all those who participated in English auction one after the other.

Modelling of Experiments

ebay.com[10] is a popular e-commerce website where the products can be put up for sale. Ebay gives the flexibility for conducting an auction for the product we want to sell. Ebay by default supports a Affiliated Valuation modified version of English auction, where the seller can keep the item for bidding with a reserve price and expiration time. At the time of expiry of auction the highest bidder will be awarded the product. During the bidding time interested buyers can keep on bidding until the auction get expired. As a part of Real world Experiment, We conducted the English auction on Ebay. As Apple iTunes gift cards have a huge demand in the market, we selected Apple iTunes gift card worth $10.00 as a product for auction. We put up the gift card for Bidding on Ebay with a reserve price of $4.99 and a validity of 7 days. During the first few days several people watched the item and book marked, but bids received during the first five days compared to the last 2 days were very low. At the end of the auction the winning bid was $12.00 which yielded a revenue of $2.00 . After the end of English Auction we contacted all the bidders and informed them about a First Price sealed Bid Auction and asked them to submit their bids to an email id before a specific time. Most of the bidders participated in First price auction, in which the bids were ranging from $2.00 to $8.00.

After that we conducted a Second Price sealed Bid auction in the similar fashion, in which the highest bid was $9.50 and the second highest was $8.80. Here English Auction dominated the other two auction formats in terms of revenue generated for the seller. It is not possible to conclude any results from this unless the experiments are repeated for a number of times.

Summary of Results

Now let us see what are all the violations of benchmark happened here in the case of real world experiment. As there are repeated auctions being conducted by several sellers for the same kind of items on Ebay, bidders usually have dependent valuations. When the bidders were bidding for relatively cheap products, they are unlikely to be highly sensitive to

minor variations in the price. Even though we cannot conclude any major results from these experiments, it is matching with the simulation experiment. During English auction bidders are able to realize that they are not going to win the auction when someone bids higher than them, So they may bid again if they really want the product. This attitude of bidders is one of the reasons behind the increased revenue from the English auction. In the case of First price sealed bid and Second price sealed bid auction bidders will know about the winning bid only after the auction completes. Under these conditions - Dependent Valuations or Affiliated valuations - English auction may perform better than First price sealed bid auction and Second price sealed bid auction in terms of generated revenue to the seller.

7. Conclusion and Future Work

First part of our work mostly included the simulation experiments to validate the equivalences of four different types of auctions, conforming to the benchmark model at large. At times, we had to put in diligent and deliberate efforts to model the agents' behaviour, strictly coinciding with the benchmark assumptions. However, it is highly unlikely that all the four conditions would simultaneously hold in a typical practical scenario. One cannot rule out the possibility of one auction turning out to be better to the seller, should some benchmark conditions be violated. In fact, in the latter part of our work, we largely focused on experiments to closely examine the deviations in revenue equivalence theorem, when the bench mark conditions don't necessarily hold together. We were able to ex- tract some interesting information and this has been elaborated in prior sections. Setting up a field study to validate the revenue equivalence under the bench mark conditions turned out to be infeasible, given the limited time frame and minimal resources. The behaviour of laymen may not always be precisely predictable and may actually turn out to be weird at times, as was evident from our Ebay experiment. Ideally, in any real world experiment to validate the revenue equivalence theorem, a sizeable number of rational agents should be engaged and the experiment itself should be repeated for a sufficiently large number of times. Albeit these stringent requirements and constraints, we made our best efforts to discover and characterize the deviations under different conditions, using carefully designed simulation experiments and techniques. Furthermore, the results obtained were religiously validated to rule out any possible error induced at hap hazard.

When too many auctions are being conducted for the same kind of item over a small period of time the players will keep on bidding low values due to the over- supply to the demand, for taking care of this issue we need a reasonable time.

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