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* Derivations in Nearrings

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Abstract: ***Let N be a non-commutative prime nearing, U be a nonzero semi group ideal of N, and D≠0, a * derivation associated with D of N such that D [x, y]- [x, y]=0 for all x, yϵU. Then F is trivial and D [x, y]+ [x, y]=0, for all x, yϵU. Then D is trivial.***

Keywords: derivations, * derivations, nearrings, associative and prime nearrings

1. Introduction

When Posner [1] proved that the existence of a nonzero centralizing derivation on a prime ring forces the ring. In view of [2], Hvala [3] introduced the concept of * derivation. Familiar examples of * derivations are derivations, * inner derivations and later includes left multipliers, that is an additive mapping d: R→R satisfy d(xy)=d(x)y* for all x, yϵR.The sum the two * derivations is a * derivation, every map of the form D(x)=cx*+d(x); where c is a fixed element of R and D a derivation of R is a * derivation ; and if R has 1.

In this paper, N will denote a zero symmetric right abelian nearring with multiplicative center Z(N), for all x, yϵN. [x, y]=xy-yx and x0y=xy+yx, denote the well-known Lie and Jordan products.

A nonempty subset U of N will be called a semi group right ideal (resp. left ideal). If UN⊂U(NU⊂U). Finally, U is called a semi group ideal if it is a right as well as a left semi group ideal. A nearring N is called a prime, if aNb={0}⇒a=0 or b=0, for all a, bϵN.

An additive mapping D:N→N is said to be a right * derivation associated with D if D(xy)=D(x)y*+xD(y), for all x, yϵN. (1) and is said to be a left * derivation associate with D if D(xy)=xD(y*)+D(y)x, for all x, yϵN. (2)

Here D is said to be * derivation associated with D. If it is a right as well as a left * derivation associated with D.

So many authors [4, 2, 3, 5] studied the commutativity in prime and semi prime rings admit with derivations and * derivations. On the other wise, many results assure that prime nearrings with certain constrained derivations have ring.

In this section we investigate some results of nearrings satisfying certain identities involving * derivation.

2. Main Theorems

Lemma 1: Let N be prime nearring and D be a * derivation on N associated with D on N, then

a(D(b)c*+bD(c))=aD(b)c*+abD(c), for all a, b, cϵN. (3)

Proof: Clearly, D(a(bc))=D(a)(bc)*+aD(bc)
=D(a)(bc)*+a(D(b)c*+bD(c))
=D(a)b*c*+a(D(b)c*+bD(c))
On the other hand,
D((ab)c)=D(ab)c*+abD(c)
=(D(a)b*+aD(b))c*+abD(c)
=D(a)b*c*+aD(b)c*+abD(c)

Comparing these two expressions for D(abc) gives the desired conclusion. ◊

Lemma 2: Let N be a prime nearring and U≠{0} a semi group ideal of N. If D is a * derivation on N such that D(U)=0 then D=0.

Proof: From the hypothesis, we obtain
0=D(ux)=D(u)x*+uD(x), ∀ uϵU, xϵN. (4)
=uD(x), ∀ uϵU, xϵN
=UD(x)
That is UD(x)=0, ∀ xϵN. (5)
⇒D(x)=0, ∀ xϵN.
⇒D=0. ◊

Lemma 3: Let N be a prime nearring and let U≠{0} a semi group ideal of N. If x b an element of N such that xU=0 0r Ux=0, then x=0.

Theorem 1: Let N be a non commutative prime nearring, U is a nonzero semi group ideal of N, and D≠0, a * derivation associated with D of N such that D [x, y]- [x, y]=0 for all x, yϵU. Then D is trivial.

Proof: From the hypothesis, we have
D [x, y]= [x, y] (6)
Substitute y with yx in (6) and using it, we get
D [x, yx]= [x, yx]
D(xy)-D(yx)=xy-yx
D(x)y*+xD(y)-D(y)x*-yD(x)=xy-yx
Replace y by yx then
D(x)y*x*+xD(yx)-D(yx)x*-yxD(x)=xy-yx
D(x)y*x*+xD(y)x*+xyD(x)-D(y)x*x*-yD(x)x*-yxD(x)=xy-yx
(D(x)y*+xD(y))x*-(D(y)x*-yD(x))x*=xy-yx
(D(xy)-D(yx))x*=xy-yx
D(xy-yx)x*+(xy-yx)D(x)=xy-yx
D [x, y]x*+(xy-yx)D(x)= [x, y]
xy D(x)=yxD(x), for all x, yϵU. (7)
Again substitute y in nz in (7) and using it, we get
[x, n]z D(x)={0}, for all x, zϵU, nϵN. (8)
x nz D(x)= nz x D(x)

(xn z-nz x)D(x)=0
(xnz-nxz)D(x)=0
(xn-nx)z D(x)=0
(x, n)z D(x)=0, for all x, zϵU, nϵN.
(x, n)U D(x)=0
That is [x, n]U D(x)={0}, for all xϵU, nϵN. (9)
Since N is prime either [x, n]=0 or D(x)=0 for all xϵU, nϵN.

Therefore by the Lemma 3, in view of hypothesis, if N be a prime nearring and let U≠{0} be a semi group ideal of N. If U⊂Z(N), then N is commutative, contradiction D(U)=0 and so D=0 by Lemma 2,
Hence, our hypothesis
D(xy)-D(yx)=xy-yx
(D(x)y*+xD(y)-D(y)x*-yD(x)=(xy-yx)
(D(x)y*-xy)=(D(y)x*-yx) (10)
Replace y* by y and x* by x
let B(x)=D(x)-x for all xϵU, and so
B(xy)=B(x)y, for all x, yϵU. Then the last equality can be written as,
B(x)y=B(y)x, for all x, yϵU. (11)
Taking zn^{l} instead of x in (11) and using Lemma 1. we find
B(z) $[y, n^{l}]=0$, for all y, zϵU, $n^{1}\epsilon N$. (12)
Substitute z with yn in the last equality, we obtain
B(y)N [y, n]=0, for all y, zϵU, n, $n^{1}\epsilon N$. (13)
It follows that
B(y)N $[y, N^{1}]=0$, for all yϵU, $n^{1}\epsilon N$. (14)
Then we conclude that, by primeness of N, that either B(y)=0 or $|y, n|$ =0 for all yϵU, nϵN, that is U⊆Z(N).
If B(y)≠0, then U⊆Z(z) implies N is commutative.
If B(y)=0, then which is contradiction.
Which complete the proof. ◊

Theorem 2: Let N be a non-commutative prime nearring, U a nonzero semigroup ideal of N and N admits a * derivation D associated with D such that D [x, y]+ [x, y]=0 for all x, yϵU. Then D is trivial.

Proof: If D=0, then we have the desired conclusion.
Now we consider D≠0,
We reach [x, y] D(x)=0 for all x, yϵU (by above Theorem)
Take yn in y in the last relation, we get [x, y]ND(x)=0 for all x, yϵN, nϵN
Since N is prime, we get the required result by hypothesis.
The similar argument can be adapted in the D [x, y]+ [x, y]=0 for all x, yϵN. ◊

Theorem 3: Let N be a non commutative prime nearring, U a nonzero semi group ideal of N, and N admits a * derivation D associated with D such that D(x0y)-(x0y)=0 for all x, yϵU. Then D is trivial.

Proof: From hypothesis, we have
d(x)y*+xd(y)+d(y)x*+yd(x)-x0y=0, for all x, yϵU. (15)
Substitute y by yx in (15), we get that
d(x)(yx)*+xd(yx)+d(yx)x*+yxd(x)-x0yx=0
d(x)y*x*+xd(y)x*+xyd(x)+d(y)x*x*+yd(x)x*+yxd(x)-x(yx)-(yx)x=0
x [d(yx)-yx]=0
xy d(y)= -yxd(x), for all x, yϵU. (16)

Substitute y by nz in (16) and using it, we reach [x, n]zD(x)=0 for all x, zϵU, nϵN
That is [x, n]UD(x)={0}, for all xϵU, nϵN. (17)
Since N is prime nearring either [x, n]=0 or D(x)=0, for all x, ϵU, nϵN.
If [x, n]=0, then N is commutative, which is contradiction to hypothesis.
Therefore D(x)=0 that implies D=0.

Theorem: let N be a non-commutative prime nearing, U a nonzero semi group ideal of N, and admit a * derivation D associated with D such that D(x0y)+x0y=0 for all x, yϵU. Then D is trivial.

Proof: For any x, yϵN,
We have D(x0y)-xoy=0
The same technique as follow in proof of Theorem 3,
We reach (x0y) D(x)=0, for all x, yϵN.
Take yz in y use in last relation, we get [x, y] z D(x)=0, for all x, yϵN.
This similar results holds in case D(x0y)+x0y=0, for all x, yϵN. ◊

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