

* Derivations in Narrings

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Abstract: Let N be a non-commutative prime narring, U be a nonzero semi group ideal of N , and $D \neq 0$, a * derivation associated with D of N such that $D[x, y] - [x, y] = 0$ for all $x, y \in U$. Then F is trivial and $D[x, y] + [x, y] = 0$, for all $x, y \in U$. Then D is trivial.

Keywords: derivations, * derivations, narrings, associative and prime narrings

1. Introduction

When Posner [1] proved that the existence of a nonzero centralizing derivation on a prime ring forces the ring. In view of [2], Hvala [3] introduced the concept of * derivation. Familiar examples of * derivations are derivations, * inner derivations and later includes left multipliers, that is an additive mapping $d: R \rightarrow R$ satisfy $d(xy) = d(x)y^*$ for all $x, y \in R$. The sum of the two * derivations is a * derivation, every map of the form $D(x) = cx^* + d(x)$; where c is a fixed element of R and D a derivation of R is a * derivation; and if R has 1.

In this paper, N will denote a zero symmetric right abelian narring with multiplicative center $Z(N)$, for all $x, y \in N$. $[x, y] = xy - yx$ and $x \circ y = xy + yx$, denote the well-known Lie and Jordan products.

A nonempty subset U of N will be called a semi group right ideal (resp. left ideal). If $UN \subset U$ ($NU \subset U$). Finally, U is called a semi group ideal if it is a right as well as a left semi group ideal. A narring N is called a prime, if $aNb = \{0\} \Rightarrow a = 0$ or $b = 0$, for all $a, b \in N$.

An additive mapping $D: N \rightarrow N$ is said to be a right * derivation associated with D if $D(xy) = D(x)y^* + xD(y)$, for all $x, y \in N$. (1) and is said to be a left * derivation associate with D if $D(xy) = xD(y^*) + D(y)x$, for all $x, y \in N$. (2)

Here D is said to be * derivation associated with D . If it is a right as well as a left * derivation associated with D .

So many authors [4, 2, 3, 5] studied the commutativity in prime and semi prime rings admit with derivations and * derivations. On the other wise, many results assure that prime narrings with certain constrained derivations have ring.

In this section we investigate some results of narrings satisfying certain identities involving * derivation.

2. Main Theorems

Lemma 1: Let N be prime narring and D be a * derivation on N associated with D on N , then

$$a(D(b)c^* + bD(c)) = aD(b)c^* + abD(c), \text{ for all } a, b, c \in N. \quad (3)$$

Proof: Clearly, $D(a(bc)) = D(a)(bc)^* + aD(bc)$
 $= D(a)(bc)^* + a(D(b)c^* + bD(c))$

$$= D(a)b^*c^* + a(D(b)c^* + bD(c))$$

On the other hand,

$$D((ab)c) = D(ab)c^* + abD(c)$$

$$= (D(a)b^* + aD(b))c^* + abD(c)$$

$$= D(a)b^*c^* + aD(b)c^* + abD(c)$$

Comparing these two expressions for $D(abc)$ gives the desired conclusion. \diamond

Lemma 2: Let N be a prime narring and $U \neq \{0\}$ a semi group ideal of N . If D is a * derivation on N such that $D(U) = 0$ then $D = 0$.

Proof: From the hypothesis, we obtain

$$0 = D(ux) = D(u)x^* + uD(x), \quad \forall u \in U, x \in N. \quad (4)$$

$$= uD(x), \quad \forall u \in U, x \in N$$

$$= UD(x)$$

$$\text{That is } UD(x) = 0, \quad \forall x \in N. \quad (5)$$

$$\Rightarrow D(x) = 0, \quad \forall x \in N.$$

$$\Rightarrow D = 0. \quad \diamond$$

Lemma 3: Let N be a prime narring and let $U \neq \{0\}$ a semi group ideal of N . If x be an element of N such that $xU = 0$ or $Ux = 0$, then $x = 0$.

Theorem 1: Let N be a non commutative prime narring, U is a nonzero semi group ideal of N , and $D \neq 0$, a * derivation associated with D of N such that $D[x, y] - [x, y] = 0$ for all $x, y \in U$. Then D is trivial.

Proof: From the hypothesis, we have

$$D[x, y] = [x, y] \quad (6)$$

Substitute y with yx in (6) and using it, we get

$$D[x, yx] = [x, yx]$$

$$D(xy) - D(yx) = xy - yx$$

$$D(x)y^* + xD(y) - D(y)x^* - yD(x) = xy - yx$$

Replace y by yx then

$$D(x)y^*x^* + xD(yx) - D(yx)x^* - yxD(x) = xy - yx$$

$$D(x)y^*x^* + xD(y)x^* + xyD(x) - D(y)x^*x^* - yD(x)x^* - yxD(x) = xy - yx$$

$$(D(x)y^* + xD(y))x^* - (D(y)x^* - yD(x))x^* = xy - yx$$

$$(D(xy) - D(yx))x^* = xy - yx$$

$$D(xy - yx)x^* + (xy - yx)D(x) = xy - yx$$

$$D[x, y]x^* + (xy - yx)D(x) = [x, y]$$

$$xyD(x) = yxD(x), \text{ for all } x, y \in U. \quad (7)$$

Again substitute y in nz in (7) and using it, we get

$$[x, n]zD(x) = \{0\}, \text{ for all } x, z \in U, n \in N. \quad (8)$$

$$xnzD(x) = nzxD(x)$$

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$$\begin{aligned} (xn - nz)x D(x) &= 0 \\ (xnz - nxz) D(x) &= 0 \\ (xn - nx)z D(x) &= 0 \\ (x, n)z D(x) &= 0, \text{ for all } x, z \in U, n \in N. \end{aligned}$$

$(x, n)U D(x) = 0$
That is $[x, n]U D(x) = \{0\}$, for all $x \in U, n \in N$. (9)
Since N is prime either $[x, n] = 0$ or $D(x) = 0$ for all $x \in U, n \in N$.

Therefore by the Lemma 3, in view of hypothesis, if N be a prime nearring and let $U \neq \{0\}$ be a semi group ideal of N . If $U \subseteq Z(N)$, then N is commutative, contradiction $D(U) = 0$ and so $D = 0$ by Lemma 2,

Hence, our hypothesis

$$\begin{aligned} D(xy) - D(yx) &= xy - yx \\ (D(x)y^* + xD(y) - D(y)x^* - yD(x)) &= (xy - yx) \\ (D(x)y^* - xy) &= (D(y)x^* - yx) \end{aligned} \quad (10)$$

Replace y^* by y and x^* by x

let $B(x) = D(x) - x$ for all $x \in U$, and so

$B(xy) = B(x)y$, for all $x, y \in U$. Then the last equality can be written as,

$$B(x)y = B(y)x, \text{ for all } x, y \in U. \quad (11)$$

Taking zn^1 instead of x in (11) and using Lemma 1. we find

$$B(z)[y, n^1] = 0, \text{ for all } y, z \in U, n^1 \in N. \quad (12)$$

Substitute z with yn in the last equality, we obtain

$$B(y)N[y, n] = 0, \text{ for all } y, z \in U, n, n^1 \in N. \quad (13)$$

It follows that

$$B(y)N[y, N^1] = 0, \text{ for all } y \in U, n^1 \in N. \quad (14)$$

Then we conclude that, by primeness of N , that either $B(y) = 0$

or $[y, n] = 0$ for all $y \in U, n \in N$, that is $U \subseteq Z(N)$.

If $B(y) \neq 0$, then $U \subseteq Z(z)$ implies N is commutative.

If $B(y) = 0$, then which is contradiction.

Which complete the proof. \diamond

Theorem 2: Let N be a non-commutative prime nearring, U a nonzero semigroup ideal of N and N admits a $*$ derivation D associated with D such that $D[x, y] + [x, y] = 0$ for all $x, y \in U$. Then D is trivial.

Proof: If $D = 0$, then we have the desired conclusion.

Now we consider $D \neq 0$,

We reach $[x, y] D(x) = 0$ for all $x, y \in U$ (by above Theorem)

Take yn in y in the last relation, we get $[x, y]ND(x) = 0$ for all $x, y \in N, n \in N$

Since N is prime, we get the required result by hypothesis.

The similar argument can be adapted in the $D[x, y] + [x, y] = 0$ for all $x, y \in N$. \diamond

Theorem 3: Let N be a non commutative prime nearring, U a nonzero semi group ideal of N , and N admits a $*$ derivation D associated with D such that $D(x0y) - (x0y) = 0$ for all $x, y \in U$. Then D is trivial.

Proof: From hypothesis, we have

$$d(x)y^* + xd(y) + d(y)x^* + yd(x) - x0y = 0, \text{ for all } x, y \in U. \quad (15)$$

Substitute y by yx in (15), we get that

$$d(x)(yx)^* + xd(yx) + d(yx)x^* + yxd(x) - x0yx = 0$$

$$d(x)y^*x^* + xd(y)x^* + xyd(x) + d(y)x^*x^* + yd(x)x^* + yxd(x) - x(yx) - (yx)x = 0$$

$$x[d(yx) - yx] = 0$$

$$xy d(y) = -yxd(x), \text{ for all } x, y \in U. \quad (16)$$

Substitute y by nz in (16) and using it, we reach $[x, n]zD(x) = 0$ for all $x, z \in U, n \in N$

That is $[x, n]UD(x) = \{0\}$, for all $x \in U, n \in N$. (17)

Since N is prime nearring either $[x, n] = 0$ or $D(x) = 0$, for all $x, e \in U, n \in N$.

If $[x, n] = 0$, then N is commutative, which is contradiction to hypothesis.

Therefore $D(x) = 0$ that implies $D = 0$.

Theorem: let N be a non-commutative prime nearring, U a nonzero semi group ideal of N , and admit a $*$ derivation D associated with D such that $D(x0y) + x0y = 0$ for all $x, y \in U$. Then D is trivial.

Proof: For any $x, y \in N$,

We have $D(x0y) - x0y = 0$

The same technique as follow in proof of Theorem 3,

We reach $(x0y) D(x) = 0$, for all $x, y \in N$.

Take yz in y use in last relation, we get $[x, y]z D(x) = 0$, for all $x, y \in N$.

This similar results holds in case $D(x0y) + x0y = 0$, for all $x, y \in N$. \diamond

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