Mathematical Modeling of the Biological Pest Control of Rice Stem Borer using Logistic Model and its Equilibrium Points

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Abstract: Many agricultural problems have important ecological dimensions. Diseases, pests and epidemics are often treated more 'ecologically' it implies more 'holistically' and less mechanistically. The models are Ecological models often use more ‘holistic’ concepts. The differential equations and difference equations are considered. In this model can be age- Structured logistical model is presented. A simple mathematical model of interaction between rice stems borer (Scirpophaga Incertulas) and its egg parasitoid Trichogramma Japanicum. In this model the rice stem borer is represented by egg and larval stages. The parasitoid is considered in terms of parasitized eggs. Linear feedback control strategy is proposed to indicate how many natural enemies introduced into the environment. Using this control strategy directs the system to the stable equilibrium point.

Keywords: Mathematical model, rice stems borer, Trichogramma Japanicum, equilibrium point

1. Introduction

The yellow stem borer (Scirpophaga Incertulas) is the most important rice pest in Tamil Nadu at the rainy season. The yellow stem borer builds internal galleries in the paddy field causing direct damages of weight loss. One of the challenges of the improvements in the farming and harvesting of rice is the biological pest control. A good strategy in biological pest control can increase the yield and also the benefits of agricultural problems. In this model applied to the problems of biological pest control of rice stem borer, the parasitoid is considered in terms of parasitized eggs. Linear feedback control strategy is proposed to indicate how many natural enemies introduced into the environment. Using this control strategy directs the system to the stable equilibrium point.

Consider two main stages of development of the rice stem borer egg and larval stages. There exists only one egg parasitoid (Trichogramma Japanicum) in a common environment. The rate at which the infestation is acquired is proportional to the number of encounters between Trichogramma Japanicum and rice stem borers. That means the net rate of parasitism is $\gamma x z$, (R.M.Aderson and R.M.May)

$x$ is the number of uninfected individuals.
$z$ the number of parasitoids egg.
$\gamma$ the rate of parasitism.

Consider the assumption for the rice stem borer and parasitoid (Trichogramma Japanicum) interaction and also using the logistic growth for egg population based on this equation the rate of change of the egg population density $N_1$ as

$$\frac{dN_1}{dt} = \beta \left(1 - \frac{N_1}{K}\right)N_1 - a_1 N_1 - b_1 N_1 - \gamma N_1 z$$ -(1)

Here $N_1$ is egg density of rice borer
$z$ the number of adult parasitoids
$\beta$ the net reproductivity rate
$K$ the carrying capacity
$a_1$ the fraction of eggs from which the larvae emerge at time $t$
$\gamma$ rate of parasitism
$b_1$ natural mortality rate of the egg population

Next stage of rice stem borer

The rate of change of the parasitized egg $N_2$

$$\frac{dN_2}{dt} = \gamma N_1 z - a_2 N_2 - b_2 N_2$$ -(2)

$a_2$ mortality rate of the parasitized egg
$b_2$ the fraction of the parasitized eggs from adult parasitoids emerge at time $t$.

Next stage of rice stem borer is emerge larvae population

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266 of 267
The rate of change of larvae population density $N_3$ 
\[
\frac{dN_3}{dt} = b_2 N_2 - a_3 N_3 - b_3 N_3 \tag{3}
\]
\(a_3\) mortality rate of larvae population which moults into pupal stage at time t.

In the adult parasitoid infects eggs of rice stem borer at the beginning of second generation life is 
\[
z = b_2 N_2 \tag{4}
\]
$\delta$ the number of parasitoids emerged from one parasitized egg($\delta=1$ for Trichogramma Japanicum)

Substitute the equation (5) in (1), (2) and (3)
\[
\begin{aligned}
\frac{dN_1}{dt} &= \beta \left(1 - \frac{N_1}{k}\right) N_1 - a_1 N_1 - b_1 N_1 - \gamma N_1 b_2 N_2 \\
\frac{dN_2}{dt} &= \gamma N_1 b_2 N_2 - a_2 N_2 - b_2 N_2 \\
\frac{dN_3}{dt} &= b_4 N_1 - a_3 N_3 - b_3 N_3 
\end{aligned} \tag{6}
\]
The above equation to use the logistical model and interaction between the rice stem borer and its parasitoid Trichogramma Japanicum.

3. Equilibrium Points

The above set of modeling equation tends to equilibrium point is right hand side of the equation (6) tends to zero.
\[
\begin{aligned}
N_1^* \left( \beta \left(1 - \frac{N_1}{k}\right) - a_1 - b_1 - \gamma b_2 N_2^* \right) &= 0 \tag{7} \\
N_2^* \left( \gamma N_1^* b_2 - a_2 - b_2 \right) &= 0 \tag{8} \\
b_4 N_1^* - N_3^* (a_3 - b_3) &= 0 \tag{9}
\end{aligned}
\]
Solve the equation (7), (8) and (9)
\[
(7) \Rightarrow N_1^* = 0 \\
(8) \Rightarrow N_2^* = 0 \\
N_1^* = 0 and N_2^* = 0 in (9) \Rightarrow N_3^* = 0 \\
N_1^* = 0, N_2^* = 0 and N_3^* = 0 is a trivial solution
\]
In equation (8) \(\Rightarrow N_2^* = 0\)
\[
N_2^* = 0 in equation (7) \Rightarrow N_1^* \left( -a_1 - b_1 + \beta \right) \frac{k}{\beta} = (\beta - a_1 - b_1) \frac{k}{\beta}
\]
\[
N_3^* = \frac{b_4 k}{\beta} \left( \beta - a_1 - b_1 \right) \frac{k}{\beta} = \frac{b_4 k}{\beta (a_3 + b_3)} (\beta - a_2 - b_2)
\]
In equation (8) \(\Rightarrow\)
\[
N_1^* = \frac{a_2 + a_3 b_2}{\gamma b_2} \quad N_2^* = \frac{1}{\gamma b_2} \left( \beta - \frac{a_2 + b_2}{\gamma b_2} \right) - a_1 - b_1 \\
N_3^* = \frac{b_1 (a_2 + b_2)}{\gamma b_2 (a_1 + b_2)}
\]
The Equilibrium points are 
\(EP_1 = (0, 0, 0)\)

\[\text{EP}_2 = \left( \frac{b_2 k}{\beta}, 0, \frac{b_2 k}{\beta} \left( \beta - a_1 - b_1 \right) \right)\]

\[\text{EP}_3 = \left( \frac{a_2 + b_2}{\gamma b_2}, 1, \frac{b_4 (a_2 + b_2)}{b_2 (a_1 + b_1)} \right)\]
The equilibrium point \(EP_1\) is a trivial point.
For \(EP_2\) the condition \(\beta > a_1 + b_1\) ensures the non-negativity of larvae population.
For \(EP_3\) the condition for non-negativity of parasitized egg population is \(\beta > a_1 + b_1 + \frac{b_2 k}{\gamma b_2} \left( \frac{a_2 + b_2}{\gamma b_2} \right)\)

4. Result and Discussions

The mathematical model for biological pest control of rice stem borer using logistical growth model there are three stages of stability analysis and its equilibrium points. The first equilibrium point is a trivial point. The second equilibrium point is non negativity of larvae population. The net reproductivity rate is more than the sum of the fraction of eggs from which the larvae emerge at time t and natural mortality rate of the egg population. The third equilibrium point is the condition for non negativity of parasitized egg population of the net reproductivity is more than the sum of the fraction of eggs from which the larvae emerge at time t and natural mortality rate of the egg population and a small positive quantity. In the linear feedback control strategy is proposed to indicate the equilibrium condition is not satisfied natural enemies introduced into the environment. Using this control strategy directs the system to the stable equilibrium point.

References


