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Lattice Controlled Hot Electron Mobility in 2DEG at Low Temperature

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Abstract: ***Hot electron mobility characteristics in non-degenerate two-dimensional electron gas (2DEG) formed in semiconductor interfaces is investigated considering simple electron temperature model at low lattice temperatures when the free electrons are dominantly scattered by the deformation potential acoustic phonons. The field dependence of electron temperature and mobility characteristics in Si(100) 2DEG are estimated and compared with other theoretical results.***

Keywords: Semiconductor, 2DEG, Phonon, Hot Electron, Mobility

1. Introduction

Over the last few decades, the transport properties of two-dimensional electron gas (2DEG) confined in a triangular well are in the focus of intense research both from the basic physics as well as technological aspects. A triangular quantum well is generally formed in semiconductor inversion and accumulation layers of metal-oxide-semiconductor field-effect-transistor (MOSFET), and in the triangular well only the quantum mechanical ground state is populated by free electrons at low lattice temperatures ($T_L < 100$ K), making the system one of the best experimental realization of a 2DEG in nature [1].

To study the behavior of the transport characteristics in 2DEG systems various theoretical models are applied and the suitability of these models depends upon the prevalent condition of lattice temperature and surface concentration of free carriers in the system. Again depending upon lattice temperature T_L and free carrier concentration N_i the transport in a 2DEG system is controlled by different type of scattering mechanisms in a particular material. The optical and intervalley phonon scattering can be important only at high temperatures when an appreciable number of corresponding phonons is excited or in the presence of a high electric field when the non-equilibrium electrons can emit high energy phonons. In conventional heterostructure-based 2DEGs the impurity and acoustic phonon scattering are dominating scattering mechanisms at low lattice temperatures ($T_L < 20$ K). In contrast to impurity scattering which can be suppressed by, e.g., dielectric engineering, scattering by acoustic phonons is intrinsic to the semiconductor and cannot be eliminated. The intrinsic mobility determined by acoustic phonon scattering alone therefore provides an important upper limit for the achievable mobility at low lattice temperatures [2-14]. The free electrons in a covalent semiconductor interact dominantly only with acoustic phonons through deformation potential. On the other, if a semiconductor consists of dissimilar atoms such as GaAs, the bonds are partly ionic and the unit cell does not contain a center of symmetry and electrons interact with acoustic phonons due to both deformation potential as well as piezoelectric coupling [6-8]. The energy of the free carrier ensemble in semiconducting system is perturbed when their drift energy becomes comparable to their thermal energy because of the presence of electric field E. In principle, the high-field energy distribution can be derived from a direct solution of the Boltzmann transport equation taking the scattering mechanism into account. But the analytical solution of the transport equation under high-field condition is usually an intractable task and as such one has to take recourse to some simplifying assumptions which, more often than not, may compromise with the physical validity of the theoretical results. However, the inter carrier collisions are frequent enough to randomize the energy and momentum of the electrons. Thus the distribution function in the momentum space can be approximated by Maxwell-Boltzmann distribution at a field-dependent effective electron temperature $T_e(E)$ which is always greater than the lattice temperature T_L. The field-dependent electron temperature can be obtained from the solution of the energy balance equation of the electron-phonon system and the solution depends on the scattering rates of the free electrons [6,7,15]. Shinba et al [16] developed a comprehensive theory to study the hot electron effect in Si(100) inversion layer at low lattice temperatures. The conductivity of electrons in Si inversion layer as a function of electric field strength was studied by Hess et al [17] at room temperature. The electric field dependence of electron temperature in 2DEG considering the equipartition law of phonon distribution has been calculated by the present author [18]. In the present article the field dependence of electron temperature and electron mobility in 2DEG at low lattice temperature has been investigated considering the full phonon distribution with the help of the effective electron temperature model. The results thus obtained are compared with the results obtained by Shinba et al [16] and the lacunas of the present theory in interpreting the actual results are discussed.

2. Theory

For mobile electrons accelerated by an applied electric field, a steady state is reached when the consumed electric power is balanced with the dissipated power. The dissipated power given to the electron system is transferred to the phonon system. When an electric field E is applied the electron gains energy at the rate of $e\mu E^2$, where e is the electronic charge and μ is the mobility of hot electrons. At low lattice temperature, a steady state may reach when the average rate of energy loss $\langle \frac{d\epsilon}{dt} \rangle_{ac}$ of the non-equilibrium electron due to

acoustic phonon scattering is equal to the rate of energy gain from the field. Thus

$$e\mu_{ac}E^2 = \langle \tfrac{d\epsilon}{dt} \rangle_{ac}. \qquad (1)$$

Once we know the values of μ_{ac} and $\langle \frac{d\epsilon}{dt} \rangle_{ac}$ in terms of electron temperature T_e in 2DEG, we may determine the electron temperature or electron mobility due to the application of electric field in semiconductor interface at low lattice temperatures.

2.1 Mobility of Hot Electrons

The high-field electron mobility μ_{ac} can be deduced in terms of electron temperature T_e in a 2DEG using a simple model valid for Maxwell-Boltzmann distribution as [7]

$$\mu_{ac} = \frac{e}{m_\mu^*} \frac{\langle \epsilon \tau_{ac} \rangle}{\langle \epsilon \rangle}, \qquad (2)$$

where m_μ^* is the mobility effective mass of the electron and $\tau_{ac}(\epsilon)$ is the momentum relaxation time due to free carrier scattering with the acoustic phonons and ϵ is the electron energy. The average value of a parameter $\mathcal{G}$ for a two-dimensional non-degenerate ensemble is given by

$$\langle \mathcal{G} \rangle = \int_0^\infty \mathcal{G} f_0(\epsilon) d\epsilon, \qquad (3)$$

where the hot electron distribution function $f_0(\epsilon)$ is given by the Maxwell-Boltzmann distribution at an effective electron temperature T_e as [7]

$$f_0(\epsilon) = \frac{N_i}{N_C^{2D}} \exp(-\epsilon/k_B T_e).$$

Here k_B is the Boltzmann constant and the effective density of state N_C^{2D} in 2DEG can be given as[19]

$$N_C^{2D} = \frac{m_\parallel^*}{\pi\hbar^2} k_B T_e,$$

where $m_\parallel^*$ is the effective mass of the electron parallel to the interface and $\hbar$ is the Dirac constant.

At low lattice temperature the electrons become hot and highly energetic under the application of even a relatively weak electric field and hence the electron energy is much higher than the phonon energy. Thus the electron-phonon collisions may be considered to be quasi-elastic. Again at low temperature the phonon distribution cannot be approximated to equipartition law and the true distribution may be expressed by the truncated Laurent's expression as [20].

$$N_q(x) = \sum_{m=0}^{\infty} \frac{B_m}{m!} x^{m-1} \quad ; \quad x \le \bar{x},$$
$$= \exp(-x) \quad ; \quad x > \bar{x}, \qquad (4)$$

where $x = \hbar q u_l / k_B T_L$, q is the magnitude of phonon wave vector and u_l is the acoustic velocity. B_m's are Bernoulli numbers and $\bar{x} < 2\pi$. For the practical purpose $\bar{x}$ may be taken to be 3.5.

Under the condition of quasi-elastic collisions and the phonon distribution given by Eq.(4) Ghorai et al [9] already reported the momentum relaxation time of free carriers due to electron-phonon scattering in a 2DEG as

$$\frac{1}{\tau_{ac}(\epsilon)} = \mathcal{A}_{ac}\, \lambda\, G(\epsilon). \qquad (5)$$

Where

$$\mathcal{A}_{ac} = \left(\frac{\mathcal{E}_a^2 m_\parallel^{*3/2}}{4\sqrt{2}\hbar^3}\right)\left(\frac{1}{\pi d \rho_v u_l}\right)\left(\frac{k_B T_L}{\sqrt{\varepsilon_s}}\right)^2,\ \varepsilon_s = \frac{1}{2} m_\parallel^* u_l^2,\ \lambda = \frac{4\sqrt{\varepsilon_s}}{k_B T_L}.$$

Here $\mathcal{E}_a$ is the effective deformation potential constant of semiconductor surface layer, the parameter d is the width of the layer of lattice atoms with which the electrons can interact and ρ_v is the mass density.

The function $G(\epsilon)$ assumes different forms in the different range of electron energy as

$$G(\epsilon) = \sum_{m=0}^{\infty} \frac{2B_m}{m!} \lambda^m [\, I_m(\pi/2) - I_m(0)]\, \epsilon^{m/2} + \lambda\sqrt{\epsilon}$$
$$= G_1(\epsilon), \qquad \text{for } \lambda\sqrt{\epsilon} \le \bar{x},$$

and

$$G(\epsilon) = \sum_{m=0}^{\infty} \frac{2B_m}{m!} \lambda^m [\, I_m(\bar{\theta}) - I_m(0)]\, \epsilon^{m/2}$$
$$+ \sum_{m=1}^{\infty} \frac{2(-1)^{m-1}}{(m-1)!} \lambda^m [\, I_m(\pi/2) - I_m(\bar{\theta})] \epsilon^{m/2} + \lambda\sqrt{\epsilon}$$
$$= G_2(\epsilon), \qquad \text{for } \lambda\sqrt{\epsilon} > \bar{x}.$$

Here

$$I_m(\theta) = -\frac{\sin^{m-1}\theta \cos\theta}{m} + \frac{m-1}{m} I_{m-2}(\theta),$$
$$I_0(\theta) = \theta, \qquad \bar{\theta} = \sin^{-1}(\bar{x}/\lambda\sqrt{\epsilon}).$$

Using Eqs.(2), (3) and (5) we get the mobility of hot electrons in a 2DEG as

$$\mu_{ac}(T_e) = \frac{e}{m_\mu^* \mathcal{A}_{ac} \lambda (k_B T_e)^2} \left[\int_0^{\epsilon_1} \frac{\epsilon}{G_1(\epsilon)} e^{-\epsilon/k_B T_e} d\epsilon \right.$$
$$\left. + \int_{\epsilon_1}^{\infty} \frac{\epsilon}{G_2(\epsilon)} e^{-\epsilon/k_B T_e} d\epsilon \right]. \qquad (6)$$

Here $\epsilon_1 = (\bar{x}/\lambda)^2$. It is obvious that the Eq.(6) is not amenable to analytical evaluation and as such it has to be evaluated numerically.

2.2 Energy Loss per Electron to Phonons

The average rate of energy loss $\langle \frac{d\epsilon}{dt} \rangle_{ac}$ of an electron due to acoustic phonon scattering in a 2DEG system can be calculated using the equation [7]

$$\langle \tfrac{d\epsilon}{dt} \rangle_{ac} = -\frac{1}{N_i s} \sum_{\vec{q}} \hbar u_l q \left(\frac{\partial N_{\vec{q}}}{\partial t}\right), \qquad (7)$$

The rate of increase in the number of phonons $\left(\frac{\partial N_{\vec{q}}}{\partial t}\right)$ can be determined by the perturbation theory. The calculation for the average rate of energy loss of electron in 2DEG as a function of T_e has already been done by the present author [13] and the result for low lattice temperature condition can be given as

$$\langle \tfrac{d\epsilon}{dt} \rangle_{ac} = \mathcal{B}_{ac} \left[\frac{\sqrt{\pi} T_e}{2p^{3/2}} + \frac{\sqrt{T_e}}{p^2} + \sum_{m=0}^{\infty} \frac{B_m}{m!} \frac{T_e^{m/2}}{p^{(m+3)/2}} \right.$$
$$\left. \times \left\{ \Gamma\left(\frac{m+3}{2}\right) - \Gamma\left(\frac{m+3}{2}, \frac{p\bar{x}^2}{T_e}\right) \right\} \right], \qquad (8)$$

where $\mathcal{B}_{ac} = \frac{\mathcal{E}_a^2 \sqrt{\varepsilon_s} k_B^{5/2} T_L^4}{8\sqrt{\pi}\hbar^3 d \rho_v u_l^4}$, $\quad p = \frac{k_B T_L^3}{16\varepsilon_s}$.

3. Result and Discussion

To study the field dependence of the electron temperature and the mobility characteristics in a 2DEG ensemble as obtained by the above theory, an n-channel (100) oriented Si inversion layer is taken with the material parameter values [16]: $\mathcal{E}_a = 12$ eV, $u_l = 9.04 \times 10^5$ cm s^{-1}, $\rho_v = 2.33$ gm cm^{-3}, ϵ_{sc}= 11.7, longitudinal effective mass $m_l^* = 0.916m_0$, transverse effective mass $m_t^* = 0.190m_0$, m_0 being the free electron mass. At low lattice temperatures one may consider presumably the electrons occupy only the lowest subband when the layer thickness d is given by $(\hbar^2\epsilon_{sc}/2m_\perp^* e^2 N_i)^{1/3}\gamma_0$. For the (100) surface of Si the six valleys are not equivalent. The two equivalent valleys for which $m_\parallel^* = m_t^*$, $m_\perp^* = m_l^*$, $m_\mu^* = m_l^*$ occupy the lower subband than the other four equivalent valleys [2,5].

The mobility of the hot electrons in terms of the electron temperature may be evaluated by Eq.(6) and as the integrals involved in the Eq.(6) are analytically intractable, however, these integrals can be calculated numerically [21] for a range of electron temperature at particular values of lattice temperature and carrier concentration. Thus using Eqs.(1), (6) and (8) we may obtain the field dependence of electron temperature and electron mobility in Si(100) surface layer at low temperature.

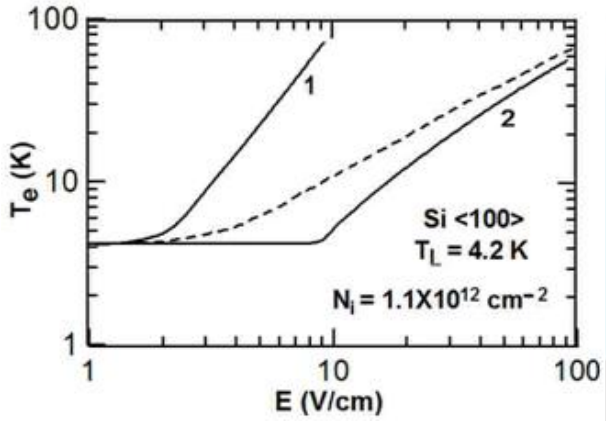

Figure 1: Dependence of electron temperature upon electric field for Si (100) 2DEG. Dashed curve is the result from Shinba et al [16]. The curve marked 1 is the result [18] due to equipartition law of the phonon distribution and the curve marked 2 is the result from the present theory.

The electron temperature in Si(100) vs electric field strength characteristics are shown in Fig.1. The corresponding dependence as obtained by Shinba et al [16] which nearly confirms the experimental data is also plotted in the same figure for a comparison. It is seen from the figure that there is huge quantitative as well as qualitative discrepancy between the result of Shinba et al and the result obtained [18] considering the equipartition law of phonon distribution. But in the present theory when the true phonon distribution is taken into account, which is rather appropriate at low lattice temperature, both quantitative as well as qualitative agreement is better and the agreement is further better at the higher field regions.

In Fig.2 the mobility characteristics of the electrons due to the application of electric field in Si(100) are shown for two different surface concentrations at lattice temperature of 4.2K. From the figure it is seen that the qualitative agreement between the results of Shinba et al and the present theory is excellent though the quantitative agreement is poor.

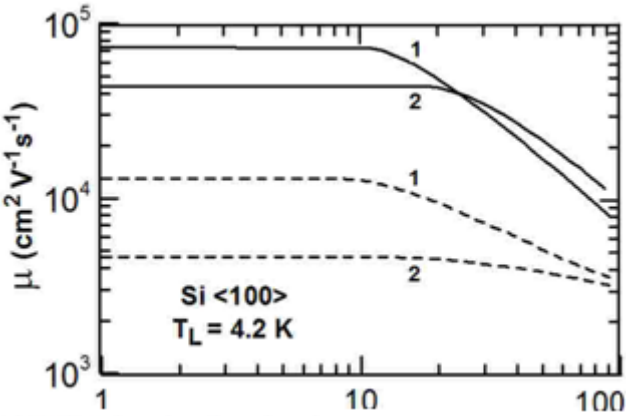

Figure 2: Electron mobility in Si(100) inversion layer vs electric field strength. Dashed curves are the results of Shinba et al [16] and the solid curves are from the present theory. The curves marked 1 and 2 are for $N_i = 1.1 \times 10^{12}$ cm^{-2} and $N_i = 5.3 \times 10^{12}$ cm^{-2} respectively.

For a ready calculation, the results of the field dependence of the effective temperature of the non-equilibrium electrons having concentration of $N_i = 1.1 \times 10^{12}$ cm^{-2} in Si(100) surface layer at $T_L = 4.2$ K as obtained by the present theory may be expressed with the help of the non-linear least-squares fit technique [21] as

$T_e = 4.2$ K, for $E \leq 8$ V/cm;
$= -2.3 + 0.75E - 0.0013E^2$ K, for $E > 8$ V/cm;

The above expression provides electron temperature values as a function of applied electric field in Si(100) surface layer at lattice temperature of 4.2 K and for surface concentration of 1.1×10^{12} cm^{-2} with an error not exceeding 1%.

The figures reveal that there is disagreement particularly in quantitative aspect between the results of the present theory and the theory of Shinba et al in the hot electron transport characteristics of 2DEG system at low lattice temperatures. Though the true phonon distribution is considered here in the electron-phonon scattering mechanism, a number of effects which are particularly dominant at low temperatures are omitted or approximated. To mention in particular, the effect of degeneracy is not considered and the effect of finite phonon energy in the energy balance equation of electron-phonon collision is approximated in the present theory. Apart from these, other scattering mechanisms which may also be dominant at low temperatures are not considered in the present theory.

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