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Properties of Operations on Total Regular Intuitionistic Fuzzy Graphs

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Abstract: ***In this paper, some properties of union and join on total regular intuitionistic fuzzy graphs are derived, and also theorems related to these concepts are stated and proved.***

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1. Introduction

In 1975, Rosenfeld [15], Yeh and Banh [17] have carried out experiments on the applications of fuzzy sets to develop the structure of Fuzzy Graphs (FGs). Intuitionistic fuzzy graph theory was introduced by K.T.Atanassov [3] in 1999. Research on the theory of Intuitionistic fuzzy sets [IFS] has been witnessing an exponential growth in mathematics and its application. M. G. Karunambigai and R.Parvathi [7] introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. A.Nagoor Gani and S.Sajitha Begum [12] defined degree, order and size in intuitionistic fuzzy graphs and extends the properties. This leads to the consideration of the operations on IFGs. In this paper, properties of union and join on total regular IFGs are presented.

2. Basic Definitions

Definition 2.1:
An *intuitionistic fuzzy graph (IFG)* is of the form $G:(V,E)$ where

(i) The function $\mu_1: V \rightarrow [0,1]$ & $\gamma_1: V \rightarrow [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively. such that $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V$, (i = 1, 2,…., n).

(ii) The function $\mu_2: V \times V \rightarrow [0,1]$ & $\gamma_2: V \times V \rightarrow [0,1]$ are defined by

$$\mu_2(v_i, v_j) \leq \mathrm{Min}[\mu_1(v_i), \mu_1(v_j)]$$
$$\gamma_2(v_i, v_j) \leq \mathrm{Max}[\gamma_1(v_i), \gamma_1(v_j)]$$

Definition 2.2:
An IFG $G = (V,E)$ is said to be *regular IFG*, if there is a vertex which is adjacent to vertices with same degrees.

Definition 2.3:
An intuitionistic fuzzy graph G is said to be *strong IFG*, if $\mu_{2ij} = \mathrm{Min}(\mu_1(v_i), \mu_1(v_j))$ and $\gamma_{2ij} = \mathrm{Max}(\gamma_1(v_i), \gamma_1(v_j))$, $\forall (v_i, v_j) \in E$.

Definition 2.4:
An IFG $G = (V,E)$ is said to be a *complete IFG* if $\mu_{2ij} = \mathrm{Min}(\mu_1(v_i), \mu_1(v_j))$ and $\gamma_{2ij} = \mathrm{Max}(\gamma_1(v_i), \gamma_1(v_j))$ for every $(v_i, v_j) \in E$.

Definition 2.5:
Let $G = \{(\mu_1, \gamma_1), (\mu_2, \gamma_2)\}$ be an IFG. Then the *degree of a vertex* v_i is defined as

$$d(v_i) = \left[\textstyle\sum_{(v_i,v_j)\in E.} \mu_2(v_i, v_j) \,;\, \textstyle\sum_{(v_i,v_j)\in E.} \gamma_2(v_i, v_j)\right].$$

Where $\mu_2(v_i, v_j) = \gamma_2(v_i, v_j) = 0$ Whenever there exists no edge between $v_i \& v_j$.

Definition 2.6:
Let $G = (V,E)$ be an IFG. If $(d_\mu(v), d_\gamma(v)) = (k_1, k_2)$ for all $v \forall V$ that is if each vertex has same membership degree k_1 and same nonmembership degree k_2 then G is said to be a *regular intuitionistic fuzzy graph*.

Definition 2.7:
Let $G = (V,E)$ be an IFG. Then the total degree of a vertex $u \in V$ is defined by,

$$td(u) = (td_\mu(u), td_\gamma(u)) = (\textstyle\sum_{u\neq v} \mu_2(u,v) + \mu_1(u) \,, \textstyle\sum_{u\neq v} \gamma_2(u,v) + \gamma 1 u),$$
$$= (d_\mu(u) + \mu_1(u), d_\gamma(u) + \gamma_1(u))$$

If each vertex of G has same membership total degree k_1 and same nonmembership total degree k_2, then G is said to be a *total regular IFG.*

Definition 2.8:
Union on Intuitionistic Fuzzy Graph:
Let $G_1:(V_1, E_1)$ and $G_2:(V_2, E_2)$ be two intuitionistic fuzzy graphs with $V_1 \cap V_2 \neq \emptyset$ and $G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$. Then the union of IFGs G_1 and G_2 is an IFG with the condition that $\gamma_2' \leq \mathrm{Min}\,(\gamma_1)$ and $\gamma_2 \leq \mathrm{Min}\,(\gamma_1')$ is defined by ,

$$(\mu_1 \cup \mu_1')(v) = \begin{cases} \mu_1(v) & , \quad \text{if } v \in V_1 \\ \mu_1'(v) & , \quad \text{if } v \in V_2 \\ \mu_1(v) \vee \mu_1'(v) & , \quad \text{if } v \in V_1 \cap V_2 \end{cases}$$

$$(\gamma_1 \cup \gamma_1')(v) = \begin{cases} \gamma_1(v) & , \quad \text{if } v \in V_1 \\ \gamma_1'(v) & , \quad \text{if } v \in V_2 \\ \gamma_1(v) \wedge \gamma_1'(v) & , \quad \text{if } v \in V_1 \cap V_2 \end{cases}$$

$$(\mu_2 \cup \mu_2')(v_iv_j) = \begin{cases} \mu_{2ij} & , \text{ if } e_{ij} \in E_1 \\ \mu_{2ij}' & , \text{ if } e_{ij} \in E_2 \\ \mu_{2ij} \vee \mu_{2ij}' & , \text{ if } e_{ij} \in E_1 \cap E_2 \end{cases}$$

$$(\gamma_2 \cup \gamma_2')(v_iv_j) = \begin{cases} \gamma_2(u_iu_j) & , \text{ if } e_{ij} \in E_1 \\ \gamma_2'(u_iu_j) & , \text{ if } e_{ij} \in E_2 \\ (\gamma_2 \wedge \gamma_2')(u_iu_j) & , \text{ if } e_{ij} \in E_1 \cap E_2 \end{cases}$$

Where (μ_1, γ_1) and (μ_1', γ_1') are the vertex membership and nonmembership of G_1 and G_2, (μ_2, γ_2) and (μ_2', γ_2') are the edge membership and nonmembership of G_1 and G_2 respectively.

Definition 2.9:
Join on Intuitionistic Fuzzy Graph:
The join of two intuitionistic fuzzy graphs G_1 and G_2 is an intuitionistic fuzzy graph, $G = G_1 + G_2 = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$ where E' is the set of all edges joining the vertices of V_1 with vertices of V_2 defined by

$$(\mu_1 + \mu_1')(v) = (\mu_1 \cup \mu_1')(v), \text{ if } v \in V_1 \cup V_2$$
$$(\gamma_1 + \gamma_1')(v) = (\gamma_1 \cup \gamma_1')(v), \text{ if } v \in V_1 \cup V_2$$
$$(\mu_2 + \mu_2')(v_iv_j) = \begin{cases} (\mu_2 \cup \mu_2')(v_iv_j), \text{ if } v_iv_j \in E_1 \cup E_2 \\ \mu_1(v_i) \wedge \mu_1'(v_j), \text{ if } v_iv_j \in E' \end{cases}$$
$$(\gamma_2 + \gamma_2')(v_iv_j) = \begin{cases} (\gamma_2 \cup \gamma_2')(v_iv_j), \text{ if } v_iv_j \in E_1 \cup E_2 \\ \gamma_1(v_i) \vee \gamma_1'(v_j), \text{ if } v_iv_j \in E' \end{cases}$$

3. Properties of Union on Total Regular Intuitionistic Fuzzy Graphs

The union of two total regular Intuitionistic fuzzy graphs need not be a total regular intuitionistic fuzzy graph.

Theorem 3.1:
Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $V_1 \cap V_2 = \emptyset$. Then $G_1 \cup G_2$ is a (k_1, k_2) total regular intuitionistic fuzzy graph if and only if G_1 and G_2 are (k_1, k_2) total regular intuitionistic fuzzy graphs.

Proof:
Since $V_1 \cap V_2 = \emptyset$.

$$td_{G_1 \cup G_2}(u) = \begin{cases} td_{G_1}(u), \text{ if } u \in V_1 \\ td_{G_2}(u), \text{ if } u \in V_2 \end{cases}$$

$$td_{G_1 \cup G_2}(u) = (k_1, k_2) \iff td_{G_1}(u) = (k_1, k_2) \text{ and } td_{G_2}(u) = (k_1, k_2)$$

Hence $G_1 \cup G_2$ is a (k_1, k_2) total regular intuitionistic fuzzy graph if and only if G_1 and G_2 are (k_1, k_2) total regular intuitionistic fuzzy graphs.

Theorem 3.2:
Let G_1 be a intuitionistic fuzzy subgraph of G_2. Then $G_1 \cup G_2$ is a (k_1, k_2) total regular intuitionistic fuzzy graph if and only if G_2 is a (k_1, k_2) total regular intuitionistic fuzzy graph.

Proof:
Let $G_1 = ((\mu_1, \gamma_1), (\mu_2, \gamma_2))$ be an intuitionistic fuzzy subgraph of $G_2 = ((\mu_1', \gamma_1'), (\mu_2', \gamma_2'))$.
Then $\mu_1 \leq \mu_1', \gamma_1 \geq \gamma_1'$ and $\mu_2 \leq \mu_2', \gamma_2 \geq \gamma_2'$

Therefore $\mu_1 \cup \mu_1' = \mu_1', \gamma_1 \cup \gamma_1' = \gamma_1'$ and $\mu_2 \cup \mu_2' = \mu_2', \gamma_2 \cup \gamma_2' = \gamma_2'$

Hence $G_1 \cup G_2 = ((\mu_1', \gamma_1'), (\mu_2', \gamma_2')) = G_2$

That is $G_1 \cup G_2$ is (k_1, k_2) total regular intuitionistic fuzzy graph if and only if G_2 is a (k_1, k_2) total regular intuitionistic fuzzy graph.

Theorem 3.3:
Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $V_1 \cap V_2 \neq \emptyset$, and none of V_1 and V_2 is a subset of the other. If $G_1 \cup G_2$ is a total regular intuitionistic fuzzy graph, then G_1 and G_2 are not total regular intuitionistic fuzzy graphs.

Proof:
Let $G_1: (V_1, E_1)$ and $G_2: (V_2, E_2)$ be two intuitionistic fuzzy graphs. (μ_1, γ_1) and (μ_1', γ_1') are the vertex membership and non membership of G_1 and G_2, (μ_2, γ_2) and (μ_2', γ_2') are the edge membership and nonmembership of G_1 and G_2.

Given $V_1 \cap V_2 \neq \emptyset$, hence the intersection of edge sets may or may not be empty.
We prove this result by the following two cases.

Case 1: $E_1 \cap E_2 = \emptyset$.
Given $G_1 \cup G_2$ is a total regular intuitionistic fuzzy graph.
(i.e) $td_{G_1 \cup G_2}(u) = (k_1, k_2)$

$$td_{G_1 \cup G_2}(u) = \begin{cases} td_{G_1}(u) & , \text{if } u \in V_1 - V_2 \\ td_{G_2}(u) & , \text{if } u \in V_2 - V_1 \\ \begin{pmatrix} td_{G_1}(u) + td_{G_2}(u) - \\ (\mu_1(u) \wedge \mu_1'(u), \gamma_1(u) \vee \gamma_1'(u)) \end{pmatrix} & , \text{if } u \in V_1 \cap V_2 \end{cases}$$

Since none of V_1 and V_2 is a subset of the other, and $V_1 \cap V_2 \neq \emptyset$, both $V_1 - V_2$ and $V_2 - V_1$ are non empty.
Let $u \in V_1 - V_2$, then $td_{G_1}(u) = (k_1, k_2)$ and if $v \in V_2 - V_1$, then $td_{G_2}(v) = (k_1, k_2)$.
Let $w \in V_1 \cap V_2$.

Then $td_{G_1}(w) + td_{G_2}(w) - (\mu_1(w) \wedge \mu_1'(w), \gamma_1(w) \vee \gamma_1'(w)) = (k_1, k_2)$
$\Rightarrow td_{G_1}(w) < (k_1, k_2)$ and $td_{G_2}(w) < (k_1, k_2)$
Therefore $td_{G_1}(u) \neq td_{G_1}(w)$ and $td_{G_2}(v) \neq td_{G_2}(w)$
Hence G_1 and G_2 are not total regular intuitionistic fuzzy graph.

Case 2: $E_1 \cap E_2 \neq \emptyset$.
There must be atleast one vertex u such that atleast one edge incident at u is not in $E_1 \cap E_2$. For otherwise we will have $V_1 = V_2$,
which is contradiction to above statement.

Let u be such a vertex.
If no edge incident at u is in $E_1 \cap E_2$, then proceed as in case 1.
If some edge incident at u lies in $E_1 \cap E_2$, then

$$td_{G_1\cup G_2}(u) = td_{G_1}(u) + td_{G_2}(u) - \left(\mu_1(u) \wedge \mu_1'(u), \gamma_1(u) \vee \gamma_1'(u)\right) - \left(\sum_{uv\in E_1\cap E_2} \mu_2(uv) \wedge \mu_2'(uv), \sum_{uv\in E_1\cap E_2} \gamma_2(uv) \vee \gamma_2'(uv)\right)$$

$$(k_1, k_2) = td_{G_1}(u) + td_{G_2}(u) - \left(\mu_1(u) \wedge \mu_1'(u), \gamma_1(u) \vee \gamma_1'(u)\right) - \left(\sum_{uv\in E_1\cap E_2} \mu_2(uv) \wedge \mu_2'(uv), \sum_{uv\in E_1\cap E_2} \gamma_2(uv) \vee \gamma_2'(uv)\right)$$

Therefore , we have $(k_1, k_2) > td_{G_1}(u)$ and $(k_1, k_2) > td_{G_2}(u)$

If $v \in V_1 - V_2$ and $w \in V_2 - V_1$, then

$td_{G_1}(v) = (k_1, k_2)$ and $td_{G_2}(w) = (k_1, k_2)$

We have $td_{G_1}(u) \neq td_{G_1}(v)$

as well as $td_{G_2}(u) \neq td_{G_2}(w)$

Hence G_1 *and* G_2 are not total regular intuitionistic fuzzy graph.

Theorem 3.4:

Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $V_1 \cap V_2 = \emptyset$. If G_1 and G_2 are total regular intuitionistic fuzzy graphs then $G_1 \cup G_2$ is not total regular intuitionistic fuzzy graph.

Proof:

Since $V_1 \cap V_2 = \emptyset$.

$$td_{G_1\cup G_2}(u) = \begin{cases} td_{G_1}(u), if\ u \in V_1 - V_2 \\ td_{G_2}(u), if\ u \in V_2 - V_1 \end{cases}$$

$td_{G_1\cup G_2}(u) = (k_1, k_2)$, if $u \in V_1 - V_2$ and

$td_{G_1\cup G_2}(u) = (k_3, k_4)$, if $u \in V_2 - V_1$

That is, $td_{G_1}(u) = (k_1, k_2)\ , \forall\ u \in V_1$ and

$td_{G_2}(u) = (k_3, k_4)\ , \forall\ u \in V_2$

Hence $G_1 \cup G_2$ is not total regular intuitionistic fuzzy graph.

Theorem 3.5:

Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $V_1 \cap V_2 \neq \emptyset$. If G_1 and G_2 are total regular intuitionistic fuzzy graphs then $G_1 \cup G_2$ is not total regular intuitionistic fuzzy graph.

Proof:

Since $V_1 \cap V_2 \neq \emptyset$.

Given G_1 *and* G_2 are total regular intuitionistic fuzzy graphs,

That is, $td_{G_1}(u_1) = (k_1, k_2)\ , \forall\ u_1 \in V_1$ and

$td_{G_2}(u_2) = (k_3, k_4)\ , \forall\ u_2 \in V_2$

We know that,

$$td_{G_1\cup G_2}(u) = \begin{cases} td_{G_1}(u) & , if\ u \in V_1 - V_2 \\ td_{G_2}(u) & , if\ u \in V_2 - V_1 \\ \left(td_{G_1}(u) + td_{G_2}(u) - (\mu_1(u) \wedge \mu_1'(u), \gamma_1(u) \vee \gamma_1'(u)) \right) & , if\ u \in V_1 \cap V_2 \end{cases}$$

$td_{G_1\cup G_2}(u) = td_{G_1}(u) = (k_1, k_2)$, if $\forall\ u \in V_1 - V_2$ and

$td_{G_1\cup G_2}(u) = td_{G_2}(u) = (k_3, k_4)$, if $\forall\ u \in V_2 - V_1$

Suppose, if there is a vertex $v \in V_1 \cap V_2$

Such that $td_{G_1\cup G_2}(v) = td_{G_1}(v) + td_{G_2}(v) - (\mu_1(v) \wedge \mu_1'(v), \gamma_1(v) \vee \gamma_1'(v))$

$= (k_1, k_2) + (k_3, k_4) - (\mu_1(v) \wedge \mu_1'(v),\ \gamma_1(v) \vee \gamma_1'(v))$

$= ((k_1 + k_3), (k_2 + k_4)) - (\mu_1(v) \wedge \mu_1'(v), \gamma_1(v) \vee \gamma_1'(v))$

Hence $td_{G_1\cup G_2}(u) \neq td_{G_1\cup G_2}(v)$.

Therefore $G_1 \cup G_2$ is not total regular intuitionistic fuzzy graph.

Theorem 3.6:

Let G_1 and G_2 be two intuitionistic fuzzy graphs. If G_1 and G_2 are (k_1, k_2) total strong regular intuitionistic fuzzy graphs then $G_1 \cup G_2$ is

1) *(k_1, k_2) total strong regular intuitionistic fuzzy graph, with $V_1 \cap V_2 = \emptyset$.*
2) *Need not be total strong regular intuitionistic fuzzy graph, with $V_1 \cap V_2 \neq \emptyset$*

Proof:

Case 1: For $V_1 \cap V_2 = \emptyset$.

We know that, from theorem 3.1, if G_1 and G_2 are (k_1, k_2) total regular intuitionistic fuzzy graphs then $G_1 \cup G_2$ is (k_1, k_2) total regular intuitionistic fuzzy graph.

To prove, if G_1 and G_2 are strong intuitionistic fuzzy graph, then $G_1 \cup G_2$ is strong intuitionistic fuzzy graph. G_1 is strong intuitionistic fuzzy graph that means for every edges of G_1 we have

(i) $\mu_2(u_i u_j) = \text{Min}\{\mu_1(u_i), \mu_1(u_j)\}$
(ii) $\gamma_2(u_i u_j) = \text{Max}\{\gamma_1(u_i), \gamma_1(u_j)\}$

Similarly G_2 is strong intuitionistic fuzzy graph, hence

(i) $\mu_2'(u_i u_j) = \text{Min}\{\mu_1'(u_i), \mu_1'(u_j)\}$
(ii) $\gamma_2'(u_i u_j) = \text{Max}\{\gamma_1'(u_i), \gamma_1'(u_j)\}$, for every $u_i u_j \in E$

$V_1 \cap V_2 = \emptyset$ implies there is no edge in common.

So $G_1 \cup G_2(e) = G_1(\text{e})$ and $G_2(\text{e})$, where e be edges.

Hence $G_1 \cup G_2(u_i u_j) = G_1(u_i u_j) = (\mu_2, \gamma_2)(u_i u_j)$ and

$G_2(u_i u_j) = (\mu_2', \gamma_2')(u_i u_j)$

Therefore $G_1 \cup G_2$ is strong intuitionistic fuzzy graph.

Hence if G_1 *and* G_2 are (k_1, k_2) total strong regular intuitionistic fuzzy graph, then $G_1 \cup G_2$ is (k_1, k_2) total strong regular intuitionistic fuzzy graph such that $V_1 \cap V_2 = \emptyset$.

Case 2: $V_1 \cap V_2 \neq \emptyset$.

As from theorem 3.5, $G_1 \cup G_2$ is not (k_1, k_2) total regular intuitionistic fuzzy graph. We know that union of two strong intuitionistic fuzzy graphs need not be strong.

Hence we conclude that, for the case $V_1 \cap V_2 \neq \emptyset$, if G_1 and G_2 are (k_1, k_2) total strong regular intuitionistic fuzzy graph, then $G_1 \cup G_2$ is need not be total strong regular intuitionistic fuzzy graph.

4. Properties of Join on Total Regular Intuitionistic Fuzzy Graphs

The join of two total regular Intuitionistic fuzzy graphs need not be a total regular intuitionistic fuzzy graph.

Theorem 4.1

Let G_1 and G_2 be two (k_1, k_2) total regular intuitionistic fuzzy graphs, with $V_1 \cap V_2 = \emptyset$. Then $G_1 + G_2$ is not total regular intuitionistic fuzzy graph.

Proof:

Given G_1 and G_2 are (k_1, k_2) total regular intuitionistic fuzzy graphs.

$td_{G_1}(u) = (k_1, k_2), \forall u \in V_1$ and $td_{G_2}(v) = (k_1, k_2), \forall v \in V_2$

To prove $td_{G_1+G_2}(u) = (k_1, k_2), \forall u \in V_1 \cup V_2$

Since $V_1 \cap V_2 = \emptyset$.

$$td_{G_1+G_2}(u) = td_{G_1 \cup G_2}(u) + \sum_{uv \in E'} [\mu_1(u) \wedge \mu_1'(v), \gamma_1(u) \vee \gamma_1'(v)]$$

For every $u_1 \in V_1$,

$$td_{G_1+G_2}(u_1) = td_{G_1}(u_1) + \sum_{u_1v_2 \in E'} [\mu_1(u_1) \wedge \mu_1'(v_2), \gamma_1(u_1) \vee \gamma_1'(v_2)]$$

$$td_{G_1+G_2}(u_1) = (k_1, k_2) + \sum_{u_1v_2 \in E'} [\mu_1(u_1) \wedge \mu_1'(v_2), \gamma_1(u_1) \vee \gamma_1'(v_2)]$$

For every vertices of V_2,

$$td_{G_1+G_2}(u_2) = td_{G_2}(u_2) + \sum_{u_2v_1 \in E'} [\mu_1'(u_2) \wedge \mu_1(v_1), \gamma_1'(u_2) \vee \gamma_1(v_1)]$$

$$= (k_1, k_2) + \sum_{u_2v_1 \in E'} [\mu_1'(u_2) \wedge$$

$\mu 1 v 1, \gamma 1'(u2) \vee \gamma 1(v1)$

Hence, $td_{G_1+G_2}(u_1) \neq td_{G_1+G_2}(u_2)$

So join of two (k_1, k_2) total regular intuitionistic fuzzy graphs is not total regular intuitionistic fuzzy graph.

Theorem 4.2:

Let G_1 and G_2 be two intuitionistic fuzzy graphs. such that $V_1 \cap V_2 = \emptyset$. If $G_1 + G_2$ is (k_1, k_2) total regular intuitionistic fuzzy graphs, then G_1 and G_2 are total regular intuitionistic fuzzy graphs.

Proof:

Since $V_1 \cap V_2 = \emptyset$.

For every $u \in V_1$,

$$td_{G_1+G_2}(u) = td_{G_1}(u) + \sum_{uv \in E'} [\mu_1(u) \wedge \mu_1'(v), \gamma_1(u) \vee \gamma_1'(v)]$$

$$(k_1, k_2) = td_{(\mu_1, \gamma_1)}(u) + \sum_{uv \in E'} [\mu_1(u) \wedge \mu_1'(v), \gamma_1(u) \vee \gamma_1'(v)]$$

$$k_1 = td_{\mu_1}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)$$

Since $G_1 + G_2$ is a regular intuitionistic fuzzy graph,

We have $\sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v) = k$,

$k_1 = td_{\mu_1}(u) + k$

$\Rightarrow td_{\mu_1}(u) = k_1 - k = k_3, \forall u \in V_1$

Similarly, $k_2 = td_{\gamma_1}(u) + \sum_{uv \in E'} \gamma_1(u) \vee \gamma_1'(v)$

$\sum_{uv \in E'} \gamma_1(u) \vee \gamma_1'(v) = C$.

Therefore $k_2 = td_{\gamma_1}(u) + C$

$td_{\gamma_1}(u) = k_2 - C = k_4, \forall u \in V_1$

Hence G_1 is (k_3, k_4) total regular intuitionistic fuzzy graph.

In a similar way, we can prove for every vertices of V_2, $td_{G_2}(v) = (k_5, k_6)$.

Hence G_2 is total regular intuitionistic fuzzy graph.

5. Conclusion

In this paper, some new properties of union and join on total regular intuitionistic fuzzy graph are discussed. It will be more useful for doing further research in the field of regular IFG.

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