

On k-Super Mean Graphs

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Abstract: Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. A graph that admits a super mean labeling is called super mean graph. Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called k -super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$. A graph that admits a k -super mean labeling is called k -super mean graph. In this paper, we investigate k -super mean labeling of (nQ_3, v_1, v_2) , TP_n , $S(P_m \times P_n)$, $(P_n \text{ A } K_1) \cup T_m$, $A(T_n)$, $C_n \ominus 2P_m$, $TL_n \odot K_1$.

Keyword: k -super mean labeling, k -super mean graph, (nQ_3, v_1, v_2) , TP_n , $S(P_m \times P_n)$, $(P_n \text{ A } K_1) \cup T_m$, $A(T_n)$, $C_n \ominus 2P_m$, $TL_n \odot K_1$

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [1]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G . In this paper, we investigate k -super mean graphs of (nQ_3, v_1, v_2) , TP_n , $S(P_m \times P_n)$, $(P_n \text{ A } K_1) \cup T_m$, $A(T_n)$, $C_n \ominus 2P_m$, $TL_n \odot K_1$.

Definition 1.1

Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **super mean labeling** if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. A graph that admits a super mean labeling is called **super mean graph**.

Definition 1.2

Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **k -super mean labeling** if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$. A graph that admits a k -super mean labeling is called **k -super mean graph**.

Definition 1.3

(G_1, G_2, v_1, v_2) is the graph obtained from G_1 and G_2 by identifying the vertices v_1 and v_2 . If $G_1 = G_2$, then (G, G, v_1, v_2) is denoted by $(2G, v_1, v_2)$.

The graph $p_2 \times p_2 \times p_2$ is called cube and is denoted by Q_3 . Q_3 is a super mean graph, then $(2Q_3, v_1, v_2)$ is a super mean graph.

Definition 1.4

A triangle C_3 can be partitioned into n number of triangles by joining one vertex C_3 to the midpoint of the opposite edges and continue this process to form n triangles and it is denoted by TP_n .

Definition 1.5

A graph obtained from grid $P_m \times P_n$ by joining opposite corners (i, j) and $(i+1, j+1)$ of each cell by an edge is denoted by $S(P_m \times P_n)$ is called strong grid.

Definition 1.6

A graph obtained a single pendant edge to each vertex of a path is called a comb $(P_n \text{ A } K_1)$.

Definition 1.7

A alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, u_3, \dots, u_n$ by joining u_i and u_{i+1} (alternatively) to a new vertex v_i for $1 \leq i \leq n-1$. That is, every edge of a path is replaced by a triangle C_3 .

Definition 1.8

Bi-armed crown $C_n \ominus 2P_m$ is a graph obtained from a cycle C_n by identifying the pendent vertices of two vertex disjoint paths of same length $m-1$ at each vertex of the cycle.

Definition 1.9

A triangular ladder TL_n is a graph obtained from L_n by adding the edges $u_i v_{i+1}$, $1 \leq i \leq n-1$ where u_i and v_i are the vertices of L_n such that $u_1, u_2, u_3, \dots, u_n$ and $v_1, v_2, v_3, \dots, v_n$ are two paths of length n in the graph.

2. Main Results

Theorem 2.1:

The graph (nQ_3, v_1, v_2) is a k -Super mean graph for all $n > 1$.

Proof:

Let $V(nQ_3, v_1, v_2) = \{v_i; 1 \leq i \leq n\} \cup \{v'_i; 1 \leq i \leq n\} \cup \{v''_i; 1 \leq i \leq n\} \cup \{v'''_i; 1 \leq i \leq n\} \cup \{u_i; 1 \leq i \leq n\} \cup \{u'_i; 1 \leq i \leq n\} \cup \{u''_i; 1 \leq i \leq n\} \cup \{u'''_i; 1 \leq i \leq n\}$

and $v_i''' = v_{i+1}$.

$E(nQ_3, v_1, v_2) = \{e_i = (v_i, v'_i); 1 \leq i \leq n\} \cup \{e'_i = (v_i, v''_i); 1 \leq i \leq n\} \cup \{e''_i = (v'_i, v''_i); 1 \leq i \leq n\} \cup \{e'''_i = (v''_i, v'''_i); 1 \leq i \leq n\} \cup \{a_i = (u_i, u'_i); 1 \leq i \leq n\} \cup \{a'_i = (u_i, u''_i); 1 \leq i \leq n\} \cup \{a''_i = (u'_i, u''_i); 1 \leq i \leq n\} \cup \{a'''_i = (u''_i, u'''_i); 1 \leq i \leq n\} \cup \{b_i = (v_i, u_i); 1 \leq i \leq n\} \cup \{b'_i = (v''_i, u_i); 1 \leq i \leq n\} \cup \{b''_i = (v'_i, u'_i); 1 \leq i \leq n\} \cup \{b'''_i = (v'''_i, u'''_i); 1 \leq i \leq n\}$

be the vertices and edges of (nQ_3, v_1, v_2) respectively.

First we label the vertices of (nQ_3, v_1, v_2) as follows:

$f(v_i) = 19i + k - 19, 1 \leq i \leq n$
 $f(v'_i) = 19i + k - 17, 1 \leq i \leq n$
 $f(v''_i) = 19i + k - 2, 1 \leq i \leq n$
 $f(v'''_i) = 19i + k, 1 \leq i \leq n$
 $f(u_i) = 19i + k - 9, 1 \leq i \leq n$
 $f(u'_i) = 19i + k - 15, 1 \leq i \leq n$
 $f(u''_i) = 19i + k - 4, 1 \leq i \leq n$
 $f(u'''_i) = 19i + k - 11, 1 \leq i \leq n$

Now the induced edge labels are

$f^*(e_i) = 19i + k - 10, 1 \leq i \leq n$
 $f^*(e'_i) = 19i + k - 18, 1 \leq i \leq n$
 $f^*(e''_i) = 19i + k - 8, 1 \leq i \leq n$
 $f^*(e'''_i) = 19i + k - 1, 1 \leq i \leq n$
 $f^*(a_i) = 19i + k - 6, 1 \leq i \leq n$
 $f^*(a'_i) = 19i + k - 12, 1 \leq i \leq n$
 $f^*(a''_i) = 19i + k - 13, 1 \leq i \leq n$
 $f^*(a'''_i) = 19i + k - 7, 1 \leq i \leq n$
 $f^*(b_i) = 19i + k - 14, 1 \leq i \leq n$
 $f^*(b'_i) = 19i + k - 3, 1 \leq i \leq n$
 $f^*(b''_i) = 19i + k - 16, 1 \leq i \leq n$
 $f^*(b'''_i) = 19i + k - 5, 1 \leq i \leq n$

Here $p = 7n + 1, q = 12n, p + q = 19n + 1$

Clearly,

$f(V) \cup \{f^*(e) : e \in E(nQ_3, v_1, v_2)\} = \{k, k + 1, \dots, 19n + k\}$

So, $f(V) \cup \{f^*(e) : e \in E(nQ_3, v_1, v_2)\}$ is a k-Super mean labeling.

Hence the graph (nQ_3, v_1, v_2) is a k-Super mean graph.

Example 2.1:

315 – Super mean labeling of $(2Q_3, v_1, v_2)$ is given in figure 2.1

Theorem 2.2:

The graph TP_n is a super mean graph

Proof:

Let $V(TP_n) = \{v_i; 1 \leq i \leq n - 1\} \cup \{u_i; 1 \leq i \leq n\}$ and

$E(TP_n) = \{e_i = (v_i, u_i); 1 \leq i \leq n - 1\} \cup \{e'_i = (v_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{a_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{b_i = (v_i, v_{i+1}); 1 \leq i \leq n - 2\}$

be the vertices and edges of TP_n respectively.

First we label the vertices of TP_n as follows:

$f(v_i) = 6i + k - 4, 1 \leq i \leq n - 1$
 $f(u_i) = 6i + k - 6, 1 \leq i \leq n - 1$
 $f(u_n) = 6i + k - 7, i = n$

Now the induced edge labels are

$f^*(e_i) = 6i + k - 5, 1 \leq i \leq n - 1$
 $f^*(e'_i) = 6i + k - 2, 1 \leq i \leq n - 1$
 $f^*(a_i) = 6i + k - 3, 1 \leq i \leq n - 1$
 $f^*(b_i) = 6i + k - 1, 1 \leq i \leq n - 2$

Here $p = 2n - 1, q = 4n - 5, p + q = 6n - 6$

Clearly,

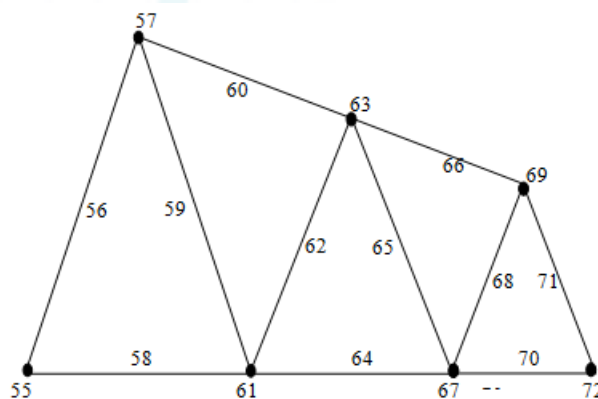
$f(V) \cup \{f^*(e) : e \in E(TP_n)\} = \{k, k + 1, \dots, 6n + k - 7\}$

So, $f(V) \cup \{f^*(e) : e \in E(TP_n)\}$ is a k-Super mean labeling.

Hence the graph TP_n is a k-Super mean graph.

Example 2.2:

55 – Super mean labeling of TP_4 is given in figure 2.2



Theorem 2.3:

The graph $S(P_m \times P_n)$ is a super mean graph.

Proof:

Let $V(S(P_m \times P_n)) = \{u_{ij}; 1 \leq i \leq m, 1 \leq j \leq n\}$ and

$E(S(P_m \times P_n)) = \{e_{ij} = (u_{ij}, u_{i(j+1)}); 1 \leq i \leq m, 1 \leq j \leq n - 1\} \cup \{a_{ij} = (u_{ij}, u_{(i+1)j}); 1 \leq i \leq m - 1, 1 \leq j \leq n\} \cup \{b_{ij} = (u_{ij}, u_{(i+1)(j+1)}); 1 \leq i \leq m - 1, 1 \leq j \leq n - 1\}$

be the vertices and edges of $S(P_m \times P_n)$ respectively.

First we label the vertices of $(P_m \times P_n)$ as follows:

$$f(u_{ij}) = (4n - 2)(i - 1) + 2j + k - 2, \quad 1 \leq i \leq m, 1 \leq j \leq n$$

Now the induced edge labels are

$$f^*(e_{ij}) = (4n - 2)(i - 1) + 2j + k - 1, \quad 1 \leq i \leq m, 1 \leq j \leq n - 1$$

$$f^*(a_{ij}) = (4n - 2)(i - 1) + (2n + j) + j + k - 3, \quad 1 \leq i \leq m - 1, 1 \leq j \leq n$$

$$f^*(b_{ij}) = (4n - 2)(i - 1) + 2n + 2j + k - 2, \quad 1 \leq i \leq m - 1, 1 \leq j \leq n - 1$$

Here $p = mn, q = 3mn - 2m - 2n + 1,$

$$p + q = 4mn - 2m - 2n + 1$$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(S(P_m \times P_n))\} = \{k, k + 1, \dots, 4mn - 2m - 2n + k\}$$

So, $f(V) \cup \{f^*(e) : e \in E(S(P_m \times P_n))\}$ is a k-Super mean labeling.

Hence the graph $(P_m \times P_n)$ is a k-Super mean graph.

Example 2.3:

545 – Super mean labeling of $(P_4 \times P_4)$ is given in figure 2.3

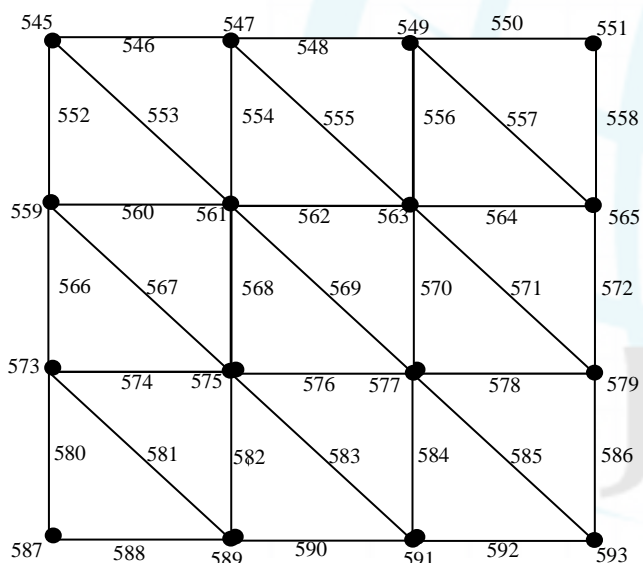


Figure 2.3: 545-Super mean labeling of $S(P_4 \times P_4)$

Theorem 2.4:

The graph $(P_n \times K_1) \cup T_m$ is a k-Super mean graph.

Proof:

$$\begin{aligned} \text{Let } V((P_n \times K_1) \cup T_m) = & \{v_i; 1 \leq i \leq n\} \cup \\ & \{u_i; 1 \leq i \leq n\} \cup \\ & \{w_i; 1 \leq i \leq m - 1\} \cup \\ & \{w'_i; 1 \leq i \leq m\} \end{aligned}$$

and

$$\begin{aligned} E((P_n \times K_1) \cup T_m) = & \{e_i = (v_i, u_i); 1 \leq i \leq n\} \cup \\ & \{e'_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \\ & \{a_i = (w_i, w'_i); 1 \leq i \leq m - 1\} \cup \\ & \{b_i = (w_i, w'_{i+1}); 1 \leq i \leq m - 1\} \cup \end{aligned}$$

$$\{c_i = (w'_i, w'_{i+1}); 1 \leq i \leq m - 1\}$$

be the vertices and edges of $(P_n \times K_1) \cup T_m$ respectively.

First we label the vertices of $(P_n \times K_1) \cup T_m$ as follows:

$$f(v_i) = 4i + k - 4, \quad 1 \leq i \leq n, \quad i \text{ is odd}$$

$$f(v_i) = 4i + k - 2, \quad 2 \leq i \leq n, \quad i \text{ is even}$$

$$f(u_i) = 4i + k - 2, \quad 1 \leq i \leq n, \quad i \text{ is odd}$$

$$f(u_i) = 4i + k - 4, \quad 2 \leq i \leq n, \quad i \text{ is even}$$

$$f(w_1) = 4n + k - 1$$

$$f(w_i) = 4n + 5i + k - 4, \quad 2 \leq i \leq m - 1$$

$$f(w'_1) = 4n + k + 1$$

$$f(w'_i) = 4n + 5i + k - 6, \quad 2 \leq i \leq m$$

Now the induced edge labels are

$$f^*(e_i) = 4i + k - 3, \quad 1 \leq i \leq n$$

$$f^*(e'_i) = 4i + k - 1, \quad 1 \leq i \leq n - 1$$

$$f^*(a_i) = 4n + 5i + k - 5, \quad 1 \leq i \leq m - 1$$

$$f^*(b_1) = 4n + k + 2$$

$$f^*(b_i) = 4n + 5i + k - 2, \quad 2 \leq i \leq m - 1$$

$$f^*(c_1) = 4n + k + 3$$

$$f^*(c_i) = 4n + 5i + k - 3, \quad 2 \leq i \leq m - 1$$

Here $p = 2n + 2m - 1, q = 2n + 3m - 4,$

$$p + q = 4n + 5m - 5$$

Clearly,

$$\begin{aligned} f(V) \cup \{f^*(e) : e \in E((P_n \times K_1) \cup T_m)\} \\ = \{k, k + 1, \dots, 4n + 5m + k - 6\} \end{aligned}$$

So, $f(V) \cup \{f^*(e) : e \in E((P_n \times K_1) \cup T_m)\}$ is a k-Super mean labeling.

Hence the graph $(P_n \times K_1) \cup T_m$ is a k-Super mean graph.

Example 2.4:

101 – Super mean labeling of $(P_4 \times K_1) \cup T_3$ is given in figure 2.4

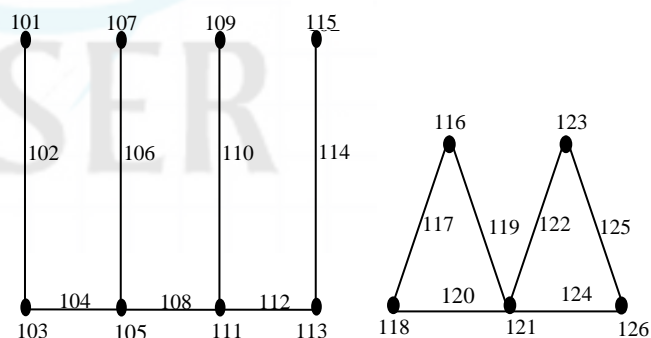


Figure 2.4: 101-Super mean labeling of $(P_4 \times K_1) \cup T_3$

Theorem 2.5:

Alternate triangular snakes $A(T_n)$ is a k-Super mean graphs.

Proof:

We consider two different cases.

Case (i):

If the alternate triangular snake $A(T_n)$ starts from $u_1,$ then we need to consider two subcases.

Subcase (i) (a): n is even

$$\text{Let } V(A(T_n)) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq \frac{n}{2}\}$$

$$\text{and } E(A(T_n)) = \{e_i = (u_{2i-1}, u_{2i}); 1 \leq i \leq \frac{n}{2}\} \cup$$

$$\{a_i = (v_i, u_{2i-1}); 1 \leq i \leq \frac{n}{2}\} \cup$$

$$\{b_i = (v_i, u_{2i}); 1 \leq i \leq \frac{n}{2}\} \cup$$

$$\{c_i = (u_{2i}, u_{2i+1}); 1 \leq i \leq (\frac{n-2}{2})\}$$

be the vertices and edges of $A(T_n)$ respectively.

First we label the vertices of $A(T_n)$ as follows:

$$f(u_{2i-1}) = 7i + k - 7, 1 \leq i \leq \frac{n}{2}$$

$$f(u_{2i}) = 7i + k - 2, 1 \leq i \leq \frac{n}{2}$$

$$f(v_i) = 7i + k - 5, 1 \leq i \leq \frac{n}{2}$$

Now the induced edge labels are

$$f^*(e_i) = 7i + k - 4, 1 \leq i \leq \frac{n}{2}$$

$$f^*(a_i) = 7i + k - 6, 1 \leq i \leq \frac{n}{2}$$

$$f^*(b_i) = 7i + k - 3, 1 \leq i \leq \frac{n}{2}$$

$$f^*(c_i) = 7i + k - 1, 1 \leq i \leq (\frac{n-2}{2})$$

Here $p = (\frac{3n}{2}), q = (\frac{4n-2}{2}), p + q = (\frac{7n-2}{2})$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(A(T_n))\} =$$

$$\{k, k + 1, \dots, (\frac{7n-2}{2}) + k - 1\}$$

So, $f(V) \cup \{f^*(e) : e \in E(A(T_n))\}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.5:

21 – Super mean labeling of $A(T_6)$ is given in figure 2.5

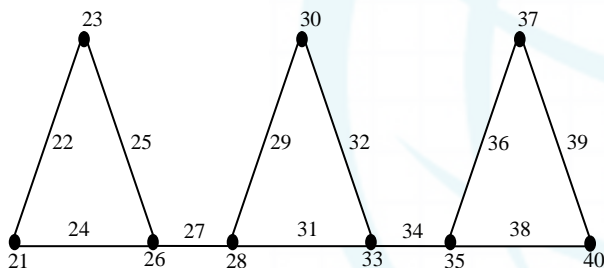


Figure 2.5: 21-Super mean labeling of $A(T_6)$

Subcase (i) (b): n is odd

Let $V(A(T_n)) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq (\frac{n-1}{2})\}$

and $E(A(T_n)) = \{e_i = (u_{2i-1}, u_{2i}); 1 \leq i \leq (\frac{n-1}{2})\} \cup$

$$\{a_i = (v_i, u_{2i-1}); 1 \leq i \leq (\frac{n-1}{2})\} \cup$$

$$\{b_i = (v_i, u_{2i}); 1 \leq i \leq (\frac{n-1}{2})\} \cup$$

$$\{c_i = (u_{2i}, u_{2i+1}); 1 \leq i \leq (\frac{n-1}{2})\}$$

be the vertices and edges of $A(T_n)$ respectively.

First we label the vertices of $A(T_n)$ as follows:

$$f(u_{2i-1}) = 7i + k - 7, 1 \leq i \leq (\frac{n-1}{2}) + 1$$

$$f(u_{2i}) = 7i + k - 2, 1 \leq i \leq (\frac{n-1}{2})$$

$$f(v_i) = 7i + k - 5, 1 \leq i \leq (\frac{n-1}{2})$$

Now the induced edge labels are

$$f^*(e_i) = 7i + k - 4, 1 \leq i \leq (\frac{n-1}{2})$$

$$f^*(a_i) = 7i + k - 6, 1 \leq i \leq (\frac{n-1}{2})$$

$$f^*(b_i) = 7i + k - 3, 1 \leq i \leq (\frac{n-1}{2})$$

$$f^*(c_i) = 7i + k - 1, 1 \leq i \leq (\frac{n-1}{2})$$

Here $p = (\frac{3n-1}{2}), q = 4(\frac{n-1}{2}), p + q = (\frac{7n-5}{2})$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(A(T_n))\} =$$

$$\{k, k + 1, \dots, (\frac{7n-5}{2}) + k - 1\}$$

So, $f(V) \cup \{f^*(e) : e \in E(A(T_n))\}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.6:

75 – Super mean labeling of $A(T_7)$ is given in figure 2.6

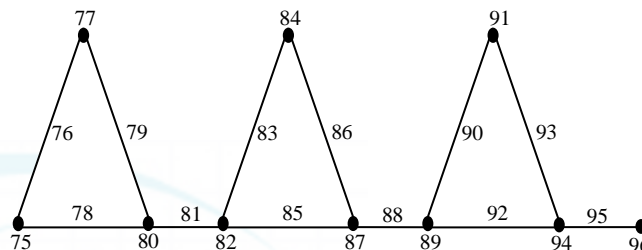


Figure 2.6: 75-Super mean labeling of $A(T_7)$

Subcase (ii) (a): n is even

Let $V(A(T_n)) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq (\frac{n-2}{2})\}$

and $E(A(T_n)) = \{e_i = (u_{2i}, u_{2i+1}); 1 \leq i \leq (\frac{n-2}{2})\} \cup$

$$\{a_i = (v_i, u_{2i}); 1 \leq i \leq (\frac{n-2}{2})\} \cup$$

$$\{b_i = (v_i, u_{2i+1}); 1 \leq i \leq (\frac{n-2}{2})\} \cup$$

$$\{c_i = (u_{2i}, u_{2i-1}); 1 \leq i \leq \frac{n}{2}\}$$

be the vertices and edges of $A(T_n)$ respectively.

First we label the vertices of $A(T_n)$ as follows:

$$f(u_{2i-1}) = 7i + k - 7, 1 \leq i \leq \frac{n}{2}$$

$$f(u_{2i}) = 7i + k - 5, 1 \leq i \leq \frac{n}{2}$$

$$f(v_i) = 7i + k - 3, 1 \leq i \leq (\frac{n-2}{2})$$

Now the induced edge labels are

$$f^*(e_i) = 7i + k - 2, 1 \leq i \leq (\frac{n-2}{2})$$

$$f^*(a_i) = 7i + k - 4, 1 \leq i \leq (\frac{n-2}{2})$$

$$f^*(b_i) = 7i + k - 1, 1 \leq i \leq (\frac{n-2}{2})$$

$$f^*(c_i) = 7i + k - 6, 1 \leq i \leq \frac{n}{2}$$

Here $p = (\frac{3n-2}{2}), q = (\frac{4n-6}{2}), p + q = (\frac{7n-8}{2})$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(A(T_n))\} =$$

$$\{k, k + 1, \dots, (\frac{7n-8}{2}) + k - 1\}$$

So, $f(V) \cup \{f^*(e) : e \in E(A(T_n))\}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.7:

56– Super mean labeling of $A(T_8)$ is given in figure 2.7

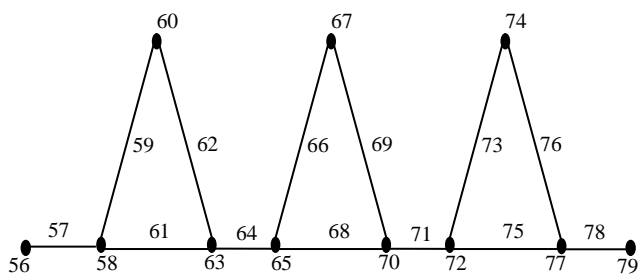


Figure 2.7: 56-Super mean labeling of $A(T_8)$

Subcase (ii) (b): n is odd

Let $V(A(T_n)) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq (\frac{n-1}{2})\}$

and $E(A(T_n)) = \{e_i = (u_{2i}, u_{2i+1}); 1 \leq i \leq (\frac{n-1}{2})\} \cup$

$\{a_i = (v_i, u_{2i}); 1 \leq i \leq (\frac{n-1}{2})\} \cup$

$\{b_i = (v_i, u_{2i+1}); 1 \leq i \leq (\frac{n-1}{2})\} \cup$

$\{c_i = (u_{2i}, u_{2i-1}); 1 \leq i \leq (\frac{n-1}{2})\}$

be the vertices and edges of $A(T_n)$ respectively.

First we label the vertices of $A(T_n)$ as follows:

$$f(u_{2i-1}) = 7i + k - 7, \quad 1 \leq i \leq (\frac{n-1}{2}) + 1$$

$$f(u_{2i}) = 7i + k - 5, \quad 1 \leq i \leq (\frac{n-1}{2})$$

$$f(v_i) = 7i + k - 3, \quad 1 \leq i \leq (\frac{n-1}{2})$$

Now the induced edge labels are

$$f^*(e_i) = 7i + k - 2, \quad 1 \leq i \leq (\frac{n-1}{2})$$

$$f^*(a_i) = 7i + k - 4, \quad 1 \leq i \leq (\frac{n-1}{2})$$

$$f^*(b_i) = 7i + k - 1, \quad 1 \leq i \leq (\frac{n-1}{2})$$

$$f^*(c_i) = 7i + k - 6, \quad 1 \leq i \leq (\frac{n-1}{2})$$

Here $p = (\frac{3n-1}{2}), q = 4(\frac{n-1}{2}), p + q = (\frac{7n-5}{2})$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(A(T_n))\} = \{k, k + 1, \dots, (\frac{7n-5}{2}) + k - 1\}$$

So, $f(V) \cup \{f^*(e) : e \in E(A(T_n))\}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.8:

98 – Super mean labeling of $A(T_7)$ is given in figure 2.8

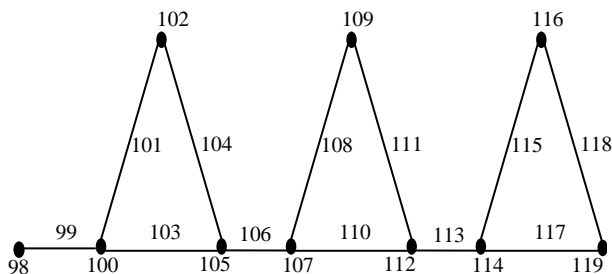


Figure 2.8: 98-Super mean labeling of $A(T_7)$

Theorem 2.6:

The bi-armed crown $C_n \odot 2P_m$ is a k-Super mean graph for all odd $n \geq 3$ and $m \geq 2$.

Proof:

$$\text{Let } V(C_n \odot 2P_m) = \{u_i; 1 \leq i \leq n\} \cup \{v_{i1}^w; 1 \leq i \leq n, 1 \leq w \leq m\} \cup \{v_{i2}^w; 1 \leq i \leq n, 1 \leq w \leq m\}$$

$$\text{and } v_{i1}^m = v_{i2}^m = u_i$$

$$E(C_n \odot 2P_m) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup$$

$$\{e_{i1}^w = (v_{i1}^w, v_{i1}^{w+1}); 1 \leq i \leq n, 1 \leq w \leq m - 1\} \cup$$

$$\{e_{i2}^w = (v_{i2}^w, v_{i2}^{w+1}); 1 \leq i \leq n, 1 \leq w \leq m - 1\}$$

$$\text{and } e_n = (u_n, u_1)$$

be the vertices and edges of $C_n \odot 2P_m$ respectively.

First we label the vertices of $C_n \odot 2P_m$ as follows:

Let $n = 2t + 1$ for some t .

$$f(v_{j1}^i) = 4(j - 1)m - 2j + 2i + k, \quad 1 \leq j \leq t + 1, 1 \leq i \leq m$$

$$f(v_{j2}^{m+1-i}) = 2(2j - 1)m - 2j + 2i + k - 2, \quad 1 \leq j \leq t, 2 \leq i \leq m$$

$$f(v_{(t+1)2}^{m-1}) = 2(2t + 1)m - (2t + 2) + k + 3$$

$$f(v_{(t+1)2}^{m-1-i}) = 2(2t + 1)m - 2t + 2i + k + 1, \quad 1 \leq i \leq m - 2$$

$$f(v_{(t+1+j)1}^i) = 4(j + t)m - 2(t + j) + 2i + k - 1, \quad 1 \leq j \leq t, 1 \leq i \leq m$$

$$f(v_{(t+1+j)2}^{m+1-i}) = (4j + 4t + 2)m - (t + 2j + 2i) + 4i - 2t + k, \quad 1 \leq j \leq t, 2 \leq i \leq m$$

Now the induced edge labels are

$$f^*(e_{j1}^i) = 4(j - 1)m - 2j + 2i + k + 1, \quad 1 \leq j \leq t + 1, 1 \leq i \leq m - 1$$

$$f^*(e_{j2}^{m-i}) = 2(2j - 1)m - 2j + 2i + k - 1, \quad 1 \leq j \leq t, 1 \leq i \leq m - 1$$

$$f^*(e_{(t+1)2}^{m-1}) = 2(2t + 1)m + k - 6$$

$$f^*(e_{(t+1)2}^{m-1-i}) = 2(2t + 1)m - 2t + 2i + k, \quad 1 \leq i \leq m - 2$$

$$f^*(e_{(t+1+j)1}^i) = 4(j + t)m - 2(t + j) + 2i + k, \quad 1 \leq j \leq t, 1 \leq i \leq m - 1$$

$$f^*(e_{(t+1+j)2}^{m+1-i}) = (4j + 4t + 2)m - (t + 2j + 2i) + 4i - 2t + k - 1, \quad 1 \leq j \leq t, 2 \leq i \leq m$$

$$f^*(e_i) = f(v_{i2}^1) - [f(v_{i1}^1) - 1] + k, \quad 1 \leq i \leq \frac{n+(n-2)}{2}$$

$$f^*(e_i) = f(v_{n1}^m) - [f(v_{i1}^1) - 1] - (\frac{i-1}{2}) [f^*(e_1) - k + 1] + k - 1, \quad i = m$$

Here $p = 2mn - n, q = 2mn - n, p + q = 4mn - 2n$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(C_n \odot 2P_m)\} = \{k, k + 1, \dots, 4mn - 2n + k - 1\}$$

So, $f(V) \cup \{f^*(e) : e \in E(C_n \odot 2P_m)\}$ is a k-Super mean labeling.

Hence the graph $C_n \odot 2P_m$ is a k-Super mean graph.

Example 2.9:

94 – Super mean labeling of $C_7 \odot 2P_3$ is given in figure 2.9

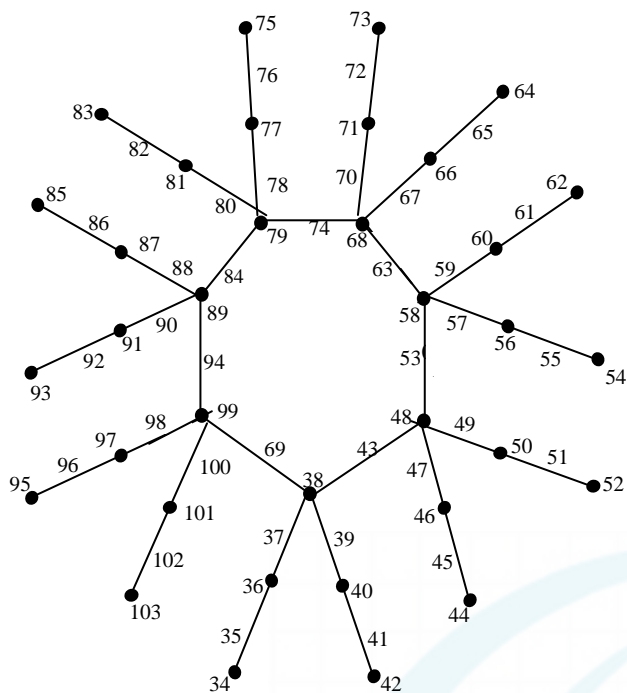


Figure 2.9: 34-Super mean labeling of $C_7 \ominus 2P_3$

Theorem 2.7:

A graph $TL_n \odot K_1$ is a super mean graph, for every n .

Proof:

Let $V(TL_n \odot K_1) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq n\} \cup$

$$\{w_i; 1 \leq i \leq n\} \cup \{z_i; 1 \leq i \leq n\}$$

and

$$E(TL_n \odot K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup \{e'_i = (u_i, v_i); 1 \leq i \leq n\} \cup \{e''_i = (v_i, v_{i+1}); 1 \leq i \leq n-1\} \cup \{e'''_i = (u_i, v_{i+1}); 1 \leq i \leq n-1\} \cup$$

$$\{a_i = (v_i, z_i); 1 \leq i \leq n\} \cup$$

$$\{a'_i = (w_i, u_i); 1 \leq i \leq n\}$$

be the vertices and edges of $TL_n \odot K_1$ respectively.

First we label the vertices of $TL_n \odot K_1$ as follows:

$$f(u_i) = 10i + k - 8, 1 \leq i \leq n$$

$$f(v_i) = 10i + k - 6, 1 \leq i \leq n$$

$$f(w_i) = 10i + k - 10, 1 \leq i \leq n$$

$$f(z_i) = 10i + k - 4, 1 \leq i \leq n$$

Now the induced edge labels are

$$f^*(e_i) = 10i + k - 3, 1 \leq i \leq n - 1$$

$$f^*(e'_i) = 10i + k - 7, 1 \leq i \leq n$$

$$f^*(e''_i) = 10i, 1 \leq i \leq n - 1$$

$$f^*(e'''_i) = 10i + k - 2, 1 \leq i \leq n - 1$$

$$f^*(a_i) = 10i + k - 5, 1 \leq i \leq n$$

$$f^*(a'_i) = 10i + k - 9, 1 \leq i \leq n$$

$$\text{Here } p = 4n, q = 3n + 3(n - 1),$$

$$p + q = 10n - 3$$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(TL_n \odot K_1)\} = \{k, k + 1, \dots, 10n + k - 4\}$$

So, $f(V) \cup \{f^*(e) : e \in E(TL_n \odot K_1)\}$ is a k -Super mean labeling.

Hence the graph $TL_n \odot K_1$ is a k -Super mean graph.

Example 2.10:

2000 – Super mean labeling of $TL_4 \odot K_1$ is given in figure 2.10

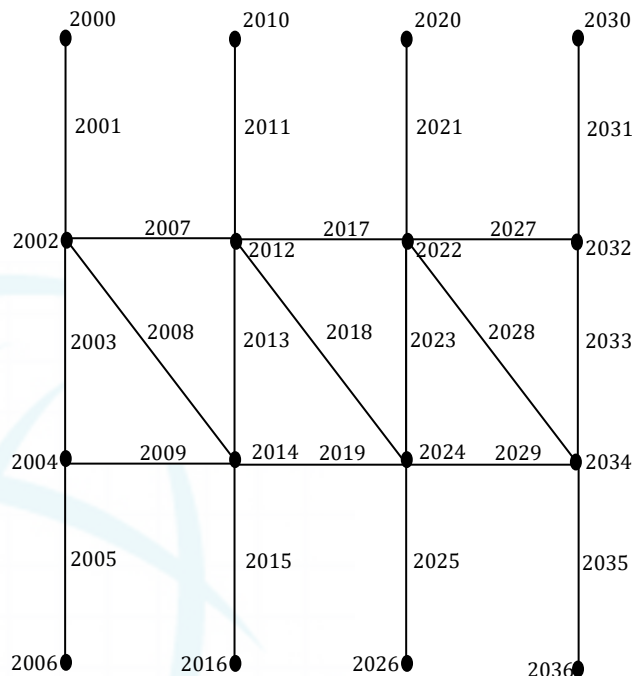


Figure 2.10: 2000-Super mean labeling of $TL_4 \odot K_1$

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