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On k-Super Mean Graphs

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Abstract: ***Let G be a (p, q) graph and $f: V(G) \to \{1, 2, 3, \ldots p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \ldots p+q\}$. A graph that admits a super mean labeling is called super mean graph. Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \ldots p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called k - super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \ldots p+q+k-1\}$. A graph that admits a k - super mean labeling is called k - super mean graph. In this paper, we investigate k-super mean labeling of (nQ_3, v_1, v_2), TP_n, $S(P_m \times P_n)$, $(P_n\, A\, K_1) \cup T_m$, $A(T_n)$, $C_n \ominus 2P_m$, $TL_n \odot K_1$.***

Keyword: k - super mean labeling, k - super mean graph, (nQ_3, v_1, v_2), TP_n, $S(P_m \times P_n)$, $(P_n\, A\, K_1) \cup T_m$, $A(T_n)$, $C_n \ominus 2P_m$, $TL_n \odot K_1$

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [1]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G. In this paper, we investigate k-super mean graphs of (nQ_3, v_1, v_2), TP_n, $S(P_m \times P_n)$, $(P_n\, A\, K_1) \cup T_m$, $A(T_n)$, $C_n \ominus 2P_m$, $TL_n \odot K_1$.

Definition 1.1
Let G be a (p, q) graph and $f: V(G) \to \{1, 2, 3, \ldots p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called **super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \ldots p+q\}$. A graph that admits a super mean labeling is called **super mean graph.**

Definition 1.2
Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \ldots p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called **k - super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \ldots p+q+k-1\}$. A graph that admits a k - super mean labeling is called **k - super mean graph.**

Definition 1.3
(G_1, G_2, v_1, v_2) is the graph obtained from G_1 and G_2 by identifying the vertices v_1 and v_2. If $G_1 = G_2$, then (G, G, v_1, v_2) is denoted by $(2G, v_1, v_2)$.
The graph $p_2 \times p_2 \times p_2$ is called cube and is denoted by Q_3. Q_3 is a super mean graph, then $(2Q_3, v_1, v_2)$ is a super mean graph.

Definition 1.4
A triangle C_3 can be partitioned into n number of triangles by joining one vertex C_3 to the midpoint of the opposite edges and continue this process to form n triangles and it is denoted by TP_n.

Definition 1.5
A graph obtained from grid $P_m \times P_n$ by joining opposite corners (i, j) and $(i+1, j+1)$ of each cell by an edge is denoted by $S(P_m \times P_n)$ is called strong grid.

Definition 1.6
A graph obtained a single pendant edge to each vertex of a path is called a comb $(P_n\, A\, K_1)$.

Definition 1.7
A alternate triangular snake $A(T_n)$is obtained from a path $u_1, u_2, u_3 \ldots u_n$ by joining u_i and u_{i+1}(alternatively) to a new vertex $v_i$for $1 \leq i \leq n-1$. That is, every edge of a path is replaced by a triangle C_3.

Definition 1.8
Bi–armed crown $C_n \ominus 2P_m$ is a graph obtained from a cycle $C_n$by identifying the pendent vertices of two vertex disjoint paths of same length m – 1at each vertex of the cycle.

Definition 1.9
A triangular ladder TL_n is a graph obtained from L_n by adding the edges $u_i v_{i+1}$, $1 \leq i \leq n-1$ where u_i and v_i are the vertices of $L_n$such that $u_1, u_2, u_3 \ldots u_n$ and $v_1, v_2, v_3 \ldots v_n$ are two paths of length n in the graph.

2. Main Results

Theorem 2.1:
The graph (nQ_3, v_1, v_2) is a k-Super mean graph for all n >1.

Proof:

Let $V(nQ_3, v_1, v_2) = \{v_i ; 1 \le i \le n\} \cup \{v_i' ; 1 \le i \le n\} \cup \{v_i'' ; 1 \le i \le n\} \cup \{v_i''' ; 1 \le i \le n\} \cup \{u_i ; 1 \le i \le n\} \cup \{u_i' ; 1 \le i \le n\} \cup \{u_i'' ; 1 \le i \le n\} \cup \{u_i''' ; 1 \le i \le n\}$

and $v_i''' = v_{i+1}$.

$$E(nQ_3, v_1, v_2) = \{e_i = (v_i, v_i''); 1 \le i \le n\} \cup \{e_i' = (v_i, v_i'); 1 \le i \le n\} \cup \{e_i'' = (v_i', v_i'''); 1 \le i \le n\} \cup \{e_i''' = (v_i'', v_i'''); 1 \le i \le n\} \cup \{a_i = (u_i, u_i''); 1 \le i \le n\} \cup \{a_i' = (u_i, u_i'); 1 \le i \le n\} \cup \{a_i'' = (u_i', u_i'''); 1 \le i \le n\} \cup \{a_i''' = (u_i'', u_i'''); 1 \le i \le n\} \cup \{b_i = (v_i, u_i); 1 \le i \le n\} \cup \{b_i' = (v_i'', u_i''); 1 \le i \le n\} \cup \{b_i'' = (v_i', u_i'); 1 \le i \le n\} \cup \{b_i''' = (v_i''', u_i'''); 1 \le i \le n\}$$

be the vertices and edges of (nQ_3, v_1, v_2) respectively.

First we label the vertices of (nQ_3, v_1, v_2) as follows:

$f(v_i) = 19i + k - 19, \ 1 \le i \le n$
$f(v_i') = 19i + k - 17, \ 1 \le i \le n$
$f(v_i'') = 19i + k - 2, \ 1 \le i \le n$
$f(v_i''') = 19i + k, \ 1 \le i \le n$
$f(u_i) = 19i + k - 9, \ 1 \le i \le n$
$f(u_i') = 19i + k - 15, \ 1 \le i \le n$
$f(u_i'') = 19i + k - 4, \ 1 \le i \le n$
$f(u_i''') = 19i + k - 11, \ 1 \le i \le n$

Now the induced edge labels are

$f^*(e_i) = 19i + k - 10, 1 \le i \le n$
$f^*(e_i') = 19i + k - 18, 1 \le i \le n$
$f^*(e_i'') = 19i + k - 8, 1 \le i \le n$
$f^*(e_i''') = 19i + k - 1, \ 1 \le i \le n$
$f^*(a_i) = 19i + k - 6, 1 \le i \le n$
$f^*(a_i') = 19i + k - 12, \ 1 \le i \le n$
$f^*(a_i'') = 19i + k - 13, \ 1 \le i \le n$
$f^*(a_i''') = 19i + k - 7, \ 1 \le i \le n$
$f^*(b_i) = 19i + k - 14, \ 1 \le i \le n$
$f^*(b_i') = 19i + k - 3, 1 \le i \le n$
$f^*(b_i'') = 19i + k - 16, 1 \le i \le n$
$f^*(b_i''') = 19i + k - 5, 1 \le i \le n$

Here $p = 7n + 1, \ q = 12n, \ p + q = 19n + 1$

Clearly,

$f(V) \in \{f^*(e): e \in E(nQ_3, v_1, v_2)\} = \{k, k+1, \dots 19n + k\}$

So, $f(V) \cup \{f^*(e) : e \in E(nQ_3, v_1, v_2)\}$ is a k-Super mean labeling.

Hence the graph (nQ_3, v_1, v_2) is a k-Super mean graph.

Example 2.1:

315 – Super mean labeling of $(2Q_3, v_1, v_2)$ is given in figure 2.1

Theorem 2.2:
The graph TP_n is a super mean graph

Proof:

Let $V(TP_n) = \{v_i ; 1 \le i \le n - 1\} \cup \{u_i ; 1 \le i \le n\}$ and

$$E(TP_n) = \{e_i = (v_i, u_i); \ 1 \le i \le n - 1\} \cup \{e_i' = (v_i, u_{i+1}); \ 1 \le i \le n - 1\} \cup \{a_i = (u_i, u_{i+1}); \ 1 \le i \le n - 1\} \cup \{b_i = (v_i, v_{i+1}); \ 1 \le i \le n - 2\}$$

be the vertices and edges of TP_n respectively.

First we label the vertices of TP_n as follows:

$f(v_i) = 6i + k - 4, \ 1 \le i \le n - 1$
$f(u_i) = 6i + k - 6, \ 1 \le i \le n - 1$
$f(u_i) = 6i + k - 7, \ i = n$

Now the induced edge labels are

$f^*(e_i) = 6i + k - 5, \ 1 \le i \le n - 1$
$f^*(e_i') = 6i + k - 2, \ 1 \le i \le n - 1$
$f^*(a_i) = 6i + k - 3, \ 1 \le i \le n - 1$
$f^*(b_i) = 6i + k - 1, \ 1 \le i \le n - 2$

Here $p = 2n - 1, q = 4n - 5, p + q = 6n - 6$

Clearly,

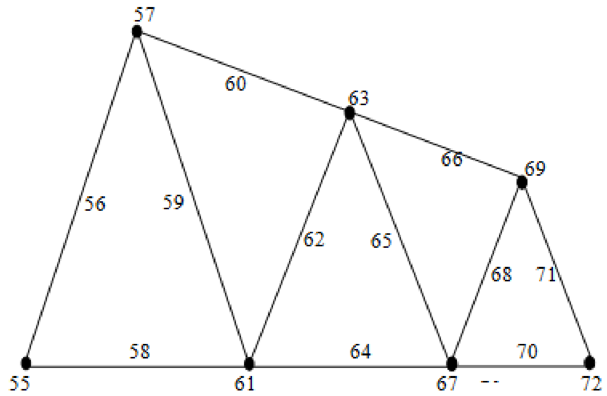
$f(V) \cup \{f^*(e) : e \in E(TP_n)\} = \{k, k+1, \dots 6n + k - 7\}$

So, $f(V) \cup \{f^*(e) : e \in E(TP_n)\}$ is a k-Super mean labeling.

Hence the graph TP_n is a k-Super mean graph.

Example 2.2:

55 – Super mean labeling of TP_4 is given in figure 2.2



Theorem 2.3:
The graph $S(P_m \times P_n)$ is a super mean graph.

Proof:

Let $V(S(P_m \times P_n)) = \{u_{ij} ; 1 \le i \le m, \ 1 \le j \le n\}$ and

$$E(S(P_m \times P_n)) = \{e_{ij} = (u_{ij}, u_{i(j+1)}); 1 \le i \le m, 1 \le j \le n - 1\} \cup \{a_{ij} = (u_{ij}, u_{(i+1)j}); 1 \le i \le m - 1, 1 \le j \le n\} \cup \{b_{ij} = (u_{ij}, u_{(i+1)(j+1)}); 1 \le i \le m - 1, 1 \le j \le n - 1\}$$

be the vertices and edges of $S(P_m \times P_n)$ respectively.

First we label the vertices of $(P_m \times P_n)$ as follows:

$f(u_{ij}) = (4n-2)(i-1) + 2j + k - 2,$
$1 \le i \le m, 1 \le j \le n$

Now the induced edge labels are

$f^*(e_{ij}) = (4n-2)(i-1) + 2j + k - 1,$
$1 \le i \le m,\ 1 \le j \le n-1$

$f^*(a_{ij}) = (4n-2)(i-1) + (2n + j) + j + k - 3,$
$1 \le i \le m-1,\ 1 \le j \le n$

$f^*(b_{ij}) = (4n-2)(i-1) + 2n + 2j, + k - 2,$
$1 \le i \le m-1,\ 1 \le j \le n-1$

Here $p = mn, q = 3mn - 2m - 2n + 1,$
$p + q = 4mn - 2m - 2n + 1$

Clearly ,

$f(V) \cup \{f^*(e): e \in E(S(P_m \times P_n))\}$
$= \{k, k+1, \ldots 4mn - 2m - 2n + k\}$

So, $f(V) \cup \{f^*(e) : e \in E(S(P_m \times P_n))\}$ is a k-Super mean labeling.

Hence the graph $(P_m \times P_n)$ is a k-Super mean graph.

Example 2.3:

545 – Super mean labeling of $(P_4 \times P_4)$ is given in figure 2.3

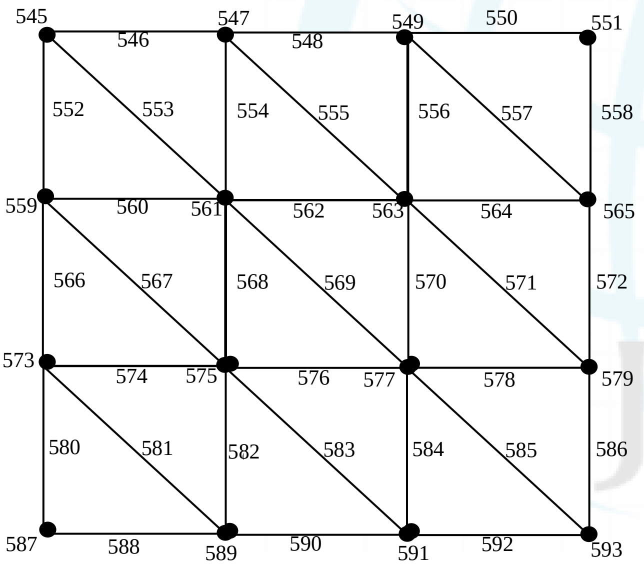


Figure 2.3: 545-Super mean labeling of $S(P_4 \times P_4)$

Theorem 2.4:

The graph $(P_n \text{ A } K_1) \cup T_m$ is a k-Super mean graph.

Proof:

Let $V((P_n \text{ A } K_1) \cup T_m) = \{v_i ; 1 \le i \le n\} \cup$
$\{u_i ; 1 \le i \le n\} \cup$
$\{w_i ; 1 \le i \le m-1\} \cup$
$\{w_i' ; 1 \le i \le m\}$

and

$E((P_n \text{ A } K_1) \cup T_m) = \{e_i = (v_i, u_i) ; 1 \le i \le n\} \cup$
$\{e_i' = (u_i, u_{i+1}) ; 1 \le i \le n-1\} \cup$
$\{a_i = (w_i, w_i') ; 1 \le i \le m-1\} \cup$
$\{b_i = (w_i, w_{i+1}') ; 1 \le i \le m-1\} \cup$
$\{c_i = (w_i', w_{i+1}') ; 1 \le i \le m-1\}$

be the vertices and edges of $(P_n \text{ A } K_1) \cup T_m$ respectively.

First we label the vertices of $(P_n \text{ A } K_1) \cup T_m$ as follows:

$f(v_i) = 4i + k - 4,\ 1 \le i \le n,\ i \text{ is odd}$
$f(v_i) = 4i + k - 2,\ 2 \le i \le n,\ i \text{ is even}$
$f(u_i) = 4i + k - 2,\ 1 \le i \le n, i \text{ is odd}$
$f(u_i) = 4i + k - 4,\ 2 \le i \le n, i \text{ is even}$
$f(w_1) = 4n + k - 1$
$f(w_i) = 4n + 5i + k - 4,\ 2 \le i \le m-1$
$f(w_1') = 4n + k + 1$
$f(w_i') = 4n + 5i + k - 6,\ 2 \le i \le m$

Now the induced edge labels are

$f^*(e_i) = 4i + k - 3\ ,\ 1 \le i \le n$
$f^*(e_i') = 4i + k - 1\ ,\ 1 \le i \le n-1$
$f^*(a_i) = 4n + 5i + k - 5\ ,\ 1 \le i \le m-1$
$f^*(b_1) = 4n + k + 2$
$f^*(b_i) = 4n + 5i + k - 2\ ,\ 2 \le i \le m-1$
$f^*(c_1) = 4n + k + 3$
$f^*(c_i) = 4n + 5i + k - 3\ ,\ 2 \le i \le m-1$

Here $p = 2n + 2m - 1, q = 2n + 3m - 4,$
$p + q = 4n + 5m - 5$

Clearly ,

$f(V) \cup \{f^*(e): e \in E((P_n \text{ A } K_1) \cup T_m)\}$
$= \{k, k+1, \ldots .4n + 5m + k - 6\}$

So, $f(V) \cup \{f^*(e) : e \in E((P_n \text{ A } K_1) \cup T_m)\}$ is a k-Super mean labeling.

Hence the graph $(P_n \text{ A } K_1) \cup T_m$ is a k-Super mean graph.

Example 2.4:

101 – Super mean labeling of $(P_4 \text{ A } K_1) \cup T_3$ is given in figure 2.4

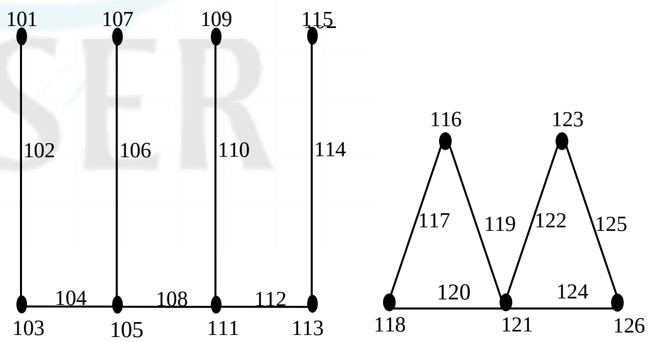


Figure 2.4: 101-Super mean labeling of $(P_4 \text{ A } K_1) \cup T_3$

Theorem 2.5:

Alternate triangular snakes $A(T_n)$is a k-Super mean graphs.

Proof:

We consider two different cases.

Case (i):

If the alternate triangular snake $A(T_n)$ starts from u_1, then we need to consider two subcases.

Subcase (i) (a): n is even

Let $V(A(T_n)) = \{u_i ; 1 \le i \le n\} \cup \{v_i ; 1 \le i \le \frac{n}{2}\}$

and $E(A(T_n)) = \{e_i = (u_{2i-1}, u_{2i}) ; 1 \le i \le \frac{n}{2}\} \cup$

$\{a_i = (v_i\ , u_{2i-1})\ ; 1 \le i \le \frac{n}{2}\} \cup$
$\{b_i\ = (v_i, u_{2i}\ )\ ; 1 \le i \le \frac{n}{2}\} \cup$
$\{c_i\ = (u_{2i}, u_{2i+1}\ )\ ; 1 \le i \le (\frac{n-2}{2})\ \}$

be the vertices and edges of $A(T_n)$ respectively.
First we label the vertices of $A(T_n)$ as follows:

$f(u_{2i-1}) = \ 7i\ +\ k - 7,\ \ 1 \le\ i\ \le\ \frac{n}{2}$
$f(u_{2i}) =\ 7i\ +\ k - 2,\ \ 1 \le\ i \le \frac{n}{2}$
$f(v_i)\ =\ 7i\ +\ k - 5,\ \ 1 \le\ i \le \frac{n}{2}$

Now the induced edge labels are

$f^*(e_i) =\ 7i\ +\ k - 4\ ,\ \ 1 \le\ i\ \le \frac{n}{2}$
$f^*(a_i) =\ 7i\ +\ k - 6\ ,\ \ 1 \le\ i\ \le \frac{n}{2}$
$f^*(b_i) =\ 7i\ +\ k - 3\ ,\ \ 1 \le\ i\ \le \frac{n}{2}$
$f^*(c_i) =\ 7i\ +\ k - 1\ ,\ \ 1 \le\ i\ \le (\frac{n-2}{2})$

Here $p\ = (\frac{3n}{2})$, $q\ = \left(\frac{4n-2}{2}\right)$, $p\ +\ q\ = (\frac{7n-2}{2})$

Clearly ,

$f(V) \cup \{f^*(e) :\ e \in E(A(T_n))\ \} =$
$$\{\ k, k + 1, \ldots..(\frac{7n-2}{2}) +\ k - 1\}$$

So, $f(V) \cup \{f^*(e) :\ e \in\ E(A(T_n))\ \ \}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.5:

21 – Super mean labeling of $A(T_6)$is given in figure 2.5

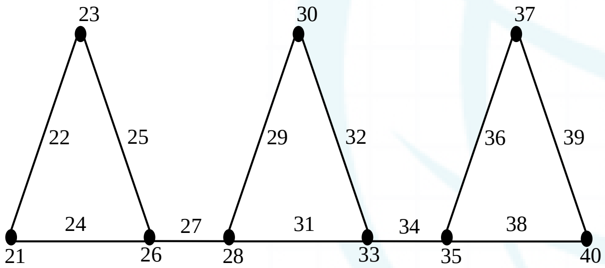


Figure 2.5: 21-Super mean labeling of $A(T_6)$

Subcase (i) (b): n is odd

Let $V(A(T_n)) = \{u_i;\ 1 \le i \le n\ \} \cup \{v_i;\ 1 \le i \le (\frac{n-1}{2})\ \}$
and $E(A(T_n)) = \{e_i\ =\ (u_{2i-1}, u_{2i});\ 1 \le i \le \left(\frac{n-1}{2}\right)\} \cup$
$\{a_i\ =\ (v_i, u_{2i-1})\ ;\ 1 \le i \le (\frac{n-1}{2})\ \} \cup$
$\{b_i\ =\ (v_i\ , u_{2i})\ ;\ 1 \le i \le (\frac{n-1}{2})\ \} \cup$
$\{c_i\ =\ (u_{2i}\ , u_{2i+1})\ ;\ 1 \le i \le (\frac{n-1}{2})\ \}$

be the vertices and edges of $A(T_n)$ respectively.
First we label the vertices of $A(T_n)$as follows:

$f(u_{2i-1})\ =\ 7i\ +\ k - 7,\ \ 1 \le\ i\ \le\ \left(\frac{n-1}{2}\right) +\ 1$
$f(u_{2i}\ )\ =\ 7i\ +\ k - 2,\ \ 1 \le\ i\ \le (\frac{n-1}{2})$
$f(v_i)\ =\ 7i\ +\ k - 5,\ \ 1 \le\ i\ \le (\frac{n-1}{2})$

Now the induced edge labels are

$f^*(e_i) =\ 7i\ +\ k - 4\ ,\ \ 1 \le\ i\ \le (\frac{n-1}{2})$
$f^*(a_i) =\ 7i\ +\ k - 6\ ,\ \ 1 \le\ i\ \le (\frac{n-1}{2})$
$f^*(b_i) =\ 7i\ +\ k - 3\ ,\ \ 1 \le\ i\ \le (\frac{n-1}{2})$
$f^*(c_i) =\ 7i\ +\ k - 1\ ,\ \ 1 \le\ i\ \le (\frac{n-1}{2})$

Here $p\ = \left(\frac{3n-1}{2}\right)$, $q\ =\ 4\,(\frac{n-1}{2})$, $p\ +\ q\ = (\frac{7n-5}{2})$

Clearly ,

$f(V) \cup \{f^*(e) :\ e \in E(A(T_n))\ \}\ =$
$$\{\ k, k\ +\ 1, \ldots..(\frac{7n-5}{2}) +\ k\ - 1\ \}$$

So, $f(V) \cup \{f^*(e) :\ e \in E(A(T_n))\ \ \}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.6:

75 – Super mean labeling of $A(T_7)$is given in figure 2.6

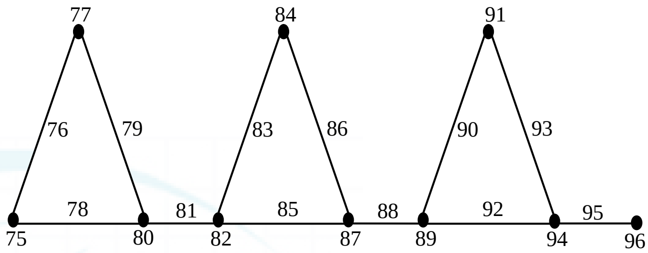


Figure 2.6: 75-Super mean labeling of $A(T_7)$

Subcase (ii) (a): n is even

Let $V(A(T_n)) = \{u_i\ ;\ 1 \le\ i\ \le n\ \} \cup \{v_i\ ;\ 1 \le\ i\ \le (\frac{n-2}{2})\ \}$
and $E(A(T_n)) = \{e_i\ = (u_{2i}\ , u_{2i+1})\ ; 1 \le i \le (\frac{n-2}{2})\ \} \cup$
$\{a_i = (v_i\ , u_{2i})\ ; 1 \le i \le (\frac{n-2}{2})\ \} \cup$
$\{b_i\ = (v_i, u_{2i+1}\ )\ ; 1 \le i \le (\frac{n-2}{2})\ \} \cup$
$\{c_i\ = (u_{2i}, u_{2i-1}\ )\ ; 1 \le i \le \frac{n}{2}\ \}$

be the vertices and edges of $A(T_n)$ respectively.
First we label the vertices of $A(T_n)$ as follows:

$f(u_{2i-1})\ =\ 7i\ +\ k - 7\ ,\ \ 1 \le\ i\ \le\ \frac{n}{2}$
$f(u_{2i})\ =\ 7i\ +\ k - 5\ ,\ \ 1 \le\ i\ \le\ \frac{n}{2}$
$f(v_i)\ =\ 7i\ +\ k - 3\ ,\ \ 1 \le\ i\ \le (\frac{n-2}{2})$

Now the induced edge labels are

$f^*(e_i) =\ 7i\ +\ k - 2\ ,\ \ 1 \le\ i\ \le (\frac{n-2}{2})$
$f^*(a_i) =\ 7i\ +\ k - 4\ ,\ \ 1 \le\ i\ \le (\frac{n-2}{2})$
$f^*(b_i) =\ 7i\ +\ k - 1\ ,\ \ 1 \le\ i\ \le (\frac{n-2}{2})$
$f^*(c_i) =\ 7i\ +\ k - 6\ ,\ \ 1 \le\ i\ \le \frac{n}{2}$

Here $p\ = \left(\frac{3n-2}{2}\right)$, $q\ = \left(\frac{4n-6}{2}\right)$, $p\ +\ q\ = (\frac{7n-8}{2})$

Clearly ,

$f(V) \cup \{f^*(e) :\ e \in E(A(T_n))\ \}\ =$
$$\{\ k, k + 1, \ldots..(\frac{7n-8}{2}) + k - 1\}$$

So, $f(V) \cup \{f^*(e) :\ e \in E(A(T_n))\ \ \}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.7:

56– Super mean labeling of $A(T_8)$is given in figure 2.7

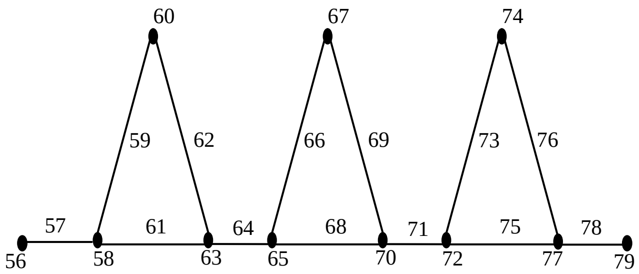


Figure 2.7: 56-Super mean labeling of $A(T_8)$

Subcase (ii) (b): n is odd

Let $V(A(T_n)) = \{u_i\ ;\ 1 \le i \le n\} \cup \{v_i\ ;\ 1 \le i \le (\frac{n-1}{2})\}$

and $E(A(T_n)) = \{e_i = (u_{2i}, u_{2i+1})\ ;\ 1 \le i \le (\frac{n-1}{2})\} \cup$

$\{a_i = (v_i, u_{2i})\ ;\ 1 \le i \le (\frac{n-1}{2})\} \cup$

$\{b_i = (v_i, u_{2i+1})\ ;\ 1 \le i \le (\frac{n-1}{2})\} \cup$

$\{c_i = (u_{2i}, u_{2i-1})\ ;\ 1 \le i \le (\frac{n-1}{2})\}$

be the vertices and edges of $A(T_n)$ respectively.

First we label the vertices of $A(T_n)$as follows:

$f(u_{2i-1}) = 7i + k - 7,\ \ 1 \le i \le \left(\frac{n-1}{2}\right) + 1$

$f(u_{2i}) = 7i + k - 5,\ \ 1 \le i \le (\frac{n-1}{2})$

$f(v_i) = 7i + k - 3,\ \ 1 \le i \le (\frac{n-1}{2})$

Now the induced edge labels are

$f^*(e_i) = 7i + k - 2,\ \ 1 \le i \le (\frac{n-1}{2})$

$f^*(a_i) = 7i + k - 4,\ \ 1 \le i \le (\frac{n-1}{2})$

$f^*(b_i) = 7i + k - 1,\ \ 1 \le i \le (\frac{n-1}{2})$

$f^*(c_i) = 7i + k - 6,\ \ 1 \le i \le (\frac{n-1}{2})$

Here $p = \left(\frac{3n-1}{2}\right)$, $q = 4\left(\frac{n-1}{2}\right)$, $p + q = \left(\frac{7n-5}{2}\right)$

Clearly ,

$$f(V) \cup \{f^*(e) : e \in E(A(T_n))\} = \{k, k + 1, \ldots.. (\frac{7n-5}{2}) + k - 1\}$$

So, $f(V) \cup \{f^*(e) : e \in E(A(T_n))\}$ is a k-Super mean labeling.

Hence the graph $A(T_n)$ is a k-Super mean graph.

Example 2.8:

98 – Super mean labeling of $A(T_7)$is given in figure 2.8

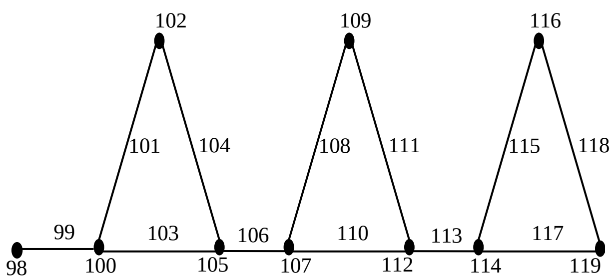


Figure 2.8: 98-Super mean labeling of $A(T_7)$

Theorem 2.6:

The bi–armed crown $C_n \ominus 2P_m$ is a k-Super mean graph for all odd $n \ge 3$ and $m \ge 2$.

Proof:

Let $V(C_n \ominus 2P_m) = \{u_i\ ;\ 1 \le i \le n\} \cup$
$\{v_{i1}^w\ ;\ 1 \le i \le n, 1 \le w \le m\} \cup$
$\{v_{i2}^w\ ;\ 1 \le i \le n, 1 \le w \le m\}$

and $v_{i1}^m = v_{i2}^m = u_i$

$E(C_n \ominus 2P_m) = \{e_i = (u_i, u_{i+1})\ ;\ 1 \le i \le n - 1\} \cup$
$\{e_{i1}^w = (v_{i1}^w, v_{i1}^{w+1})\ ;\ 1 \le i \le n, 1 \le w \le m - 1\} \cup$
$\{e_{i2}^w = (v_{i2}^w, v_{i2}^{w+1})\ ;\ 1 \le i \le n, 1 \le w \le m - 1\}$

and $e_n = (u_n, u_1)$

be the vertices and edges of $C_n \ominus 2P_m$ respectively.

First we label the vertices of $C_n \ominus 2P_m$ as follows:

Let $n = 2t + 1$ for some t.

$f(v_{j1}^i) = 4(j - 1)m - 2j + 2i + k,$
$1 \le j \le t + 1, 1 \le i \le m$

$f(v_{j2}^{m+1-i}) = 2(2j - 1)m - 2j + 2i + k - 2,$
$1 \le j \le t, 2 \le i \le m$

$f(v_{(t+1)2}^{m-1}) = 2(2t + 1)m - (2t + 2) + k + 3$

$f(v_{(t+1)2}^{m-1-i}) = 2(2t + 1)m - 2t + 2i + k + 1,$
$1 \le i \le m - 2$

$f(v_{(t+1+j)1}^i) = 4(j + t)m - 2(t + j) + 2i + k - 1,$
$1 \le j \le t, 1 \le i \le m$

$f(v_{(t+1+j)2}^{m+1-i}) = (4j + 4t + 2)m - (t + 2j + 2i) +$
$4i - 2t + k,\ \ 1 \le j \le t,\ 2 \le i \le m$

Now the induced edge labels are

$f^*(e_{j1}^i) = 4(j - 1)m - 2j + 2i + k + 1,$
$1 \le j \le t + 1, 1 \le i \le m - 1$

$f^*(e_{j2}^{m-i}) = 2(2j - 1)m - 2j + 2i + k - 1,$
$1 \le j \le t, 1 \le i \le m - 1$

$f^*(e_{(t+1)2}^{m-1}) = 2(2t + 1)m + k - 6$

$f^*(e_{(t+1)2}^{m-1-i}) = 2(2t + 1)m - 2t + 2i + k,$
$1 \le i \le m - 2$

$f^*(e_{(t+1+j)1}^i) = 4(j + t)m - 2(t + j) + 2i + k,$
$1 \le j \le t, 1 \le i \le m - 1$

$f^*(e_{(t+1+j)2}^{m+1-i}) = (4j + 4t + 2)m - (t + 2j + 2i) + 4i -$
$2t + k - 1,\ 1 \le j \le t,\ 2 \le i \le m$

$f^*(e_i) = f(v_{i2}^1) - [f(v_{11}^{'}) - 1] + k,\ 1 \le i \le \frac{n+(n-2)}{2}$

$f^*(e_i) = f(v_{n1}^m) - [f(v_{11}^{'}) - 1] - (\frac{i-1}{2})[f^*(e_1) - k + 1] +$
$k - 1,\ i = m$

Here $p = 2mn - n$, $q = 2mn - n$, $p + q = 4mn - 2n$

Clearly,

$f(V) \cup \{f^*(e) : e \in E(C_n \ominus 2P_m)\} =$
$\{k, k + 1, \ldots. 4mn - 2n + k - 1\}$

So, $f(V) \cup \{f^*(e) : e \in E(C_n \ominus 2P_m)\}$ is a k-Super mean labeling.

Hence the graph $C_n \ominus 2P_m$ is a k-Super mean graph.

mple 2.9:

54 – Super mean labeling of $C_7 \ominus 2P_3$ is given in figure 2.9

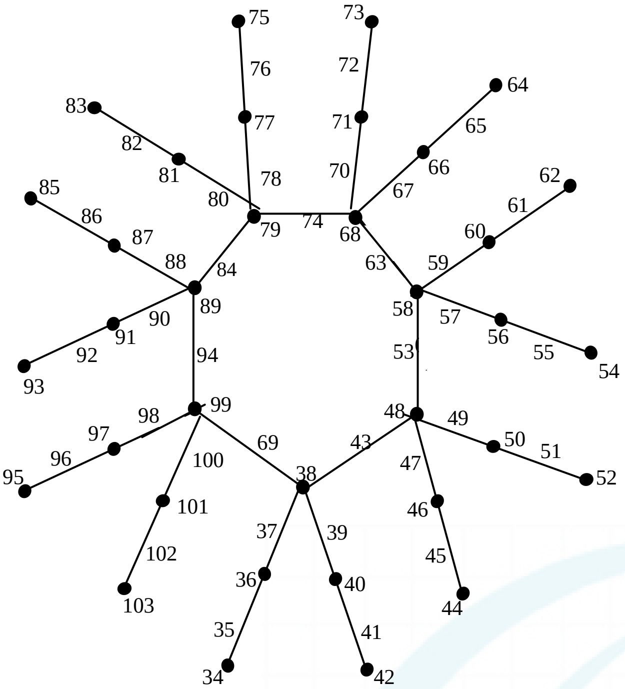


Figure 2.9: 34-Super mean labeling of $C_7 \ominus 2P_3$

Theorem 2.7:
A graph $TL_n \odot K_1$ is a super mean graph, for every n.
Proof:
Let $V(TL_n \odot K_1) = \{u_i;\ 1 \le i \le n\} \cup \{v_i;\ 1 \le i \le n\} \cup$

$$\{w_i;\ 1 \le i \le n\} \cup \{z_i;\ 1 \le i \le n\}$$

and

$$E(TL_n \odot K_1) = \{e_i = (u_i, u_{i+1});\ 1 \le i \le n-1\} \cup$$
$$\{e_i' = (u_i, v_i);\ 1 \le i \le n\} \cup$$
$$\{e_i'' = (v_i, v_{i+1});\ 1 \le i \le n-1\} \cup$$
$$\{e_i''' = (u_i, v_{i+1});\ 1 \le i \le n-1\} \cup$$

$$\{a_i = (v_i, z_i);\ 1 \le i \le n\} \cup$$
$$\{a_i' = (w_i, u_i);\ 1 \le i \le n\}$$

be the vertices and edges of $TL_n \odot K_1$ respectively.

First we label the vertices of $TL_n \odot K_1$ as follows:
$f(u_i) = 10i + k - 8,\ 1 \le i \le n$
$f(v_i) = 10i + k - 6,\ 1 \le i \le n$
$f(w_i) = 10i + k - 10,\ 1 \le i \le n$
$f(z_i) = 10i + k - 4,\ 1 \le i \le n$
Now the induced edge labels are
$f^*(e_i) = 10i + k - 3,\ 1 \le i \le n - 1$
$f^*(e_i') = 10i + k - 7,\ 1 \le i \le n$
$f^*(e_i'') = 10i,\ 1 \le i \le n - 1$
$f^*(e_i''') = 10i + k - 2,\ 1 \le i \le n - 1$
$f^*(a_i) = 10i + k - 5,\ 1 \le i \le n$
$f^*(a_i') = 10i + k - 9,\ 1 \le i \le n$
Here $p = 4n,\ q = 3n + 3(n-1)$,
$p + q = 10n - 3$
Clearly ,
$f(V) \cup \{f^*(e) : e \in E(TL_n \odot K_1)\} =$

$$\{k, k+1, \ldots 10n + k - 4\}$$

So, $f(V) \cup \{f^*(e) : e \in E(TL_n \odot K_1)\}$ is a k-Super mean labeling.

Hence the graph $TL_n \odot K_1$ is a k-Super mean graph.

Example 2.10:

2000 – Super mean labeling of $TL_4 \odot K_1$ is given in figure 2.10

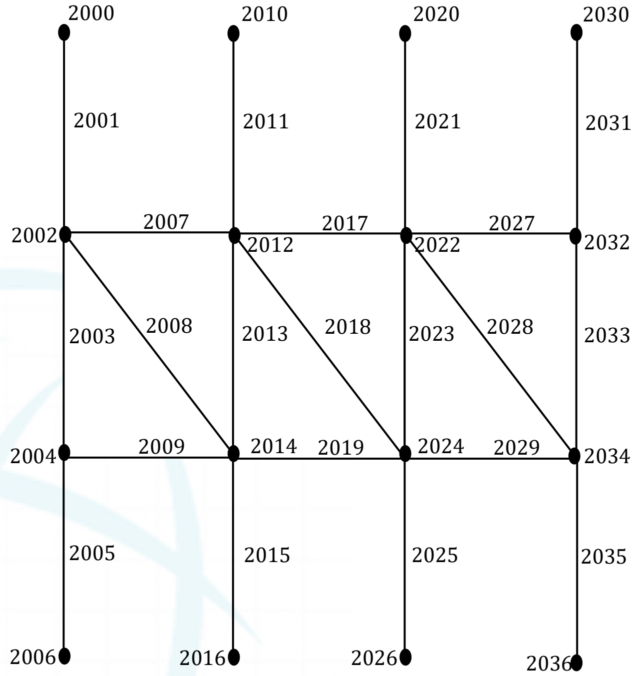


Figure 2.10: 2000-Super mean labeling of $TL_4 \odot K_1$

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