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Mathematical Dispatching Model Based on Cost Benefit in Distributed Service Network

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Abstract: ***In Distributed Service Network, the main concern of the user is mostly focused on the immediate response time of a call for service. The user evaluates the efficiency of a service network in terms of how long he or she has been waiting until a service unit arrives at the scene of a call. In same manner dispatching is an important problem, the objective of which is not only to transport resource to meet the demand as soon as possible. In this paper the mathematical model is proposed to minimize the expected response time and how to promptly monitor for the service units.***

Keywords: Distributed Service Network , Resource dispatching , Response time , Cost Benefit Analysis

1. Introduction

A distributed service network is a concept related to distribution and traveling : distribution of resources among facilities located at various locations and traveling of resources along adistributed network. There are many types of businesses and organizations that can fit intomodels of distributed service networks. In theory, almost every service provider can be modeled by means of a network even when one wanders through the long corridors of a mammoth bureaucratic organization while being transferred from one clerk to another, servicesare in fact, being received from a network.

To incorporate the term “Service” into this discussion, we must assume thatthe entity called a “ distributed service network” provides something called ‘service’ . This must be in the form of labor such as maintenance or rescue ; or in the form of equipment. Service is provided because there is a demand for it. Demand is materialized through service calls. Calls are presumed to be generated only on the nodes of the network.

Managing a distributed service network is not an easy task. It involves a variety of problems related to policy making in the long and in the short range. Here we Discussed short term policies such as “Dispatching” depend critically on the information available and the communications system operated by the dispatcher at the moment a service request arises. In Distributed service network, the main concern of the users is mostly focused on the immediate response time of a call for service. The user evaluates the efficiency of a service network in terms of how long he or she has been waiting until a service unit arrives at the scene of a call. Therefore culmination of many planning efforts lies in the dispatching rule. If this has been properly articulated, the user is satisfied. How to device a dispatching rule and how to promptly monitor the service units are main aim of the dispatching plan.

A common approach to dispatching is always to assign the service unit that is the closest one to the calling node. This assumes that the calling node will be served in the shortest response time under the particular circumstances prevailing at the moment that the call is issued. After all Figuring out who is closest server is not very complex, particularly when the dispatching center possesses advanced computing and communicating facilities.

A dispatching plan is the set of criteria guiding the decision maker in planning dispatching related activities. The criteria belongs to exponential response time minimization, imposing a minimum level response time to every mode, minimization of maximum response time , minimization of cost of dispatching . A dispatching plan is a dispatching policy which is a set of actions that ought to be taken under various circumstances e.g. always dispatch the closest available server ; when all units are busy, server calls on a first-in-first out basis. A dispatching decision is an elementary action that should be taken in response to a certain incident. The set of dispatching decisions is derived from devising a decision policy for a given set of initial assumption and initial parameters.

Dispatching plans depend on exogenous factors as well as on management preferences. The major exogenous factor is the rate of request for service .If the rate is very high most of the time, a few or even none of the service units may be available. We call such a network is a congested network and if the number of service units is very high or the rate at which calls are generated is small so there is no problem of congestion, such a network is called non-congestion network.

In a congested network management has to decide whether it always for co-operation between home node or not. Co-operation implies that a certain home nodes server help in a case, a call is issued in another unit’s territory while the unit is busy. No co-operation implies that once territories have been assigned to home node. There is no border crossing if a certain unit is busy the user of its constituency will have to be served by a reserve unit placed in a queue, regardless of the availability of unit in other territories. In this network there are periods when only few or none of the service units are available. When a call is issued, an available server is dispatching to the calling node. We assume that all the servers are busy; a reserve unit is assigned at a relatively high cost. The co-operation implies that all the service units can be dispatched to any calling node depending on this availability and the distance from the node.

2. Mathematical Formulation

Suppose there two service units that can respond to call that are generated on a network G(N,L). Calls are generated to a Poisson process, at a mean value λ calls per unit of time and at each node independently at a mean rate of λh_j (h_j being the fraction demand that come from node i) j=1,2,3,…….n. Each service unit is capable of providing μ services on the average per unit of time and service time distribution is of negative exponential.

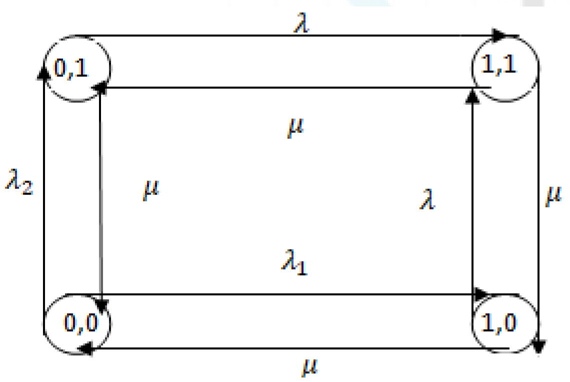
Consider that serve 1 is located While available at point $X_1 \epsilon G$ and server 2 is located while available at point $X_2 \epsilon G$. In the framework of the hypercube model the four state of the system are:-

(0,0) : Both unit are available
(0,1) : Server 1 is free and server 2 is busy
(1,0) : Server 1 is busy and server 2 is free
(1,1) : Both unit are not available

When the system is in any state accept for state (0,0) the dispatching decision is trivial. For state (0,1) and (1,0). We dispatch the only available server to any possible calls (respectively server 1 and 2). For state (1,1) we dispatch a special reserve unit. Only for state (0,0) is there a decision to make. Let N_i be the sets of nodes that are assigned to server I, when the network is in state (0,0) i = 1,2. We require $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \emptyset$. Let

$$\lambda_i = \left(\sum_{j=N_i} h_j\right)\lambda, \qquad i = 1,2.$$

Which is mean arrival rate of calls that server I faces when the system is in state (0,0). Our objective is to find $N_1$and N_2 (λ_1 and λ_2) . The hypercube diagram for this situation is illustrate below:



The steady state probabilities of the problem can be obtained by solving four equations:

$$\lambda\, P(0,0) = \mu\, P(0,1) + \mu\, P(1,0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$(\lambda + \mu)\, P(1,0) = \lambda_1\, P(0,0) + \mu\, P(1,1) \ldots\ldots\ldots\ldots\ldots(2)$$
$$(\lambda + \mu)\, P(0,1) = \lambda_2\, P(0,0) + \mu\, P(1,1) \ldots\ldots\ldots\ldots\ldots (3)$$
$$P(0,0) + P(0,1) + P(1,0) + P(1,1) = 1 \ldots\ldots\ldots\ldots\ldots(4)$$

The solutions of this set of equations is

$$P(0,0) = 1/(1 + \rho + \tfrac{\rho^2}{2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$
$$P(0,1) = P(0,0)\,(\rho - \rho_1 + \tfrac{\rho^2}{2})/\,(1 + \rho) \ldots\ldots(6)$$
$$P(1,0) = P(0,0)\,\left(\rho_1 + \tfrac{\rho_2}{2}\right)(1 + \rho) \ldots\ldots\ldots\ldots \ (7)$$
$$P(1,1) = P(0,0)\,\tfrac{\rho^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Where $\rho = \lambda/\mu\,,\ \rho_1 = \lambda_1/\mu = \left(\sum_{j=N_1} h_j\right)\rho$

The expected response time of the system is:

$$\text{ERT} = P(0,0)\left[\sum_{j\epsilon N_1} h_j\, d(\overline{X_1}, j) + \sum_{j\epsilon N_2} h_j\, d(\overline{X_2}, j)\right] + P(0,1)C_1 + P(1,0)C_2 + P(1,1)R \quad \ldots.(9)$$

Where R is the cost in time units of dispatching a special reserve unit and

$$C_i = \sum_{j=N} h_j\, d(\overline{X_i}, j), \qquad i = 1,2 \quad \ldots\ldots\ldots\ldots(10)$$

is the expected response time given that the system is respectively in states 1 & 2. Our aim is to minimize the expected response time given by equation (9). Obviously , we see that the steady state probabilities P(0,1) and P(1,0) are dependent on the set $N_1$and N_2. On the other hand $C_1$and C_2 are independent of the dispatching plan. Then expression (9) can also be written as:

ERT = P(0,0) $\left[\sum_{j\epsilon N_1} h_j\, d(\overline{X_1}, j) + \sum_{j\epsilon N} h_j\, d(\overline{X_2}, j) - \right.$ jϵN1hjdX2,j+ P0,1C1+
$P(1,0)C_2 + P(1,1)R \quad \ldots\ldots\ldots\ldots\ldots(11)$

Also,

$$\text{ERT} = P(0,0)\left[\sum_{j\epsilon N_1} h_j\,\{d(\overline{X_1}, j) - d(\overline{X_2}, j)\}\right] + P(0,0)C_2 + P(0,1)C_1 + P(1,0)C_2 + P(1,1)R \quad \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

But

$$P(0,1)C_1 + [P(0,0) + P(1,0)]C_2 + P(1,1)R = P(0,0)\left(\sum_{j=N_1} h_1\right)\frac{\rho}{1+\rho}(C_2 - C_1) + \alpha \quad \ldots\ldots.(13)$$

$\alpha = P(0,0)\left\{\left(\rho + \frac{\rho^2}{2}\right)/(1 + \rho)\right\}C_1 + \left\{\left(1 + \rho + \frac{\rho^2}{2}\right)/(1 + \right.$ ρ)C2+ P1,1R ……………..(14)

ERT can be written again as:

$$\text{ERT} = P(0,0)\left[\sum_{j\epsilon N_1} h_j\,\{d(\overline{X_1}, j) - d(\overline{X_2}, j) - S\}\right] + \alpha \quad \ldots\ldots\ldots\ldots\ldots.(15)$$

Where $S = \left[\frac{\rho}{1+\rho}\right](C_1 - C_2)$

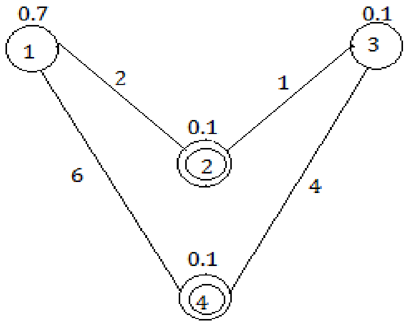
Which is independent of $N_1$and N_2. Obviously minimizing (15), we get

$$N_1 = \left[\sum_{j\epsilon N} \{d(\overline{X_1}, j) - d(\overline{X_2}, j)\}\right] < S$$
$$N_2 = N - N_1$$

This constitutes the partitioning of N between two service units at state (0,0).

3. Application

Let G be a service network . The number near the links display lengths I(i,j) and the number near the nodes represent demand fractions (h_j); double circles mark the location of service units (home nodes) .



The above network G is also given in the following tabular form:

Node Number	Distance from Node 2	Distance from Node 4	Demand Proportion (h_j)
1	2	6	0.7
2	0	5	0.1
3	1	4	0.1
4	5	0	0.1
ERT	$C_1 = 2.0$	$C_2 = 5.1$	

Let us denote at node 2 and 4 as server 1 and 2 correspondingly. The network can be one of four States, depending on the availability of the two servers. The states are listed as under:

State Number	Notation	Description
1	(0,0)	Both unit are available
2	(0,1)	Server 1 is available and server 2 is busy
3	(1,0)	Server 1 is busy and server 2 is available
4	(1,1)	Both unit are busy

The dispatching rule for states (0,1), (1,0) and (1,1) is obvious. It is to dispatch the available server in states (0,1) & (1,0) or to call for a reserve unit in state (1,1). A real dispatching problem contains only in state 1 i.e. (0,0) where we have to assign one of the two available servers to the node that initiates a request. Further, we find that the dispatching of the closest server is not necessary the optimal decision.

Let S be the following expression:

$$S = \left[\frac{\rho}{1+\rho}\right](C_1 - C_2)$$

Here $\rho = 2/0.5 = 4$

$$S = [4/5]\ (2-5.1) = -2.48$$

The table given below shows the shortest distance differences and the assignments of various servers:

Node Number	d(2,j) – d(4,j)	Assignment for S = -2.48
1	-4	Server at node 2
2	-5	Server at node 2
3	-3	Server at node 2
4	5	Server at node 4

Now, the state probabilities and ERT for the entire network can be calculated by means of hypercube model discussed above. It is obvious that the dispatching decision depend on the value of S, which in itself derive from the value of λ , μ , distances and demand proportion h_j. These are major parameters of the model. The demand proportions are:

$$h_1 = 0.05\ , \quad h_2 = 0.85\ , \ h_3 = 0.05\ , h_4 = 0.05$$

Such a change can be caused by a special temporary event which may require division of resources to another node. The dispatching center can very quickly drive a new dispatching rule. Let the new rule be:

$$C_1 = 0.05 \times (2 + 1 + 5) + 0.85 \times 0 = 0.4$$
$$C_2 = 0.05 \times (6 + 4 + 0) + 0.85 \times 5 = 4.75$$
$$S = \frac{4}{5}(0.4 - 4.75) = -3.48$$

Consequently, the dispatching decisions should be to assign the server at node 2 to calls only from node 1 and 2 when the system is in state (0,0). Since the demand foe node 2 is very large. It becomes advangeous to allow server located in node 4 to be dispatched to calls issued in node 3.

4. Conclusion

In this paper the importance of the cost of emergency management in distributive service network is discussed and the cost and the benefit of emergency management are considered in a mathematic programming model when studying the resource dispatching problem considering the potential demand based on Cost-Benefit analysis. Aiming at the successive emergencies with the probabilities of the potential emergencies, the mathematic model is proposed to reach the balance between the emergency that has happened and the potential demand to a certain degree.

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