

# Pre-Rg-open and Pre-Rg-closed Functions in Topology

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**Abstract:** Levine in 1970, introduced the concept of generalized closed ( $g$ -closed) sets in topological space and a class of topological spaces called  $T_{1/2}$  spaces. Palaniappan et al in 1993, introduced the notions of regular generalized (in brief,  $rg$ -) closed sets,  $rg$ -open sets,  $rg$ -continuity and  $rg$ -irresoluteness and in 1997, Arokia Rani et al introduced and studied the concepts of  $rg$ -openness and  $rg$ -closedness in topology. In 2013, Navalagi et al have studied the concepts of pre- $rg$ -openness and other allied  $rg$ -openness in topological spaces. The purpose of this paper is to investigate the concept of pre- $Rg$ -open functions, pre- $Rg$ -closed functions and other allied  $Rg$ -openness in topology.

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## 1. Introduction

N. Levine [5] introduced the class of  $g$ -closed sets, a super class of closed sets in 1970. Palaniappan [14] defined  $rg$ -closed sets,  $rg$ -open sets,  $rg$ -continuous functions,  $rg$ -irresolute functions between topological spaces, in 1993. Later, in 1997.

Arokarani et al [1] have defined and studied the notion of  $rg$ -open functions and  $rg$ -closed functions. Recently, G. Navalagi et al [11] have introduced and studied the concept of pre- $rg$ -open functions in topological spaces. In this paper, we introduce and study the new classes of functions namely, pre- $Rg$ -open functions and pre- $Rg$ -closed functions in topological spaces.

## 2. Preliminaries

In what follows, spaces  $X$  and  $Y$  are always topological spaces  $Cl(A)$  and  $Int(A)$  designate the closure and the interior of  $A$  which is a subset of  $X$ . A set  $A$  is said to be regular open (resp. Regular closed) if  $A = Int(Cl(A))$  (resp.  $A = Cl(Int(A))$ ).

**Definition 2.1** [6]: A subset  $A$  of a space  $X$  is said to be pre-open if  $A \subset Int(Cl(A))$ .

The family of all preopen sets in a space  $X$  is denoted by  $PO(X)$ . The complement of a preopen set of a space  $X$  is called preclosed [3].

**Definition 2.2** [7]: The union of all preopen sets contained in  $A$  is called the preinterior of  $A$  and is denoted by  $pInt(A)$ .

**Definition 2.3** [3]: The intersection of all preclosed sets containing  $A$  is called the preclosure of  $A$  and is denoted by  $pCl(A)$ .

**Definition 2.4** [5]: A subset  $A$  of a space  $X$  is called a generalized closed set ( $g$ -closed) set if  $Cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open.

Clearly, every closed set is a  $g$ -closed set. The complement of a  $g$ -closed set in  $X$  is called generalized open or  $g$ -open set. Clearly, every open set is a  $g$ -open set.

**Definition 2.5** [12]: A subset  $A$  of a space  $X$  is called a generalized preclosed set ( $gp$ -closed) set if  $pCl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open.

The complement of a  $gp$ -closed set in  $X$  is called a generalized open or  $gp$ -open set.

It is obvious that every closed set is preclosed set, every closed set is  $g$ -closed set, every  $g$ -closed set is  $gp$ -closed set and every preopen set is a  $gp$ -open set.

**Definition 2.6** [14]: A subset  $A$  of a space  $X$  is called a regular generalized closed ( $rg$ -closed) set if  $Cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular open. The complement of a  $rg$ -closed set of a space is called  $rg$ -open. The family of all  $rg$ -open sets of a space  $X$  is denoted by  $RGO(X)$ .

**Definition 2.7:** A function  $f: X \rightarrow Y$  is said to be :

- (i) preopen [7] if the image of each open set  $U$  of  $X$ ,  $f(U)$  is preopen in  $Y$ .
- (ii) preclosed [3] if the image of each closed set  $F$  of  $X$ ,  $f(F)$  is preclosed in  $Y$ .
- (iii) preirresolute [16] if the inverse image of each preopen set of  $Y$  is preopen in  $X$ .
- (iv)  $gp$ -closed [12] if the image of each closed set of  $X$  is  $gp$ -closed in  $Y$ .
- (v) pre- $gp$ -closed [12] if the image of each preclosed set of  $X$  is  $gp$ -closed in  $Y$ .
- (vi)  $gp$ -open [9] if the image of each open set of  $X$  is  $gp$ -open in  $Y$ .

(vii) always-gp-closed [9] if the image each gp-closed set of  $X$  is gp-closed in  $Y$ .

**Definition 2.8 [13]:** A function  $f: X \rightarrow Y$  is said to be almost preclosed if the image of each regular closed set of  $X$  is preclosed in  $Y$ .

**Definition 2.9 [8]:** A function  $f: X \rightarrow Y$  is said to be M-preopen if the image of each preopen set of  $X$  is preopen in  $Y$ .

**Definition 2.10[8]:** A function  $f: X \rightarrow Y$  is said to be M-preclosed if the image of each pre-closed set of  $X$  is preclosed in  $Y$ .

**Definition 2.11:** A function  $f: X \rightarrow Y$  is called

- i) rg-open[1] if image of each open set of  $X$  is rg-open in  $Y$ .
- ii) rg-closed [1] if image of each closed set of  $X$  is rg-closed in  $Y$ .

**Definition 2.12:** A function  $f: X \rightarrow Y$  is said to be

- i) rg-irresolute [14] if the inverse image of each rg-open set  $V$  of  $Y$  is rg-open in  $X$ .
- ii) perfectly rg-continuous [1] if the inverse image of each rg-open set in  $Y$  is both open and closed in  $X$ .

**Definition 2.13 [11]:** A function  $f: X \rightarrow Y$  is said to be pre-rg-open if the image of each regular open set of  $X$  is rg-open in  $Y$ .

**Definition 2.14[11]:** A function  $f: X \rightarrow Y$  is called strongly rg-open if the image of each rg-open set of  $X$  is open in  $Y$ .

**Definition 2.17[11]:** A function  $f: X \rightarrow Y$  is said to be always rg-open, if the image of each rg-open set of  $X$  is rg-open in  $Y$ .

**Definition 2.19[11]:** A function  $f: X \rightarrow Y$  is said to be prg-open if image of each preopen set of  $X$  is rg-open in  $Y$ .

**Definition 2.20[11]:** A function  $f: X \rightarrow Y$  is called strongly p-open if the image of each preopen set of  $X$  is open in  $Y$ .

Clearly, every strongly p-open function is M-preopen.

### 3. Properties of pre-Rg-open functions

**Definition 3.1:** A function  $f: X \rightarrow Y$  is called pre-Rg-open if the image of each rg-open set of  $X$  is preopen in  $Y$ .

Clearly, (i) prg-open functions and Pre-Rg-open functions are dual in nature and hence their composition yields a M-preopen function due to Mashhour et al. [8]. (ii) Every strongly-rg-open function is pre-Rg-open function. (iii) Every pre-Rg-open function is preopen.

We, characterize the prg-openness in the following.

**Theorem 3.2:** Let  $f: X \rightarrow Y$  be a map. Then the following are equivalent:

- i)  $f$  is prg-open
- ii) The image of each preopen set in  $X$  is rg-open in  $Y$
- iii) The image of each preclosed set in  $X$  is rg-closed in  $Y$

Obvious proof is omitted.

**Theorem 3.3:** If  $f: X \rightarrow Y$  is strongly rg-open and  $g: Y \rightarrow Z$  is preopen, then the composition  $g \circ f: X \rightarrow Z$  is pre-Rg-open.

**Proof:** Let  $U$  be rg-open set in  $X$ , then  $f(U)$  is open in  $Y$  since  $f$  is strongly rg-open. Since  $g$  is preopen and  $f(U)$  is open set in  $Y$ ,  $g(f(U)) = g \circ f(U)$  is preopen set in  $Z$ . This shows that  $g \circ f$  is pre-Rg-open function.

**Theorem 3.4:** If  $f: X \rightarrow Y$  is prg-open and  $g: Y \rightarrow Z$  is always-rg-open, then the composition  $g \circ f: X \rightarrow Z$  is prg-open.

Easy proof is omitted.

We, define the following.

**Definition 3.5:** A topological space  $X$  is said to be  $T_{1/2}^{**}$  iff every rg-closed set is preclosed.

**Definition 3.6:** A topological space  $X$  is said to be  $T_{rg}^*$  space iff every rg-closed set is gp-closed.

**Theorem 3.7:** Let  $X$  and  $Z$  be any topological spaces and  $Y$  be a  $T_{1/2}^{**}$  space. Then the composition  $g \circ f: X \rightarrow Z$  of the rg-closed function  $f: X \rightarrow Y$  and M-preclosed function  $g: Y \rightarrow Z$  is preclosed function.

**Proof:** Let  $F$  be any closed set in  $X$ . Since  $f$  is rg-closed function,  $f(F)$  is rg-closed in  $Y$ . But  $Y$  is  $T_{1/2}^{**}$ -space, therefore  $f(F)$  is preclosed set in  $Y$  which implies that  $g(f(F)) = g \circ f(F)$  is preclosed set in  $Z$ . This shows that  $g \circ f$  is preclosed.

**Theorem 3.8:** Let  $X$  and  $Z$  be any topological spaces and  $Y$  be a  $T_{1/2}^{**}$  space. Then, if  $f: X \rightarrow Y$  be rg-closed function and  $g: Y \rightarrow Z$  be pre-gp-closed then  $g \circ f$  is gp-closed.

**Proof:** Obvious.

**Theorem 3.9:** Let  $X$  and  $Z$  be any topological spaces and  $Y$  be a  $T_{rg}^*$ -space. Then the composition  $g \circ f: X \rightarrow Z$  of the rg-closed function  $f: X \rightarrow Y$  and always gp-closed function  $g: Y \rightarrow Z$  is gp-closed function.

**Proof:** Obvious.

We, define the following.

**Definition 3.10:** A function  $f: X \rightarrow Y$  is said to be contra rg-open if for each open set  $U$  of  $X$ ,  $f(U)$  is rg-closed set in  $Y$ .

**Definition 3.11:** A function  $f: X \rightarrow Y$  is said to be regular-rg-open if for each regular open set  $U$  of  $X$ ,  $f(U)$  is rg-open in  $Y$ .

**Definition 3.12:** A function  $f : X \rightarrow Y$  is said to be contra regular-rg-open if for each regular-open set  $U$  of  $X$ ,  $f(U)$  is rg-closed in  $Y$ .

**Definition 3.13:** A function  $f : X \rightarrow Y$  is said to be contra always-rg-open if for each rg-open set  $U$  of  $X$ ,  $f(U)$  is rg-closed in  $Y$ . We, prove the following.

**Theorem 3.14:** A surjective function  $f : X \rightarrow Y$  is pre-Rg-open if and only if for each subset  $B$  of  $Y$  and each rg-closed set  $F$  of  $X$  containing  $f^{-1}(B)$ , there exists preclosed set  $H$  of  $Y$  such that  $B \subset H$  and  $f^{-1}(H) \subset F$ .

**Proof:** Necessity: Suppose  $f$  is pre-Rg-open. Let  $B$  be any subset of  $Y$  and  $F$  is rg-closed set of  $X$  containing  $f^{-1}(B)$ . Put  $H = f(X-F)$ . Then,  $H$  is preclosed in  $Y$ ,  $B \subset H$  and  $f^{-1}(H) \subset F$ .

Sufficiency: Let  $U$  be any rg-open set in  $X$ . Put  $B = Y - f(U)$ , then we have  $f^{-1}(B) \subset X-U$  and  $X-U$  is rg-closed such that  $B = Y - f(U) \subset H$  and  $f^{-1}(H) \subset X-U$ . Therefore we obtain  $f(U) = Y - H$  and hence  $f(U)$  is preopen in  $Y$ . This shows that  $f$  is pre-Rg-open.

**Theorem 3.15:** Let  $f : X \rightarrow Y$  be a function. Then the following are equivalent.

- (i)  $f$  is pre-Rg-open
- (ii) The image of each rg-open set in  $X$  is preopen in  $Y$ .
- (iii) The image of each rg-closed set in  $X$  is preclosed in  $Y$ .

**Proof:** (i)  $\Leftrightarrow$  (ii) : it follows from definition.

(ii)  $\Leftrightarrow$  (iii) : Let  $F$  be any rg-closed set in  $X$ . Then  $X-F$  is rg-open in  $X$ . Since  $f$  is preopen,  $f(X-F)$  is preopen in  $Y$ . But  $f(X-F) = Y - f(F)$  is preclosed in  $Y$ . Therefore  $f(F)$  is preclosed in  $Y$ .

We define the following.

**Definition 3.16:** A function  $f : X \rightarrow Y$  is said to be rg-regular-open if for each rg-open set of  $X$  is regular-open in  $Y$ . Clearly, Every rg-regular-open function is pre-rg-open. We prove the following.

**Theorem 3.17:** A surjective function  $f : X \rightarrow Y$  is rg-regular-open if and only if for each subset  $B$  of  $Y$  and each rg-closed set

$F$  of  $X$  containing  $f^{-1}(B)$ , there exists regular-closed set  $H$  of  $Y$  such that  $B \subset H$  and  $f^{-1}(H) \subset F$ .

**Proof:** Similar to Th.3.14.

**Theorem 3.18:** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. If  $f$  is rg-regular-open and  $g$  is almost-preopen function, then their composition  $g \circ f : X \rightarrow Z$  is pre-Rg-open.

**Proof:** Let  $U$  be an rg-open set in  $X$ . Since  $f$  is rg-regular-open. Then  $f(U)$  is regular-open in  $Y$ . Hence  $g(f(U))$  is preopen in  $Z$  because  $g$  is almost preopen function. But  $g(f(U)) = g \circ f(U)$ . This shows that  $g \circ f$  is pre-Rg-open.

**Theorem 3.19:** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. If  $f$  is strongly-rg-open function and preopen function  $g$ , then their composition  $g \circ f : X \rightarrow Z$  is pre-Rg-open.

**Proof :** Let  $U$  be an arbitrary rg-open set in  $X$ . Since  $f$  is strongly-rg-open. Then  $f(U)$  is open in  $Y$ . Hence  $g(f(U))$  is preopen in  $Z$  because  $g$  is preopen function. But  $g(f(U)) = g \circ f(U)$ . This shows that  $g \circ f$  is pre-Rg-open.

We, state the following

**Theorem 3.20:** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions and  $g \circ f$  is pre-Rg-open function. Then,

- (i) If  $f$  is rg-irresolute and surjective, then  $g$  is pre-Rg-open.
- (ii) If  $g$  is pre-irresolute and injective, then  $f$  is pre-Rg-open.

**Theorem 3.21:** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions and  $g \circ f$  is always rg-open function. Then,

- (i) If  $f$  is rg-irresolute and surjective, then  $g$  is always rg-open.
- (ii) If  $g$  is rg-irresolute and injective, then  $f$  is always rg-open.

**Theorem 3.22 :** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. Then,

- (i)  $g \circ f$  is M-preopen, if  $f$  is prg-open and  $g$  is pre-Rg-open function.
- (ii)  $g \circ f$  is pre-Rg-open, if  $f$  is pre-Rg-open and  $g$  is M-preopen function.
- (iii)  $g \circ f$  is almost preopen, if  $f$  is pre-rg-open and  $g$  is pre-Rg-open function.
- (iv)  $g \circ f$  is pre-Rg-open, if  $f$  is always rg-open and  $g$  is pre-Rg-open function.

#### 4. Properties of pre-rg-closed functions

In this section we define the following:

**Definition 4.1:** A function  $f : X \rightarrow Y$  is called pre-Rg-closed if the image of each rg-closed set of  $X$  is preclosed in  $Y$ .

**Definition 4.2:** A function  $f : X \rightarrow Y$  is said to be contra rg-closed if for each closed set  $F$  of  $X$ ,  $f(F)$  is rg-open set in  $Y$ .

**Definition 4.3:** A function  $f : X \rightarrow Y$  is said to be contra always-rg-closed if for each rg-closed set  $F$  of  $X$ ,  $f(F)$  is rg-open in  $Y$ .

**Definition 4.4:** A function  $f : X \rightarrow Y$  is said to be rg-regular-closed if the image of each rg-closed set of  $X$  is regular-closed in  $Y$ .

We, prove the following.

**Theorem 4.5 :** A surjective function  $f : X \rightarrow Y$  is pre-Rg-closed if and only if for each subset  $B$  of  $Y$  and each rg-open set  $U$  of  $X$  containing  $f^{-1}(B)$ , there exists preopen set  $V$  of  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ .

**Proof:** Necessity: Suppose  $f$  is pre-Rg-closed. Let  $B$  be any subset of  $Y$  and  $U$  is rg-open set of  $X$  containing  $f^{-1}(B)$ . Put  $V = Y - f(X-U)$ . Then,  $V$  is preopen in  $Y$ ,  $B \subset V$  and  $f^{-1}(V) \subset U$ .

Sufficiency : Let  $F$  be any rg-closed set of  $X$ . Put  $B = Y - f(F)$ , then we have  $f^{-1}(B) \subset X-F$  and  $X-F$  is rg-open such that

$B = Y - f(F) \subset V$  and  $f^{-1}(V) \subset X - F$ . Therefore, we obtain  $f(F) = Y - V$  and hence  $f(F)$  is preclosed in  $Y$ . This shows that  $f$  is pre-Rg-closed.

**Theorem 4.6 :** Let  $f : X \rightarrow Y$  be a function. Then the following are equivalent.

- (i)  $f$  is pre-Rg-closed.
- (ii) The image of each rg-closed set in  $X$  is preclosed in  $Y$ .
- (iii) The image of each rg-open set in  $X$  is preopen in  $Y$ .

Easy proof is omitted.

**Theorem 4.7 :** A surjective function  $f : X \rightarrow Y$  is rg-regular-closed if and only if for each subset  $B$  of  $Y$  and each rg-open set  $U$  of  $X$  containing  $f^{-1}(B)$ , there exists regular-open set  $V$  of  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ .

**Proof:** Necessity: Suppose  $f$  is rg-regular-closed. Let  $B$  be any subset of  $Y$  and  $U$  is rg-open set of  $X$  containing  $f^{-1}(B)$ . Put  $V = Y - f(X - U)$ . Then,  $V$  is rg-open in  $Y$ ,  $B \subset V$  and  $f^{-1}(V) \subset U$ .

Sufficiency : Let  $F$  be any rg-closed set in  $X$ . Put  $B = Y - f(F)$ , then we have  $f^{-1}(B) \subset X - F$  and  $X - F$  is rg-open set in  $X$ . There exists regular-open set  $V$  of  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset X - F$ . Therefore we obtain  $f(F) = Y - V$  and hence  $f(F)$  is regular-closed in  $Y$ . This shows that  $f$  is rg-regular-closed.

We define the following..

**Definition 4.8 :** A function  $f : X \rightarrow Y$  is said to be strongly rg-closed if the image of each rg-closed set of  $X$  is closed in  $Y$ .

**Definition 4.9 :** A space  $X$  is said to be p-rg-normal if for any pair of disjoint rg-closed sets  $A, B$  of  $X$ , there exists disjoint preopen sets  $U, V$  such that  $A \subset U$  and  $B \subset V$ .

**Definition 4.10 :** A space  $X$  is said to be p-rg-regular if for each rg-closed set  $F$  of  $X$  and each point  $x \in X - F$ , there exist disjoint preopen sets  $U$  and  $V$  of  $X$  such that  $F \subset U$  and  $x \in V$ .

**Definition 4.11 :** A function  $f : X \rightarrow Y$  is said to be prg-closed if the image of each preclosed set of  $X$  is rg-closed in  $Y$ .

**Theorem 4.12 :** A surjective function  $f : X \rightarrow Y$  is always rg-closed if and only if for each subset  $B$  of  $Y$  and each rg-open set  $U$  of  $X$  containing  $f^{-1}(B)$ , there exists rg-open set  $V$  of  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ .

**Proof:** Necessity: Suppose  $f$  is always rg-closed. Let  $B$  be any subset of  $Y$  and  $U$  is rg-open set of  $X$  containing  $f^{-1}(B)$ . Put  $V = Y - f(X - U)$ . Then,  $V$  is rg-open in  $Y$ ,  $B \subset V$  and  $f^{-1}(V) \subset U$ .

Sufficiency : Let  $F$  be any rg-closed set in  $X$ . Put  $B = Y - f(F)$ , then we have  $f^{-1}(B) \subset X - F$  and  $X - F$  is rg-open set in  $X$ . There exists rg-open set  $V$  of  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset X - F$ . Therefore we obtain  $f(F) = Y - V$  and hence  $f(F)$  is rg-closed in  $Y$ . This shows that  $f$  is always rg-closed.

**Theorem 4.13 :** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. If  $f$  is rg-regular-closed and  $g$  is almost-preclosed

function, then their composition  $g \circ f : X \rightarrow Z$  is pre-Rg-closed.

**Proof :** Let  $F$  be an rg-closed set in  $X$ . Since  $f$  is rg-regular-closed. Then  $f(F)$  is regular-closed in  $Y$ . Hence  $g(f(F))$  is preclosed in  $Z$  because  $g$  is almost preclosed function. But  $g(f(F)) = g \circ f(F)$ . This shows that  $g \circ f$  is pre-Rg-closed.

**Theorem 4.14:** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. If  $f$  is strongly-rg-closed and  $g$  is preclosed function  $g$  then their composition  $g \circ f : X \rightarrow Z$  is pre-Rg-closed.

**Proof :** Let  $F$  be any rg-closed set in  $X$ . Since  $f$  is strongly rg-closed. Then  $f(F)$  is closed in  $Y$ . Hence  $g(f(F))$  is preclosed in  $Z$  because  $g$  is preclosed function. But  $g(f(F)) = g \circ f(F)$ . This shows that  $g \circ f$  is pre-Rg-closed.

We, state the following.

**Theorem 4.15 :** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. If  $f$  is strongly-rg-closed and  $g$  is preclosed function, then their composition  $g \circ f : X \rightarrow Z$  is pre-Rg-closed.

- (i) If  $f$  is rg-irresolute and surjective, then  $g$  is pre-Rg-closed.
- (ii) If  $g$  is preirresolute and injective, then  $f$  is pre-Rg-closed.

**Theorem 4.16 :** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions and  $g \circ f$  is always rg-closed function.

- (i) If  $f$  is rg-irresolute and surjective, then  $g$  is always rg-closed.
- (ii) If  $g$  is rg-irresolute and injective, then  $f$  is always rg-closed.

**Theorem 4.17 :** Let  $f : X \rightarrow Y$  and  $g : X \rightarrow Y$  be two functions . Then,

- (i)  $g \circ f$  is M-preclosed, if  $f$  is prg-closed and  $g$  is pre-Rg-closed function.
- (ii)  $g \circ f$  is pre-Rg-closed, if  $f$  is pre-Rg-closed and  $g$  is M-preclosed function.
- (iii)  $g \circ f$  is almost preclosed, if  $f$  is pre-rg-closed and  $g$  is pre-Rg-closed function.
- (iv)  $g \circ f$  is pre-Rg-closed, if  $f$  is always rg-closed and  $g$  is pre-Rg-closed function.

We, prove the following.

**Theorem 4.18:** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. Then  $f$  is rg-g-closed function and  $g$  is g-rg-closed and  $g \circ f$  is always-rg-closed function.

**Proof:** Let  $F$  be an arbitrary rg-closed set in  $Y$ . Since  $f$  is rg-g-closed, then  $f(F)$  is g-closed in  $Y$ . Hence  $g(f(F))$  is rg-closed in  $Z$  because  $g$  is g-rg-closed function. But  $g(f(F)) = g \circ f(F)$ . This shows that  $g \circ f$  is always rg-closed.

**Theorem 4.19:** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions . Then  $f$  is rg-regular-closed function and  $g$  is regular-preclosed function, then their composition  $g \circ f$  is pre-Rg-closed function.

**Proof :** Let  $F$  be anyrg-closed set in  $X$ . Since  $f$  is rg-regular-closed, then  $f(F)$  is regular-closed in  $Y$ . Hence  $g(f(F))$  is

preclosed in  $Z$  because  $g$  is regular-preclosed function. But  $g(f(F)) = \text{gof}(F)$ . This shows that  $\text{gof}$  is pre-Rg-closed.

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