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Pre-Rg-open and Pre-Rg-closed Functions in Topology

Govindappa Navalagi[1], Sujata Mookanagoudar[2]

[1]Department of Mathematics, KIT Tiptur-572202, Karnataka, India

[2]Department of Mathematics, Government First Grade College, Haliyal-581329, Karnataka, India

Abstract: ***Levine in 1970, introduced the concept of generalized closed (g-closed) sets in topological space and a class of topological spaces called $T_{1/2}$spaces. Palaniappan et al in 1993, introduced the notions of regular generalized (in brief, rg-) closed sets , rg-open sets, rg-continuity and rg-irresoluteness and in 1997 ,Arokia Rani et al introduced and studied the concepts of rg-openness and rg-closedness in topology. In 2013, Navalagi et al have studied the concepts of pre-rg-openness and other allied rg-openness in topological spaces.The purpose of this paper is to investigate the concept of pre-Rg-open functions,pre-Rg-closed functions and other allied Rg-openness in topology.***

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1. Introduction

N.Levine[5] introduced the class of g-closed sets, a super class of closed sets in 1970. Palaniappan [14] defined rg-closed sets, rg-open sets, rg-continuous functions, rg-irresolute functions between topological spaces, in 1993. Later, in 1997.

Arokaraniet.al [1] have defined and studied the notion of rg-open functions and rg-closed functions. Recently, G.Navalagi et al [11] have introduced and studied the concept of pre-rg-open functions in toplogical spaces. In this paper, we introduce and study the new classes of functions namely, pre-Rg-open functions and pre-Rg-closed functions in topological spaces.

2. Preliminaries

In what follows, spaces X and Y are always topological spaces Cl(A) and Int(A) designate the closure and the interior of A which is a subset of X. A set A is said to be regular open (resp. Regular closed) if A = Int (Cl(A)) (resp. A=Cl(Int(A))).

Definition 2.1 [6]**:** A subset A of a space X is said to be a pre-open if A$\subset$ Int(Cl(A)).

The family of all preopen sets in a space X is denoted by PO(X). The complement of a preopen set of a space X is called preclosed [3].

Definition 2.2 [7]**:** The union of all preopen sets contained in A is called the preinterior of A and is denoted by pInt(A) .

Definition 2.3[3]: The intersection of all preclosed sets containing A is called the preclosure of A and is denoted by pCl(A).

Definition 2.4 [5]: A subset A of a space X is called a generalized closed set (g-closed) set if Cl(A)$\subset$U whenever A $\subset$ U and U is open.

Clearly, every closed set is a g-closed set. The complement of a g-closed set in X is called generalized open or g-open set. Clearly, every open set is a g-open set.

Definition 2.5 [12]**:** A subset A of a space X is called a generalized preclosed set (gp-closed) set if pCl(A)$\subset$U whenever A$\subset$ U and U is open.

The complement of a gp-closed set in X is called a generalized open or gp-open set.

It is obvious that every closed set is preclosed set, every closed set is g-closed set, every g-closed set is gp –closed set and every preopen set is a gp-open set.

Definition 2.6 [14]**:** A subset A of a space X is called a regular generalized closed(rg-closed) set if Cl(A)$\subset$U whenever A$\subset$U and U is regular open. The complement of a rg-closed set of a space is called rg-open. The family of all rg-open sets of a space X is denoted by RGO(X).

Definition 2.7: A function f: X $\rightarrow$Y is said to be :

(i) preopen [7] if the image of each open set U of X, f(U) is preopen in Y.
(ii) preclosed[3] if the image of each closed set F of X, f(F) is preclosed in Y.
(iii) preirresolute [16] if the inverse image of each preopen set of Y is preopen in X.
(iv) gp-closed [12] if the image of each closed set of X is gp-closed in Y.
(v) pre-gp-closed [12] if the image of each preclosed set of X is gp-closed in Y.
(vi) gp-open [9] if the image of each open set of X is gp-open in Y.

(vii) always-gp-closed [9] if the image each gp-closed set of X is gp-closed in Y.

Definition 2.8 [13]: A function f: X →Y is said to be almost preclosed if the image of each regular closed set of X is preclosed in Y.

Definition 2.9 [8]: A function f: X →Y is said to be M-preopen if the image of each preopen set of X is preopen in Y.

Definition 2.10[8]: A function f: X →Y is said to be M-preclosed if the image of each pre- closed set of X is preclosed in Y.

Definition 2.11: A function f : X→ Y is called
- i) rg-open[1] if image of each open set of X is rg-open in Y.
- ii) rg-closed [1] if image of each closed set of X is rg-closed in Y.

Definition 2.12: A function f : X→ Y is said to be
- i) rg- irresolute [14] if the inverse image of each rg- open set V of Y is rg-open in X.
- ii) perfectly rg-continuous [1] if the inverse image of each rg-open set in Y is both open and closed in X.

Definition 2.13 [11]: A function f : X→Y is said to be pre-rg-open if the image of each regular open set of X is rg-open in Y.

Definition 2.14[11]: A function f : X→ Y is called strongly rg-open if the image of each rg-open set of X is open in Y.

Definition 2.17[11]: A function f : X→Y is said to be always rg-open , if the image of each rg-open set of X is rg-open in Y.

Definition 2.19[11]: A function f : X→Y is said to be prg-open if image of each preopen set of X is rg-open in Y.

Definition 2.20[11]: A function f :X→Y is called strongly p-open if the image of each preopen set of X is open in Y.

Clearly, every strongly p-open function is M-preopen.

3. Properties of pre-Rg-open functions

Definition 3.1: A function f : X → Y is called pre-Rg-open if the image of each rg-open set of X is preopen in Y.

Clearly, (i) prg-open functions and Pre-Rg-open functions are dual in nature and hence their composition yields a M-preopen function due to Mashhour et al. [8]. (ii) Every strongly-rg-open function is pre-Rg-open function. (iii) Every pre-Rg-open function is preopen.

We, characterize the prg -openness in the following.

Theorem 3.2: Let f: X → Y be a map. Then the following are equivalent:

- i) f is prg - open
- ii) The image of each preopen set in X is rg-open in Y
- iii) The image of each preclosed set in X is rg-closed in Y

Obvious proof is omitted.

Theorem 3.3: If f : X → Y is strongly rg-open and g : Y → Z is preopen , then the composition gof : X →Z is pre-Rg-open .

Proof: Let U be rg- open set in X, then f(U) is open in Y since f is strongly rg-open . Since g is preopen and f(U) is open set in Y, g(f(U))= gof(U) is preopen set in Z. This shows that gof is pre-Rg-open function.

Theorem 3.4: If f : X→Y is prg-open and g : Y→Z is alays-rg-open , then the composition gof :X →Z is prg-open .

Easy proof is omitted.

We, define the following.

Definition 3.5: A topological space X is said to be $T^{**}_{1/2}$ iff every rg-closed set is preclosed.

Definition 3.6: A topological space X is said to be T^{*}_{rg} space iff every rg-closed set is gp-closed.

Theorem 3.7: Let X and Z be any topological spaces and Y be a $T^{**}_{1/2}$ space. Then the composition gof : X → Z of the rg-closed function f : X→Y and M-preclosed function g :Y→Z is preclosed function.

Proof: Let F be any closed set in X . Since f is rg-closed function, f(F) is rg-closed in Y. But Y is $T^{**}_{1/2}$ –space, therefore f(F) is preclosed set in Y which implies that g(f(F))= gof(F) is preclosed set in Z. This shows that gof is preclosed.

Theorem 3.8: Let X and Z be any topological spaces and Y be a $T^{**}_{1/2}$ space. Then,if f : X→Y be rg-closed function and g :Y→Z be pre-gp-closed then gof is gp-closed .

Proof: Obvious.

Theorem 3.9: Let X and Z be any topological spaces and Y be a T^{*}_{rg} -space. Then the composition gof : X → Z of the rg-closed function f : X→Y and always gp-closed function g :Y→Z is gp-closed function.

Proof: Obvious.

We, define the following.

Definition 3.10: A function f : X → Y is said to be contra rg-open if for each open set U of X , f(U) is rg-closed set in Y.

Definition 3.11: A function f : X → Y is said to be regular -rg-open if for each regular open set U of X , f(U) is rg-open in Y.

Definition 3.12: A function $f : X \to Y$ is said to be contra regular-rg-open if for each regular-open set U of X, f(U) is rg-closed in Y.

Definition 3.13: A function $f : X \to Y$ is said to be contra always-rg-open if for each rg-open set U of X, f(U) is rg-closed in Y. We, prove the following.

Theorem 3.14: A surjective function $f : X \to Y$ is pre-Rg-open if and only if for each subset B of Y and each rg-closed set F of X containing $f^{-1}(B)$, there exists preclosed set H of Y such that $B \subset H$ and $f^{-1}(H) \subset F$.

Proof: Necessity: Suppose f is pre-Rg-open. Let B be any subset of Y and F is rg-closed set of X containing $f^{-1}(B)$. Put H = f(X-F). Then, H is preclosed in Y, $B \subset H$ and $f^{-1}(H) \subset F$.

Sufficiency: Let U be any rg-open set in X. Put B = Y- f(U), then we have $f^{-1}(B) \subset X\text{-}U$ and X-U is rg-closed such that B= Y-f(U) $\subset$ H and $f^{-1}(H) \subset X\text{-}U$. Therefore we obtain f(U) = Y – H and hence f(U) is preopen in Y. This shows that f is pre-Rg-open.

Theorem 3.15: Let $f : X \to Y$ be a function. Then the following are equivalent.
(i) f is pre-Rg-open
(ii) The image of each rg-open set in X is preopen in Y.
(iii) The image of each rg-closed set in X is preclosed in Y.

Proof: (i) $\Leftrightarrow$ (ii) : it follows from definition.
(ii)$\Leftrightarrow$ (iii) : Let F be any rg-closed set in X. Then X-F is rg-open in X. Since f is preopen, f(X-F) is preopen in Y. But f(X-F) = Y – f(F) is preclosed in X. Therefore f(F) is preclosed in Y.
We define the following.

Definition 3.16: A function $f : X \to Y$ is said to be rg-regular-open if for each rg-open set of X is regular-open in Y. Clearly, Every rg-regular-open function is pre-rg-open. We prove the following.

Theorem 3.17: A surjective function $f : X \to Y$ is rg-regular-open if and only if for each subset B of Y and each rg-closed set
F of X containing $f^{-1}(B)$, there exists regular-closed set H of Y such that $B \subset H$ and $f^{-1}(H) \subset F$.
Proof: Similar to Th.3.14.

Theorem 3.18: If $f : X \to Y$ and $g : Y \to Z$ be two functions. If f is rg-regular-open and g is almost-preopen function, then their composition $gof : X \to Z$ is pre-Rg-open.

Proof: Let U be an rg-open set in X. Since f is rg-regular-open. Then f(U) is regular-open in Y. Hence g(f(U)) is preopen in Z because g is almost preopen function. But g(f(U)) = gof(U). This shows that gof is pre-Rg-open.

Theorem 3.19: Let $f: X \to Y$ and $g : Y \to Z$ be two functions. If for strongly-rg-open function and preopen function g, then their composition $gof: X \to Z$ is pre-Rg-open.

Proof : Let U be an arbitrary rg-open set in X. Since f is strongly-rg-open. Then f(U) is open in Y. Hence g(f(U)) is preopen in Z because g is preopen function. But g(f(U)) = gof(U). This shows that gof is pre-Rg-open.

We, state the following

Theorem 3.20: Let $f : X \to Y$ and $g : Y \to Z$ be two functions and gof is pre-Rg-open function. Then,
(i) If f is rg-irresolute and surjective, then g is pre-Rg-open.
(ii) If g is pre-irresolute and injective, then f is pre-Rg-open.

Theorem 3.21: Let $f : X \to Y$ and $g : Y \to Z$ be two functions and gof is always rg-open function. Then,
(i) If f is rg-irresolute and surjective, then g is always rg-open.
(ii) If g is rg-irresolute and injective, then f is always rg-open.

Theorem 3.22 : Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Then,
(i) gof is M-preopen, if f is prg-open and g is pre-Rg-open function.
(ii) gof is pre-Rg-open, if f is pre-Rg-open and g is M-preopen function.
(iii) gof is almost preopen, if f is pre-rg-open and g is pre-Rg-open function.
(iv) gof is pre-Rg-open, if f is always rg-open and g is pre-Rg-open function.

4. Properties of pre-rg-closed functions

In this section we define the following:

Definition 4.1: A function $f : X \to Y$ is called pre-Rg-closed if the image of each rg-closed set of X is preclosed in Y.

Definition 4.2: A function $f : X \to Y$ is said to be contra rg-closed if for each closed set F of X, f(F) is rg-open set in Y.

Definition 4.3: A function $f : X \to Y$ is said to be contra always-rg-closed if for each rg-closed set F of X, f(F) is rg-open in Y.

Definition 4.4: A function $f : X \to Y$ is said to be rg-regular-closed if the image of each rg-closed set of X is regular-closed in Y.

We, prove the following.

Theorem 4.5 : A surjective function $f : X \to Y$ is pre-Rg-closed if and only if for each subset B of Y and each rg-open set U of X containing $f^{-1}(B)$, there exists preopen set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof: Necessity: Suppose f is pre-Rg-closed. Let B be any subset of Y and U is rg-open set of X containing $f^{-1}(B)$. Put V = Y-f(X-U). Then, V is preopen in Y, $B \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency : Let F be any rg-closed set of X. Put B = Y-f(F), then we have $f^{-1}(B) \subset X\text{-}F$ and X-F is rg-open such that

B= Y-f(F) ⊂ V and $f^{-1}(V)$ ⊂ X-F. Therefore, we obtain f(F) = Y – V and hence f(F) is preclosed in Y. This shows that f is pre-Rg-closed.

Theorem 4.6 : Let f : X → Y be a function. Then the following are equivalent.

(i) f is pre-Rg-closed.
(ii) The image of each rg-closed set in X is preclosed in Y.
(iii) The image of each rg-open set in X is preopen in Y.

Easy proof is omitted.

Theorem 4.7 : A surjective function f : X → Y is rg-regular-closed if and only if for each subset B of Y and each rg-open set U of X containing $f^{-1}(B)$, there exists regular-open set V of Y such that B ⊂ V and $f^{-1}(V)$ ⊂U.

Proof: Necessity:Suppose f is rg-regular-closed. Let B be any subset of Y and U is rg-open set of X containing $f^{-1}(B)$. Put V = Y- f(X-U). Then, V is rg-open in Y, B ⊂Vand $f^{-1}(V)$ ⊂U.

Sufficiency : Let F be any rg-closed set in X. Put B = Y-f(F), then we have $f^{-1}(B)$ ⊂ X-F and X-F is rg-open set in X. There exists regular-open set V of Y such that B= Y-f(F) ⊂ V and $f^{-1}(V)$ ⊂ X-F. Therefore we obtain f(F) = Y – V and hence f(F) is regular-closed in Y. This shows that f is rg-regular-closed.

We define the following..

Definition 4.8 : A function f : X → Y is said to be strongly rg-closed if the image of each rg-closed set of X is closed in Y.

Definition 4.9 : A space X is said to be p-rg-normal if for any pair of disjoint rg-closed sets A, B of X, there exists disjoint preopen sets U,V such that A ⊂ U and B ⊂ V.

Definition 4.10 : A space X is said to be p-rg-regular if for each rg-closed set F of X and each point x ϵ X-F, there exist disjoint preopen sets U and V of X such that F ⊂ U and x ϵ V.

Definition 4.11 :A function f : X → Y is said to be prg-closed if the image of each preclosed set of X is rg-closed in Y.

Theorem 4.12 : A surjective function f : X → Y is always rg-closed if and only if for each subset B of Y and each rg-open set U of X containing $f^{-1}(B)$, there exists rg-open set V of Y such that B ⊂ V and $f^{-1}(V)$ ⊂ U.

Proof: Necessity: Suppose f is always rg-closed. Let B be any subset of Y and U is rg-open set of X containing $f^{-1}(B)$. Put V = Y- f(X-U). Then, V is rg-open in Y, B ⊂ V and $f^{-1}(V)$ ⊂ U.

Sufficiency : Let F be any rg-closed set in X. Put B = Y-f(F), then we have $f^{-1}(B)$ ⊂ X-F and X-F is rg-open set in X. There exists rg-open set V of Y such that B= Y-f(F) ⊂ V and $f^{-1}(V)$ ⊂ X-F. Therefore we obtain f(F) = Y – V and hence f(F) is rg-closed in Y. This shows that f is always rg-closed.

Theorem 4.13 : If f : X → Y and g : Y → Z be two functions. If f is rg-regular-closed and g is almost-preclosed function, then their composition gof : X → Z is pre-Rg-closed.

Proof : Let F be an rg-closed set in X. Since f is rg-regular-closed. Then f(F) is regular-closed in Y. Hence g(f(F)) is preclosed in Z because g is almost preclosed function. But g(f(F)) = gof(F). This shows that gof is pre-Rg-closed.

Theorem 4.14: If f : X → Y and g : Y → Z be two functions. If f is strongly-rg-closed and g is preclosed function g then their compositiong of : X → Z is pre-Rg-closed.

Proof : Let F be any rg-closed set in X. Since f is strongly rg-closed. Then f(F) is closed in Y. Hence g(f(F)) is preclosed in Z because g is preclosed function. But g(f(F)) = gof(F). This shows that gof is pre-Rg-closed.

We, state the following.

Theorem 4.15 : If f : X → Y and g : Y → Z be two functions. If f is strongly-αg-closed and g is preclosed function, then their composition gof : X → Z is pre-Rg-closed.

(i) If f is rg-irresolute and surjective, then g is pre-Rg-closed.
(ii) If g is preirresolute and injective, then f is pre-Rg-closed.

Theorem 4.16 : If f : X → Y and g : Y → Z be two functions and gof is always rg-closed function.

(i) If f is rg-irresolute and surjective, then g is always rg-closed.
(ii) If g is rg-irresolute and injective, then f is always rg-closed.

Theorem 4.17 : Let f : X → Y and g : X→Y be two functions . Then,

(i) gof is M-preclosed, if f is prg-closed and g is pre-Rg-closed function.
(ii) gof is pre-Rg-closed, if f is pre-Rg-closed and g is M-preclosed function.
(iii) gof is almost preclosed, if f is pre-rg-closed and g is pre-Rg-closed function.
(iv) gof is pre-Rg-closed, if f is always rg-closed and g is pre-Rg-closed function.

We, prove the following.

Theorem 4.18: Let f : X → Y and g : Y→Z be two functions. Then f is rg-g-closed function and g is g-rg-closed and gof is always-rg-closed function.

Proof: Let F be an arbitrary rg-closed set in Y. Since f is rg-g-closed, then f(F) is g-closed in Y. Hence g(f(F)) is rg-closed in Z because g is g-rg-closed function. But g(f(F)) = gof(F). This shows that gof is always rg-closed.

Theorem 4.19: Let f : X → Y and g : Y→Z be two functions . Then f is rg-regular-closed function and g is regular-preclosed function, then their compositingof is pre-Rg-closed function.

Proof : Let F be anyrg-closed set in X. Since f is rg-regular-closed, then f(F) is regular-closed in Y. Hence g(f(F)) is preclosed in Z because g is regular-preclosed function. But g(f(F)) = gof(F). This shows that gof is pre-Rg-closed.

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