

Using Diffie-Hellman Key - Exchange in RADG

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Abstract: In this paper used a Diffie-Hellman key exchange with data of representation braid group to generation keys, encryption, and decryption method by using RADG (Reaction Automata Direct Graph) cryptosystem.

Keywords: key-exchange, RADG cryptosystem, braid group

1. Introduction

The confidentiality in Cryptography is the science of encryption and decryption depend on many mathematical concepts, such as braid groups which are infinite non-commutative groups [1]. There are many relationships between non-commutative groups and Theoretical of cryptosystems with Key Agreement Protocols based on groups, because used the conjugacy problem (or transformation problem) for a group[2], it is undecidable for many classes of groups [3], and it be used in public-key cryptography. The special case RADG (Reaction Automata Direct Graph) cryptosystem, Albermany, and Ghazanfar proposed a new cryptosystem for creation random multi-cipher text for the same plaintext[4] by use Direct graph, which is based on automata direct graph with special node called state, and other reaction states, which is by divided the state into two collection sets , the first set Q of normal set , it's have a subset set J called jump set, and the second R set called reaction set, each sets of R, and Q except J , have λ values inside the state [4]. Albermany, and Fatima Radi proposed new two method depended on RADG cryptosystem , called BRADG, and RBC, use key block cipher based on structure of unbalanced Fiestel, and new S-boxes[5]. Alwan proposed design is changeable, and faster, it's developed to RADG, by use Multi-Reaction states, called MRADG [6]. Nathim Rasool solved the problem of transition states in design by proposed system depend it on chaotic map equation, (for example logistic map equation), called CRADG [7]. Mahdi use the RADG in to development the stream cipher automata algorithm [8], Albakaa use McElliece, and Diffie-Hellman to improving RADG system [9].

The propose of the Diffie and Hellman (or Deffie-Hellman) for key exchange was one of the first public-key protocols 1976 [10],there were a lot of people have been proposed public-key cryptosystem (PKC) and broken [11] , the famous public-key cryptosystem is depend on the prime numbers such as RSA [8] and its variants.

Another approach is it use hard problems based on the braid groups such as Anshel-Anshel-Goldfeld [12], with number of cryptographic protocols using non-commutative groups including Cha-Ko-Lee-Han-Cheon braid groups [11]. The braid groups B_n is an infinite non-commutative group of n-braids, where $n > 1$.

B_n is defined as:

$$B_n = \left\langle \sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{n-1} \left| \begin{array}{l} \sigma_i \sigma_j = \sigma_j \sigma_i \text{ if } |i - j| \geq 2 \\ \sigma_i \sigma_j \sigma_i = \sigma_j \sigma_i \sigma_j \text{ if } |i - j| = 1 \end{array} \right. \right\rangle$$

In n-braid groups B_n , where n is braid index , $m = \text{floor}(n/2)$, the lower braid LB_n (also called as left braid) and upper braid UB_n (also called as right braid) are define as $LB_n = \langle \sigma_1, \sigma_2, \dots, \sigma_{m-1} \rangle$ and $UB_n = \langle \sigma_{m+1}, \sigma_{m+2}, \sigma_{m+3}, \dots, \sigma_{n-1} \rangle$, For any value of $a \in LB_n$ and $b \in UB_n$ we have $ab = ba$. The elements of B_n can be interpreted as deometric n strand braids [10] .

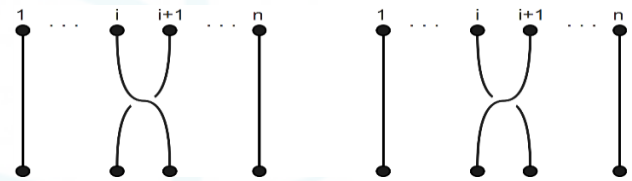


Figure 1.1: The elementary braids σ_i^{-1}, σ_i [13]

2. Burau Representation of braid Group

In 1930 Burau presented his representation for braid groups by the map

$$\gamma_n : B_n \rightarrow GL_n(\mathbb{Z}[t, t^{-1}])$$

defined by

$$\sigma_i \rightarrow I_{i-1} \oplus \begin{bmatrix} 1 & -t & t \\ & 1 & 0 \end{bmatrix} \oplus I_{n-(i+2)}$$

Where I_k denote the square identity matrix of size k [14]. The Burau representation classified into two types , first type Reducible representation where the image $\gamma_n(\sigma_i)$ of a generator σ_i of is B_n represented by the matrix irreducible representation where the images of generator σ_1, σ_{n-1} and σ_i ($2 \leq i \leq n-2$) of B_n by γ_n are represented as matrix of above description, but Birman showed the Burau representation γ_n is faithful when $n \leq 3$, In 1999 , S. Bigelow showed this map is unfaithful when $n \geq 5$, It is not known whether

$$\gamma_4 : B_n \rightarrow GL_4(\mathbb{Z}[t, t^{-1}])$$

3. Reaction Automata Direct Graph (RADG)

Mathematical model of (RADG) is effected by graph theory expressed by sextuple $\{Q, R, \Sigma, \Psi, J, T\}$ where the function

$F_Q(n,\lambda)$ is number of cases which consist of Design of the set Q which contains jump state.

The jump state in the set Q is represented with $|J| \leq \lfloor n/2 \rfloor$, it is clearly noticed that

$$F_Q(n,\lambda) \leq n^{(n-k)(\lambda-1)} (n-1)^{(n-k)}$$

where $k = 1, \dots, \lfloor n/2 \rfloor, (n-k) \geq \lambda$ [4]

4. Diffie-Hellman key exchange over braid Groups

Cheon et al. found in Ko’s research group had an idea of the possibility of using Diffie-Hellman key exchange based on braid group, There are many protocols that pertain to the original Devi-Hellman procedure can also rework this way, consider the subgroups of upper braids $UB_n = \langle \sigma_{m+1}, \sigma_{m+2}, \sigma_{m+3}, \dots, \sigma_{n-1} \rangle$, and lower braids $LB_n = \langle \sigma_1, \sigma_2, \dots, \sigma_{m-1} \rangle$, where $m = \text{floor}(n/2)$

Protocol:

- Public key: let $p \in B_n$.
- Private keys: Alice choose $x \in LB_n$, and Bob choose $y \in UB_n$
- Alice send to Bob $p' = xpx^{-1}$, and Bob send $p'' = ypy^{-1}$
- Shared secret key $K = xpyy^{-1}x^{-1}$
- K shared: Alice $K = xp''x^{-1} = xpyy^{-1}x^{-1}$
- Bob $K = yp'y^{-1} = yxpx^{-1}y^{-1}$

5. Implementation of method

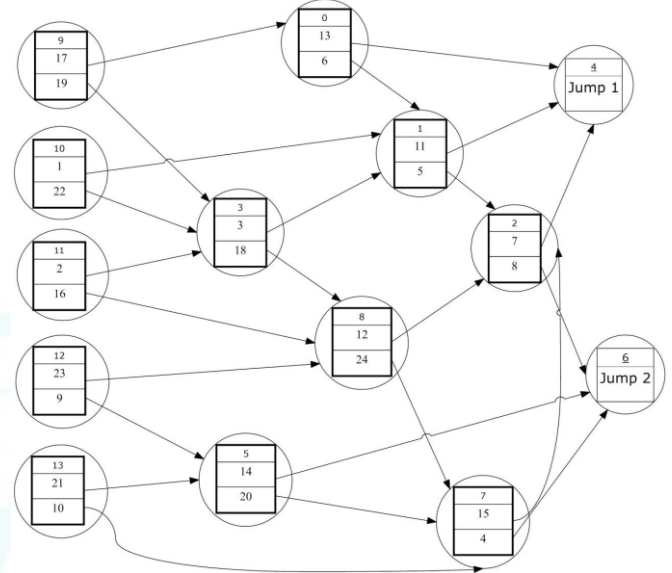
Suppose the message is “hello” the encryption steps for the first letter illustrates in the below table:

Index message	Message bit	State index	value	Jump state
0	0	3	3	
1	1	1	11	J
2	1	13	10	
3	0	7	15	J
4	1	11	16	
5	0	8	12	
6	0	2	7	
7	0	1	11	Short path

The cipher text is [14,8,19,3,20,13,15,13] the summation this values mod 256 equal to 105 represent the letter i . Start random at state number 3 , the message entered to state is “0” then select the corresponding cipher text is “3” as shown in figure below . The transition function drive to the next state which is “1” , the message enter to state number 1 is “1” ; to determine what the value choose from the values of $\lambda \{11,5\}$; and choice “5” that corresponding to the message value “1” . transition function drive to jump state which also transfer to Reaction state and choice state randomly from them. And so on ... ; at the final stage to ensure that we finish the encryption process in the set Q we apply short path . At the short path if the penultimate state is drive to Jump state, force it to choice the close state to finish in the set Q .

6. Conclusion

In this paper ,we proposed a new design is based on the concept of braid groups and RADG (Reaction Automata Direct Graph) ,the algorithm depends on Diffie-Hellman key exchange over braid group with RADG cryptosystem. The output of ciphertexts are random, to increase statistical frequency to broke of ciphertext.



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Author Profile



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