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Compound Option Pricing under Fuzzy Environment Revisit

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Abstract: ***Wang, He and Li published a paper in Journal of Applied Mathematics (Volume 2014, Article ID 875319, 9 pages) to develop a compound option pricing under fuzzy environment. However, we find that they implicitly assumed the crisp possibilistic mean value being commuting with fuzzy multiplication and fuzzy division. In this paper, we show that their assumptions are not valid to reveal that their theoretical derivations needed further improvement.***

Keywords: Compound Option Pricing, Fuzzy

1. Introduction

After Geske (1979) derived the closed form pricing formula for compound option, many authors have tried to extend his result under different conditions. For examples, Geske and Johnson (1984), Thomassen and Wouwe (2001), Lajeri-Chaherli (2002), Lin (2002), Cassimon et al. (2004), Gukhal (2004), Agliardi and Agliardi (2005), Fouque and Han (2005), Lee et al. (2008), Chiarella and Kang (2011), Griebsch (2013), Park et al. (2013) studied the valuation of compound options. Carr (1988), Paxson (2007), and Huang and Pi (2009) scrutinized sequential compound option approaches. Agliardi and Agliardi (2003), Chen (2003) considered Geske's model. Yoshida (2003), Yoshida et al. (2006), Chrysafis and Papadopoulos (2009), Thavaneswaran et al. (2009), Zmeskal (2010), Guerra et al. (2011), Thavaneswaran et al. (2013) examined options under fuzzy environment proposed by Zadeh (1965). Wu (2004) and Wu (2007) investigated Black-Scholes formula. Based on Geske (1979), for pricing formula of compound option with crisp setting, and then motivated by Wu (2004) and Nowak and Romaniuk (2010), Wang et al. (2014) developed a new method for the fuzzy price of compound option. The main purpose of this paper is to provide a detailed examination for the derivation in Wang et al. (2014) to compute the crisp possibilistic mean value of the compound option price. Wang et al. (2014) generalized compound option pricing under fuzzy environment that is an important generalization, because the interest rate and the volatility of compound option cannot decide the actual values owing to a lack of knowledge in the real stock market such that fuzziness is an appropriate method to deal with this situation. Among Wang et al. (2014) research, two papers have cited their research by Wang and He (2014) and Zhang et al. (2015). However, we find that the derivation of Wang et al. (2014) to evaluate the crisp possibilistic mean value contain questionable results. Hence, in this paper, we will first provide a recap of their derivation and then point out their unexplained findings. We will show that there are two lemmas that were used in Wang et al. (2014) without any verification that will be shown by us to be invalid.

2. Review of previous results

Based on the pricing formula for compound option of Geske (1979), Wu (2004) and Nowak and Romaniuk (2010), Wang et al. (2014) developed their compound option pricing under fuzzy environment. We directly cite their results in the next theorem. To save the precious space of the journal, we only list those related results. For a detailed derivation of Wang et al. (2014), please refer to the original paper of Wang et al. (2014).

Theorem 4 of Wang et al. (2014)

Let the interest rate and the volatility be fuzzy numbers. Then the fuzzy price of compound option is

$$\tilde{C} = \tilde{1}_{\{S\}} \otimes \tilde{N}_2\left(\tilde{d}_1, \tilde{d}_2, \tilde{1}_{\{\rho\}}\right) \pm \tilde{1}_{\{K_2\}} \otimes e^{-\tilde{r}\otimes\tilde{1}_{\{T_2\}}} \otimes \tilde{N}_2\left(\tilde{d}_3, \tilde{d}_4, \tilde{1}_{\{\rho\}}\right)$$
$$\pm \tilde{1}_{\{K_1\}} \otimes e^{-\tilde{r}\otimes\tilde{1}_{\{T_1\}}} \otimes \tilde{N}\left(\tilde{d}_3\right), \tag{1}$$

where

$$\tilde{d}_1 = \left[\tilde{1}_{\{\ln(S/S^*)\}} \oplus \left(\tilde{r} \oplus \tilde{1}_{\{1/2\}} \otimes \tilde{\sigma} \otimes \tilde{\sigma}\right) \otimes \tilde{1}_{\{T_1\}}\right] \div \left(\tilde{\sigma} \otimes \tilde{1}_{\{\sqrt{T_1}\}}\right), \tag{2}$$

$$\tilde{d}_2 = \left[\tilde{1}_{\{\ln(S/K_2)\}} \oplus \left(\tilde{r} \oplus \tilde{1}_{\{1/2\}} \otimes \tilde{\sigma} \otimes \tilde{\sigma}\right) \otimes \tilde{1}_{\{T_2\}}\right] \div \left(\tilde{\sigma} \otimes \tilde{1}_{\{\sqrt{T_2}\}}\right), \tag{3}$$

$$\tilde{d}_3 = \tilde{d}_1 \pm \left(\tilde{\sigma} \otimes \tilde{1}_{\{\sqrt{T_1}\}}\right), \tag{4}$$

$$\tilde{d}_4 = \tilde{d}_2 \pm \left(\tilde{\sigma} \otimes \tilde{1}_{\{\sqrt{T_2}\}}\right). \tag{5}$$

S^* is the unique solution of the equation

$$xN\left(d^* + M(\tilde{\sigma})\sqrt{T_2 - T_1}\right) - K_2 e^{-M(\tilde{r})(T_2 - T_1)} N\left(d^*\right) = K_1, \tag{6}$$

where

$$d^* = \frac{\ln(x/K_2) + \left(M(\tilde{r}) - (1/2)[M(\tilde{\sigma})]^2\right)(T_2 - T_1)}{M(\tilde{\sigma})\sqrt{T_2 - T_1}}, \tag{7}$$

$$M(\tilde{r}) = \int_0^1 \alpha\left(\tilde{r}_\alpha^L + \tilde{r}_\alpha^U\right) d\alpha, \tag{8}$$

$$M(\tilde{\sigma}) = \int_0^1 \alpha\left(\tilde{\sigma}_\alpha^L + \tilde{\sigma}_\alpha^U\right) d\alpha. \tag{9}$$

$\tilde{r}_\alpha^L$ and $\tilde{r}_\alpha^U$ are the left-end and right-end point of the $\alpha-$level set of $\tilde{r}$, respectively, and $\tilde{\sigma}_\alpha^L$ and $\tilde{\sigma}_\alpha^U$ are the left-end and right-end point of the $\alpha-$level set of $\tilde{\sigma}$, respectively.

From Fuller and Majlender (2003), the crisp possibilistic mean value of $\tilde{a}$ is denoted as

$$M(\tilde{a}) = \int_0^1 \alpha\left(\tilde{a}_\alpha^L + \tilde{a}_\alpha^U\right) d\alpha, \tag{10}$$

where α-cut of $\tilde{a}$ is expressed as $\left[\tilde{a}_\alpha^L, \tilde{a}_\alpha^U\right]$ and then the crisp possibilistic mean value of a triangular fuzzy number $\tilde{a} = (A - \gamma, A, A + \beta)$ is computed as

$$M(\tilde{a}) = A + \frac{\beta - \gamma}{6}. \tag{11}$$

Wang et al. (2014) computed the crisp possibilistic mean value of their compound option price. We also quote their findings in the following.

Theorem 6 of Wang et al. (2014)
The crisp possibilistic mean value of the compound option price is

$$M(\tilde{C}) = SN_2\left(M(\tilde{d}_1), M(\tilde{d}_2), \rho\right) - K_2 e^{-M(\tilde{r})T_2} N_2\left(M(\tilde{d}_3), M(\tilde{d}_4), \rho\right) - K_1 e^{-M(\tilde{r})T_1} N\left(M(\tilde{d}_3)\right), \tag{12}$$

where

$$M(\tilde{d}_1) = \frac{\ln(S/S^*) + \left(M(\tilde{r}) + (1/2)[M(\tilde{\sigma})]^2\right)T_1}{M(\tilde{\sigma})\sqrt{T_1}}, \tag{13}$$

$$M(\tilde{d}_2) = \frac{\ln(S/K_2) + \left(M(\tilde{r}) + (1/2)[M(\tilde{\sigma})]^2\right)T_2}{M(\tilde{\sigma})\sqrt{T_2}}, \tag{14}$$

$$M(\tilde{d}_3) = M(\tilde{d}_1) - \left(M(\tilde{\sigma})\sqrt{T_1}\right), \tag{15}$$

and

$$M(\tilde{d}_4) = M(\tilde{d}_2) - \left(M(\tilde{\sigma})\sqrt{T_2}\right). \tag{16}$$

Proof. From Theorem 4, Wang et al. (2014) derived that

$$M(\tilde{C}) = SM\left(\tilde{N}_2(\tilde{d}_1, \tilde{d}_2, \tilde{1}_{\{\rho\}})\right) - K_2 M\left(e^{-\tilde{r}\otimes\tilde{1}_{\{T_2\}}}\right) M\left(\tilde{N}_2(\tilde{d}_3, \tilde{d}_4, \tilde{1}_{\{\rho\}})\right) - K_1 M\left(e^{-\tilde{r}\otimes\tilde{1}_{\{T_1\}}}\right) M\left(\tilde{N}(\tilde{d}_3)\right), \tag{17}$$

$$= SN_2\left(M(\tilde{d}_1), M(\tilde{d}_2), \rho\right) - K_2 e^{-M(\tilde{r})T_2} N_2\left(M(\tilde{d}_3), M(\tilde{d}_4), \rho\right) - K_1 e^{-M(\tilde{r})T_1} N\left(M(\tilde{d}_3)\right), \tag{18}$$

where $M(\tilde{d}_i)$, $i = 1,2,3,4$, are given as Theorem 6, equations (13-16).

3. Our discussion for the theoretical derivation of Wang et al. (2014)

We study the Theorem 6 of Wang et al. (2014) to find out that they needed the following two lemmas to finish their proof. Consequently, we will point out their unexplained derivations in their proof of Theorem 6.

Lemma 1 (Implicitly used in Wang et al. (2014))

$$M(\tilde{a} \otimes \tilde{b}) = M(\tilde{a}) M(\tilde{b}). \tag{19}$$

Lemma 2 (Implicitly used in Wang et al. (2014))

$$M(\tilde{a} \div \tilde{b}) = \frac{M(\tilde{a})}{M(\tilde{b})}. \tag{20}$$

To illustrate Lammas 1 and 2 are questionable, we will use triangle fuzzy numbers to construct counterexamples. Consequently, we assume that there are two triangle fuzzy numbers $\tilde{a}$ and $\tilde{b}$ with the membership functions $\mu_{\tilde{a}}$ and $\mu_{\tilde{b}}$ where

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - \left((A - x)/\gamma_1\right), & A - \gamma_1 \le x \le A \\ 1 - \left((x - A)/\beta_1\right), & A \le x \le A + \beta_1 \\ 0, & \text{otherwise} \end{cases}, \tag{21}$$

and

$$\mu_{\tilde{b}}(x) = \begin{cases} 1 - \left((B - x)/\gamma_2\right), & B - \gamma_2 \le x \le B \\ 1 - \left((x - B)/\beta_2\right), & B \le x \le B + \beta_2 \\ 0, & \text{otherwise} \end{cases}, \tag{22}$$

under the assumption $\gamma_1 \ge 0$, $\beta_1 \ge 0$, $\gamma_2 \ge 0$, $\beta_2 \ge 0$, $0 < A - \gamma_1$ and $0 < B - \gamma_2$. Hence, from equations (21) and (22), we know that α-cut of $\tilde{a}$ as

$$\left[\tilde{a}_\alpha^L, \tilde{a}_\alpha^U\right] = \left[A - \gamma_1(1-\alpha), A + \beta_1(1-\alpha)\right], \tag{23}$$

and α-cut of $\tilde{b}$ as

$$\left[\tilde{b}_\alpha^L, \tilde{b}_\alpha^U\right] = \left[B - \gamma_2(1-\alpha), B + \beta_2(1-\alpha)\right]. \tag{24}$$

It is well known that

$$M(\tilde{a}) = A + \frac{\beta_1 - \gamma_1}{6}, \tag{25}$$

and

$$M(\tilde{b}) = B + \frac{\beta_2 - \gamma_2}{6}. \tag{26}$$

We compute that

$$\left(\tilde{a} \otimes \tilde{b}\right)_\alpha = \left[\min\left\{\tilde{a}_\alpha^L \tilde{b}_\alpha^L, \tilde{a}_\alpha^L \tilde{b}_\alpha^U, \tilde{a}_\alpha^U \tilde{b}_\alpha^L, \tilde{a}_\alpha^U \tilde{b}_\alpha^U\right\}, \max\left\{\tilde{a}_\alpha^L \tilde{b}_\alpha^L, \tilde{a}_\alpha^L \tilde{b}_\beta^U, \tilde{a}_\alpha^U \tilde{b}_\alpha^L, \tilde{a}_\alpha^U \tilde{b}_\alpha^U\right\}\right]$$

$$= \left[\tilde{a}_\alpha^L \tilde{b}_\alpha^L, \tilde{a}_\alpha^U \tilde{b}_\alpha^U\right],$$

$$= \left[AB - (\gamma_1 B + \gamma_2 A)(1-\alpha) + \gamma_1\gamma_2(1-\alpha)^2, AB + (\beta_1 B + \beta_2 A)(1-\alpha) + \beta_1\beta_2(1-\alpha)^2\right] \tag{27}$$

According to equations (10) and (27), we derive that

$$M(\tilde{a} \otimes \tilde{b}) = AB + \frac{B(\beta_1 - \gamma_1) + A(\beta_2 - \gamma_2)}{6} + \frac{\gamma_1\gamma_2 + \beta_1\beta_2}{12}. \tag{28}$$

On the other hand, we evaluate

$$M(\tilde{a}) M(\tilde{b}) = \left(A + \frac{\beta_1 - \gamma_1}{6}\right)\left(B + \frac{\beta_2 - \gamma_2}{6}\right),$$

$$= AB + \frac{B(\beta_1 - \gamma_1) + A(\beta_2 - \gamma_2)}{6} + \frac{(\beta_1 - \gamma_1)(\beta_2 - \gamma_2)}{36}. \tag{29}$$

If we compare equations (28) and (29), we find that

$$M(\tilde{a} \otimes \tilde{b}) \ge M(\tilde{a}) M(\tilde{b}). \tag{30}$$

Moreover, if $M(\tilde{a} \otimes \tilde{b}) = M(\tilde{a}) M(\tilde{b})$, then

$$\gamma_1\gamma_2 + \beta_1\beta_2 - \gamma_1\beta_2 - \gamma_2\beta_1 = 3(\gamma_1\gamma_2 + \beta_1\beta_2). \tag{31}$$

We rewrite equation (31) as

$$\gamma_1\gamma_2 + \beta_1\beta_2 + (\gamma_1 + \beta_1)(\gamma_2 + \beta_2) = 0, \tag{32}$$

to imply that $\gamma_1 + \beta_1 = 0$ or $\gamma_2 + \beta_2 = 0$, since γ_1 and β_1 are the right and left spreads of $\tilde{a}$ with $\gamma_1 \ge 0$ and $\beta_1 \ge 0$. Therefore, if $M(\tilde{a} \otimes \tilde{b}) = M(\tilde{a}) M(\tilde{b})$, we obtain that $\tilde{a}$ or $\tilde{b}$ must be degenerated to a crisp number. We conclude that in general, Lemma 1 (Implicit in Wang et al. (2014)) does not hold for arbitrary fuzzy numbers.

We compute that

$$\left(\tilde{a} \div \tilde{b}\right)_\alpha = \left[\min\left\{\frac{\tilde{a}_\alpha^L}{\tilde{b}_\alpha^L}, \frac{\tilde{a}_\alpha^L}{\tilde{b}_\alpha^U}, \frac{\tilde{a}_\alpha^U}{\tilde{b}_\alpha^L}, \frac{\tilde{a}_\alpha^U}{\tilde{b}_\alpha^U}\right\}, \max\left\{\frac{\tilde{a}_\alpha^L}{\tilde{b}_\alpha^L}, \frac{\tilde{a}_\alpha^L}{\tilde{b}_\alpha^U}, \frac{\tilde{a}_\alpha^U}{\tilde{b}_\alpha^L}, \frac{\tilde{a}_\alpha^U}{\tilde{b}_\alpha^U}\right\}\right]$$

$$= \left[\frac{\tilde{a}_\alpha^L}{\tilde{b}_\alpha^U}, \frac{\tilde{a}_\alpha^U}{\tilde{b}_\alpha^L}\right],$$

$$= \left[\frac{A - \gamma_1(1-\alpha)}{B + \beta_2(1-\alpha)}, \frac{A + \beta_1(1-\alpha)}{B - \gamma_2(1-\alpha)}\right]. \tag{33}$$

Using $\dfrac{A - \gamma_1(1-\alpha)}{B + \beta_2(1-\alpha)} = \dfrac{-\gamma_1}{\beta_2} + \dfrac{(\beta_2 A + \gamma_1 B)/\beta_2}{B + \beta_2(1-\alpha)}$ and $\dfrac{\alpha}{B + \beta_2(1-\alpha)} = \dfrac{-1}{\beta_2} + \dfrac{(\gamma_2 - B)/\gamma_2}{B + \beta_2(1-\alpha)}$, we find that

$$M(\tilde{a} \div \tilde{b}) = \frac{\gamma_2 A + \beta_1 B}{\gamma_2}\left(\frac{1}{\gamma_2} + \left(\frac{\gamma_2 - B}{(\gamma_2)^2}\right)\ln\left(\frac{B}{B - \gamma_2}\right)\right),$$

$$-\frac{1}{2}\left(\frac{\gamma_1}{\beta_2} + \frac{\beta_1}{\gamma_2}\right) + \frac{\beta_2 A + \gamma_1 B}{\beta_2}\left(\frac{-1}{\beta_2} + \left(\frac{\beta_2 + B}{(\beta_2)^2}\right)\ln\left(\frac{B + \beta_2}{B}\right)\right). \tag{34}$$

On the other hand, we know that

$$\frac{M(\tilde{a})}{M(\tilde{b})} = \frac{6A + (\beta_1 - \gamma_1)}{6B + (\beta_2 - \gamma_2)}. \tag{35}$$

Now, we compare equations (34) and (35) to reveal that Lemma 2 (Implicit in Wang et al. (2014)) to use $M(\tilde{a} \div \tilde{b}) = \dfrac{M(\tilde{a})}{M(\tilde{b})}$ that is invalid.

4. Our discussion for the sensitivity analysis of Wang et al. (2014)

At last, for completeness, we will point out another questionable result in the sensitivity analysis of Wang et al. (2014). They compared (a) the underlying asset price, S_* and the compound option price, C under the Black-Scholes model in their Table 1, and (b) the corresponding S^* and $M(\tilde{C})$ of their model under fuzzy environment in their Table 2. We observe their Tables 1 and 2 to know that $S_* \geq S^*$ and $C \leq M(\tilde{C})$. Our observation was contradicted with assertion mentioned in Wang et al. (2014).

For completeness, we quote their results, except the index number of referred two articles are modified to be consistent within this paper, "From Tables 1 and 2, the compound option prices derived from the Black-Scholes model are slightly lower than the prices derived from the crisp possibilistic mean value with the same parameters. This seems to be consistent with our intuition that the crisp possibilistic mean value model contains more uncertainty than the Black-Scholes model (see Xu et al. (2009), Zhang et al. (2012)). But this intuition is not necessarily true, which one is bigger between C and $M(\tilde{C})$ is related to the selected parameters. Similarly, from Tables 1 and 2, we notice that S_* is slightly higher than S^*; this conclusion is not surely true. For example, when $S=100$, $K_1=5$, $K_2=90$, $T_1=0.5$, $T_2=1$, $\tilde{r}=(0.049, 0.05, 0.052)$ and $\tilde{\sigma}=(0.28, 0.3, 0.31)$, then the computing result is $S_*=82.8336$, $C=15.2744$, $S^*=82.9162$, and $M(\tilde{C})=15.2290$; obviously, $S_* < S^*$ and $C > M(\tilde{C})$."

From our partially cited Tables 1 and 2 of Wang et al. (2014) as our table 1, we find that $S_*=82.8336$ and $C=15.2744$ as reported in the above citation.

However, on the contrary, in their Table 2 (cited in our table 1), $S^*=82.7509$ and $M(\tilde{C})=15.3199$ such that their claim of $S_* < S^*$ and $C > M(\tilde{C})$ contains questionable findings.

Table 1. Partially cited results from Tables 1 and 2 of Wang et al. (2014)

Table 1			Table 2		
T_2	S_*	C	T_2	S^*	$M(\tilde{C})$
0.75	88.2795	13.6596	0.75	88.2253	13.7005
1	82.8336	15.2744	1	82.7509	15.3199
1.25	78.3652	16.8882	1.25	78.2638	16.9375

$S=100$, $K_1=5$, $K_2=90$, $T_1=0.5$, $r=0.05$, $\sigma=0.3$, $r_c=0.05$, and $\sigma_c=0.3$.

5. Conclusions

Based on our above discussion, we prove that $M(\tilde{a}\otimes\tilde{b})=M(\tilde{a})M(\tilde{b})$ if and only if $\tilde{a}$ or $\tilde{b}$ are crisp numbers. Moreover, for triangular fuzzy numbers, in general, $M(\tilde{a}\otimes\tilde{b})\neq M(\tilde{a})M(\tilde{b})$. On the other hand, based on equations (34) and (35), $M(\tilde{a}\div\tilde{b})$ and $\frac{M(\tilde{a})}{M(\tilde{b})}$ are completely different so that the proof of Wang et al. (2014) for the crisp possibilistic mean value of their compound option price requires revisions.

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