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Domination in Quasi Strong Fuzzy Planar Graph

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Abstract: ***In this paper we introduced the concept of "Domination in Quasi Strong and Isomorphic Fuzzy Planar Graph". It is combination of domination and fuzzy planar graph. We discussed various properties like Domination in Fuzzy Planar graph(γ_{FP}), Domination of Quasi Strong Fuzzy Planar Graph(γ_{QSFP}), Domination Isomorphic Fuzzy Planar Graph (γ_{IFP}), Domination in fuzzy planar graph with planarity value.***

Keywords: Fuzzy graph, Fuzzy planar graph, domination in fuzzy planar graph, domination in strong fuzzy planar graph, Quasi Strong Fuzzy planar graph

1. Introduction

Fuzzy graph and Fuzzy planar graph are the sub classes of Graph theory. Combination of these two graphs is called "Fuzzy Planar Graph". One of the fastest growing areas within graph theory is the study of domination.

We discussed the concept of dominating graph were we introduced by V. R. Kuli and Bidarhali Janakiraman [4]. We used the concept of Fuzzy Planar Graph with using Planarity value is introduced by Sovan Samantha, Anitha Pal, Madhumangal [9] and Domination in Fuzzy Planar graph as introduced by A. Somasundaram and S. Somasundram [10]. Here we introduced the concept of Doimnation in Quasi strong fuzzy graph, Isomorphic fuzzy planar Graph.

2. Preliminaries

Definition: 2.1
A ***finite Graph*** is a graph $G = (V, E)$ such that V and E are called vertices and edges in finite sets.

Definition: 2.2
An ***infinite graph*** is one with an infinite graph set or edges or both.

Definition: 2.3
If more than one edge is joining two vertices are allowed, the resulting object is a ***multi graph***[1]. Edges joining the same vertices are called multiple lines.

Definition: 2.4
A drawing of a geometric representation of a graph on any surface such that no edges intersect is called ***embedding.***

Definition: 2.5
A graph G is ***planar*** [1] if it can be drawn in the plane with its edges only intersecting at vertices of G, so the graph is ***non-planar*** if it cannot be drawn without crossings.

Definition: 2.6
A ***fuzzy set A*** [5] on a universal set X is characterized by a mapping $m: X \to [0, 1]$, which is called the membership function . A fuzzy set is donated by"$F = (X, n)$".

Definition: 2.7
A ***fuzzy graph*** [5] $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of function $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that for all $u, v \in V, \mu(u,v) \leq \sigma(u)^\wedge\sigma(v)$ where $\sigma(u)$ and $\mu(u,v)$ represent the membership values of the vertex u and of the edge (u,v) in G respectively.

Definition: 2.8
The fuzzy graph G = (V, σ, μ), an edge (u,v) is called ***strong*** [3] if
$\frac{1}{2}\{\,\sigma(u)^\wedge\sigma(v) \leq\ \mu(u,v)\}$ and weak otherwise .

Definition: 2.9
The fuzzy graph G = (V, σ, μ), an edge (u,v) is called ***quasi- strong*** if, $\frac{1}{4}\{\,\sigma(u)^\wedge\sigma(v)\} \leq\ \mu(u,v)\}$and quasi-weak otherwise .

Definition: 2.10
The neighborhood of u can be written as $N(u) = \{v \in V\,|\mu(uv) = \sigma(u)^\wedge\sigma(v)\}$ is called and $N[u] = N(u) \cup \{u\}$ is called the ***closed neighborhood of u*** [10].

Definition: 2.11
The ***orders*** [10] and ***size t*** [10] of a fuzzy graph $G = (\sigma, \mu)$ are defined to be
$s = \sum_{u \in V} \sigma(u)$ and $= \sum_{u,v \in E} \mu(u,v)$.

Definition: 2.12
If an edge (u,v) of a fuzzy graph satisfies the condition $\mu(u,v) = \sigma(u)^\wedge\sigma(v)$, then this edge is called an ***effective edge*** [9].

Definition: 2.13
Two vertices are said to be an ***effective adjacent*** [9] if they are the end vertices of the same effective edge.

Definition: 2.14
The ***minimum neighborhood degree*** is denoted by δ_N and ***maximum neighborhood degree*** is denoted by Δ_N .

Definition: 2.15
A ***homomorphism*** [9] between fuzzy graph G and G' is a map $h: S \to S'$ which satisfies $\sigma(u) \leq \sigma'(h(u))$ for all $u \in S$and $\mu(u,v) = \mu'(h(u), h(v))$ for all $u, v \in S$ where S is set of vertices of G and S' is that of G'.

Definition: 2.16
An ***isomorphism*** [9] between fuzzy graph is a bijective homomorphism, $h: S \to S'$ which satisfies $\sigma(u) \leq \sigma'(h(u))$ for all $u \in S$ and $\mu(u, v) = \mu'(h(u), h(v))$ for all $\in S$.

Definition: 2.17
Let G be a ***fuzzy planar graph*** with planarity value f, [9] where $= \frac{1}{1+\{I_{P1}+I_{P2}+\cdots\ldots+I_{Pn}\}}$. The range of f is $0 < f \leq 1$.
Here $P_1, P_2 \ldots\ldots P_n$be the points intersection between the edges.
In a graph $= (V, \sigma, E)$, E contains two edges $\mu(u, v)$ and $\mu(c, d)$, which are intersected at a point P.
Strength of the fuzzy edge $I_{(u,v)} = \frac{\mu(u,v)}{\sigma(u) \wedge \sigma(v)}$.
The intersecting point at P is $I_P = \frac{I_{(u,v)}+I_{(c,d)}}{2}$.

Results: 2.18
- If there is ***no point of intersection*** for a geometrical representation of a fuzzy planar graph, then ***its fuzzy planarity value is 1.***
- If $\mu(w, u) = 1$ (or near to 1) and $\mu(v, z) = 0$ (near to 0), then we say that the fuzzy graph has no crossing. Then the crossing will not be important for planarity.
- If $\mu(w, u) = 1$ (or near to 1) and $\mu(v, z) = 1$ (near to 1), then the crossing will be an important for planarity.
- Quasi-strong fuzzy planar graph if $f \geq 0.25$ otherwise quasi-weak.

Definition: 2.19
A dominating set for a graph $G = (V, E)$ is a subset of the vertex set denote the set of vertices in which are in or adjacent to a vertex in G. If then said to be dominating set.

Definition: 2.20
The ***Dominating set*** $\boldsymbol{\gamma(G)}$[4] of G is the minimum cardinality of a dominating set.

Definition: 2.21
A Dominating set D is a ***minimal dominating set*** [4] if no proper subset $D' \subset D$ is a dominating set of G.

Definition: 2.22
Let $G = (V, \sigma, \mu)$ be a fuzzy graph on V. Let u, v $\in V$, if u dominates V in G if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. A subset D of V is called a ***dominating set*** [10] in G if for every v $\notin D$, there exists u $\in D$ such that u dominates v of a dominating set in G is called the dominating number .

Definition: 2.23
The minimum fuzzy cardinality of a dominating set in G is called the ***dominating number*** [10] of G and is denoted by $\gamma(G)$or γ .

Definition: 2.24
Let G be a fuzzy graph without isolated vertices. A subset D of V is said to be a ***total dominating set*** [10] if every vertex in V is dominated by a vertex in D.

The ***total dominating number*** of G is denoted by δ_t .

Domination in Quasistrong Fuzzy Planar Graph

Definition: 3.1
If a graph G is said to be ***domination in fuzzy planar graph*** if
- $G = (V, \sigma, \mu)$ be a fuzzy planar graph with planarity value f
- Let $u, v \in V$, u dominates v in G then $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$
- A subset D of V is called a dominating set in G if for every v $\notin D$, there exist u $\in D$ such that u dominates v.
- The minimum fuzzy cardinality of a dominating set in G is called the dominating set in G is called the dominating number of G and is denoted by γ_{FP} .

Example: 3.2

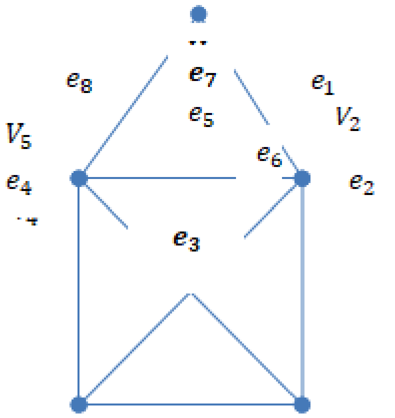

Figure 1: δFP in G with intersection

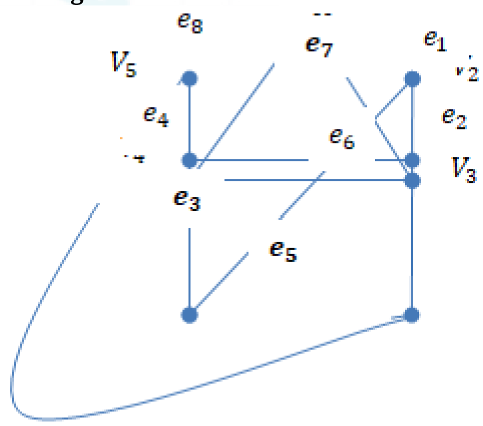

Figure 2: δFP in G without intersection

G_1 and $G_2$are the same fuzzy planar graphplanarity value also there are same.
Dominating set= $\{V_1, V_4\}$, $\{V_2, V_4\}$, $\{V_3\}$, $\{V_5\}$
Minimum dominating number $\gamma_{FP} = 1$

Definition: 3.3
If a graph G is said to be domination in ***quasi-strong fuzzy planar graph*** if
- $G = (V, \sigma, \mu)$ be a quasi-fuzzy planar graph with planarity value f$\geq$ 0.25
- A subset D of V is called a dominating set in G if for every v$\notin D$, there exist u $\in D$ such that u dominates v.
- The minimum fuzzy cardinality of a dominating set in G is called the dominating number in G and is denoted by γ_{QSFP} .

Definition: 3.4
If a graph G_1 and G_2 is said to be domination in ***isomorphic fuzzy planar graph*** if

- G_1 and G_2 be an isomorphic fuzzy planar graph.
- The minimum fuzzy cardinality of a dominating set in G is called the dominating number of G and it is denoted by γ_{IFP} .

Theorem: 3.5
Fuzzy planar graph, $s-t \leq \gamma_{FP} \leq s-\delta_E$, where$\gamma_{FP}$ be the minimum dominating number in fuzzy planar graph and s is the order, t is size and δ_E is minimum effective incident degree of G.

Proof:
Let G be a fuzzy planar. D be a dominating set and γ_{FP} be a minimum dominating number in G.

Then the scalar cardinality of V-D is γ_{FP} and the scalar cardinality of$V \times V$ is $s-t$.

$s-t \leq \gamma_{FP}$ …………………. (1)

Let u be the vertex with minimum effective incident degree δ_E . Clearly $V-\{u\}$ is a dominating set and

$\gamma_{FP} \leq u-\delta_E$ ……………………. (2)

From (1) and (2)
$s-t \leq \gamma_{FP} \leq s-\delta_E$.

Remark: 3.6
If all the vertices having the same membership value, then $s-t \leq \gamma_{FP} \leq s-\Delta_E$.

Theorem: 3.7
Every dominating fuzzy planar graph (with condition $\mu(u,v)=\sigma(u)^\wedge\sigma(v)$) is the γ_{QSFP} according to calculating its strength.

Proof:
Let G be a γ_{FP}. Then u dominates v if $\mu(u,v)=\sigma(u)^\wedge\sigma(v)$.
Hence the membership value of edge will be equal to the membership value of minimum vertex. Then the strength of an edge will be 1.

Strength $I_{(u,v)}=\frac{\mu(u,v)}{\sigma(u)^\wedge\sigma(v)}$

A fuzzy planar graph is γ_{QSFP} if the planarity ≥ 0.25

Then the strength of a γ_{QSFP} is 1, then the planarity will be 0.25.

Hence every dominating fuzzy planar graph is the h.

Theorem: 3.8
Let G be a quasi-strong fuzzy planar graph with the minimum dominating set γ_{QSFP}. v be a vertex $\gamma_{QSFP} \geq s-\Delta_N$.

Proof:
Let G be a quasi-strong fuzzy planar graph with the minimum dominating set γ_{QSFP} .

Let v be a vertex such that$dN(v)=\Delta_N$. Then $V\backslash N(v)$ is a dominating set of G.

Hence $\gamma_{QSFP} \geq s-\Delta_N$.

Theorem: 3.9
For any fuzzy planar graph G, total dominating set of a fuzzy planar graph $\gamma_{tFP}=s$
iff every vertex of G has a unique neighbor.

Proof:
Let G be a γ_{FP} with unique neighbor.
If every vertex of G has unique neighbor then D is the only total dominating set of G.

So that $\gamma_{tFP}=s$
Conversely, suppose $\gamma_{tFP}=s$
If there exists a vertex V with two neighbor's u and v then $V-\{u\}$ is a total dominating set of G.
So that $\gamma_{tFP}<s$ which is contradiction.
Hence every vertex of fuzzy planar has unique neighbor.

Theorem: 3.10
Let G_1 and G_2 be the two γ_{IFP} with the minimum dominating setγ_{IFP_1} and γ_{IFP_2}. Let f_1 and f_2 be the planarity values. Then

a) $\gamma_{IFP_1}=\gamma_{IFP_2}$
b) $f_1=f_2$

Proof:
Let G_1 and G_2 be then γ_{IFP}.
i.e G_1 is isomorphic to G_2. Now isomorphismpreserves size weight of the edges and vertex
ofa γ_{FP}.

Hence, weight of the edges and vertex of G_1 and G_2 are similarly.

Hence, $\gamma_{IFP_1}=\gamma_{IFP_2}$and $f_1=f_2$.

3. Conclusion

The domination in fuzzy planar graph (γ_{FP}), domination in quasi-strong fuzzy planar graph (γ_{QSFP}), domination in isomorphic fuzzy planar graph (γ_{IFP}) are discussed. We defined the quasi-strong fuzzy planar, Using the concept of finding strength of an edges .we defined relationship between maximum neighborhood degree andγ_{QSFP}. Then we described the relation between the fuzzy planar graph with the planarity and domination in an γ_{IFP}.

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