International Journal of Scientific Engineering and Research (IJSER)
ISSN (Online): 2347-3878
Impact Factor (2018): 5.426

Determinant of a Fuzzy Neutrosophic Matrix

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Abstract: ***Determinants have wonderful algebraic properties and occupied the proud place in linear algebra. It provides extremely efficient tools for thinking about problems of linear algebra, including those in practical application. The purpose of this paper is to present a new way of expanding the determinant of a Neutrosophic fuzzy matrix and its properties. Also we define the concept of trace and adjoint of a Neutrosophic fuzzy matrix and the basic properties of this matrix are discussed. Numerical examples are also included.***

Keywords: Neutrosophic set, Neutrosophic fuzzy matrix, Adjoint of NFM, Determinant of NFM, Trace of NFM

1. Introduction

Neutrosophic set is a modern mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In 1988, Florentin smarandache introduce the concept of a neutrsophic set from a philosophical point of view. The neutrosophic set is a powerful general framework that generalizes the concept of fuzzy set and intutionistic fuzzy set. Each element had three associated defining functions, namely the membership function (T), indeterminacy function (I), the non-membership function (F) defined on the universe of discourse X, the three functions are completely independent.

The determinants are very important in forming the inverse and also it perhaps the linear dependence or independence. Fuzzy matrices play an important role in real-life problems. Fuzzy matrices were introduced by M.G. Thomasan [7] to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. In 2004, W.B.V.Kandasamy and F.Smarandache[5] introduced fuzzy relational maps and neutrosophic relational maps. In 2014, F.Smarandache[6] introduced a type of neutrosophic matrices, called Square neutrosophic matrices, whose entries are of the form a + I b (neutrosophic number), where a, b are the elements of [0, 1] and I is an indeterminate such that In = I, n being a positive integer.

The rest of the paper has been organized as follows: Section 2 deals with the preliminaries of Neutrosophic fuzzy set and the new definition of trace, operations of NFM. Section 3 provides some basic properties of NFMs. Section 4 provides the process of finding determinant of a NFM and some properties of determinant NFMs which are presented with the help of numerical examples. Section 5 gives our conclusions.

2. Preliminaries

Definition 2.1 Neutrosophic Fuzzy Set [16]

Let X be a non - empty set. A neutrosophic fuzzy set A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A: X \to [0, 1]$, $\sigma_A: X \to [0, 1]$, $\nu_A: X \to [0, 1]$ denote the degree of membership function, degree of indeterminacy function, degree of non-membership function respectively of each element $x \in X$ to the set A and $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$, for each $x \in X$.

Definition 2.2 Empty Neutrosophic Fuzzy Set [10]

A Neutrosophic fuzzy set A over the universe X is said to be null or empty neutrosophic fuzzy set if $\mu_A(x) = 0, \sigma_A(x) = 0, \nu_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.3 Absolute Neutrosophic Fuzzy Set [10]

A Neutrosophic fuzzy set A over the universe X is said to be absolute neutrosophic fuzzy set if $\mu_A(x) = 1$, $\sigma_A(x) = 1, \nu_A(x) = 0$ for all $x \in X$. It is denoted by I_N.

Definition 2.4 Sum of a Neutrosophic Fuzzy Set [10]

Let X be a non - empty set. A neutrosophic fuzzy sets A and B is of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ then, the sum, difference and product of two Neutrosophic fuzzy sets is defined by,

$$A + B = \{ x, (\mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x))\}$$

Definition 2.5 Difference of a Neutrosophic Fuzzy Set [10]

Let X be a non - empty set. A neutrosophic fuzzy sets A and B is of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ then, the difference of two Neutrosophic fuzzy sets is defined by,

$$A - B = \{ x, (\mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x))\}.$$

Definition 2.6 Complement of a Neutrosophic Fuzzy Set [10]

Let X be a non - empty set. A neutrosophic fuzzy set A is of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ then the complement of a fuzzy neutrosophic set is defined by $A^C = \{x, (\nu_A(x), 1 - \sigma_A(x), \mu_A(x))\}$.

Definition 2.7 Product of Neutrosophic Fuzzy Set

Let X be a non - empty set. A neutrosophic fuzzy sets A and B is of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ then the product of two Neutrosophic fuzzy sets is defined by,

$$AB = \{x, (\mu_A(x) \wedge \mu_B(x), 1 - \sigma_A(x) \wedge 1 - \sigma_B(x), \nu_A(x) \vee \nu_B(x))\}$$

Definition 2.8 Neutrosophic fuzzy matrix

Neutrosophic fuzzy matrix of order $m \times n$ is defined as $A = (a_{ij})\, m \times n$ where $(a_{ij}) = (\mu_a(x), \sigma_a(x), \gamma_a(x))$ is the ij th element of A where $\mu_a(x)$ denote the degree of membership function, $\sigma_a(x)$ denote the degree of indeterminacy function and $\gamma_a(x)$ denote the degree of non membership function respectively.

Definition 2.9 Null Neutrosophic fuzzy matrix

Neutrosophic fuzzy matrix is said to be Null Neutrosophic fuzzy matrix if all its entries are zero. i.e., all elements are (0, 0, 1).

Definition 2.10 Unit Neutrosophic fuzzy matrix

A square Neutrosophic fuzzy matrix is said to be unit Neutrosophic fuzzy matrix if $a_{ii} = (1, 1, 0)$ and $a_{ij} = (0, 0, 1)$
$i \neq j$, for all $i = j$. It is denoted by I.

Definition 2.11 Symmetric Neutrosophic fuzzy matrix

A square Neutrosophic fuzzy matrix is said to be symmetric Neutrosophic fuzzy matrix if $a_{ij} = a_{ji}$.

Definition 2.12 Triangular Neutrosophic fuzzy matrix

A square Neutrosophic fuzzy matrix is said to be triangular Neutrosophic fuzzy matrix if either a_{ij} = (0, 0, 1) for all $i > j$ or a_{ij} = (0, 0, 1) for all $i < j$: i, j =1, 2, …, nz.

A square Neutrosophic fuzzy matrix is said to be upper triangular Neutrosophic fuzzy matrix if either a_{ij} = (0, 0, 1) for all $i > j$ and is said to be lower triangular Neutrosophic fuzzy matrix if aij = (0, 0, 1) for all $i < j$: i, j =1, 2, …, n.

Definition 2.13 Addition of a NFM

Let $A = (a_{ij})$) is $n \times m$ and $B = (b_{ij})$ is $m \times p$ matrix be two Neutrosophic fuzzy matrix, then the addition of a NFMs can be done when the same dimensions of NFMs. Adding one NFM by another NFM is obtained by just adding the corresponding entries of the NFMs

Note that, the NFM addition is not defined when the NFMs do not have same dimension.

Definition 2.14 Product of a NFM

If $A = (a_{ij})$ is $n \times m$ matrix and $B = (b_{ij})$ is $m \times p$ NFM, their product AB is an $n \times p$ NFM, in which the m entries across a row of A are multiplied with the m entries down a column of B and summed to produce an entry of AB. Thus the product of a NFMs AB is defined if and only if the number of columns in A equals the number of rows in B.

Definition 2.15 Trace of a square NFM

The trace of a square NFM $A = (a_{ij})$ denoted by tr (A), is

the sum of the principal diagonal elements. In otherwords, $tr(A) = \sum_{i=1}^{n} a_{ii}$.

3. Basic Properties of a Neutrosophic fuzzy Matrices

Property 3.1 For any three NFMs A, B, C of order m×n we have

(i) $A + B = B + A$.
(ii) $A + (B + C) = (A + B) + C$.
(iii) $A + 0 = A$.

Property 3.2 If A and B be two NFMs such that A+B and AB are defined then,

(i) $(A')' = A$.
(ii) $(A + B)' = A' + B'$.
(iii) $(AB)' = B' A'$.

Property 3.3 Let A be a symmetric NFM then,

(i) (AA') and $(A'A)$ are both symmetric.
(ii) $A + A'$ is symmetric.

Property 3.4 Let $A = (a_{ij})$ and $B = (b_{ij})$ be any two square NFMs of order $n \times n$ then,

(i) $tr(A) + tr(B) = tr(A + B)$
(ii) $tr(A) = tr(A')$
(iii) $tr(AB) = tr(BA)$

4. Determinant of a Neutrosophic fuzzy Matrices

Definition 4.1 Determinant of a Neutrosophic fuzzy Matrices

The determinant of a Neutrosophic fuzzy matrix A of order $n \times n$ is denoted by |A| or det (A) and is defined as,

$$|A| = \sum_{\sigma \in S_n} \operatorname{sgn} \sigma\big(\mu_{1\,\sigma(1)}(x), \sigma_{1\,\sigma(1)}(x), \nu_{1\,\sigma(1)}(x)\big) \ldots (\mu_{n\,\sigma(n)}(x), \sigma_{n\,\sigma(n)}(x), \nu_{n\,\sigma(n)}(x)$$

$$= \sum_{\sigma \in S_n} \operatorname{sgn} \sigma \prod_{i=1}^{n} a_{i\sigma(i)}$$

$a_{i\sigma(i)} = \mu_{i\,\sigma(i)}(x), \sigma_{i\,\sigma(i)}(x), \nu_{i\sigma(i)}(x)$ are NFNs and S_n denotes the symmetric group of all permutations of the indices {1, 2, …, n} and sgn σ = 1 if the permutation is even or odd.

Example 4.1

$$A = \begin{bmatrix} (0.3, 0.1, 0.2) & (0.1, 0.4, 0.2) & (0.3, 0.5, 0.1) \\ (0.2, 0.4, 0.6) & (0.4, 0.2, 0.1) & (0.7, 0.2, 0.1) \\ (0.9.0.1, 0.2) & (0, 0.1, 0.2) & (0.1, 0.4, 0.9) \end{bmatrix}$$

$|A| =$ (0.3, 0.1, 0.2)[(0.1, 0.6, 0.9) +(0, 0.8, 0.2)]
+ (0.1, 0.4, 0.2) [(0.2, 0.6, 0.9) + (0.7, 0.8, 0.2)]

+(0.3, 0.5, 0.1) [(0, 0.6, 0.6) +(0.4, 0.8, 0.2)]

$|A| =$ (03, 0.6, 0.2).

Definition 4.2 Minor of an element

Let A = (a_{ij}) be a square NFM of order n × n. The minor of an element aij in det (A) is a determinant of order $(n-1) \times (n-1)$, which is obtained by deleting the ith row and the jth column from A and is denoted by M_{ij}.

Definition 4.3 Co factor of an element

Let A = (a_{ij}) be a square NFM of order $n \times n$. The cofactor of an element aij in A is denoted by A_{ij} and is defined as, A_{ij} = $(1)^{i+j}\, M_{ij}$.

Definition 4.4 Adjoint of a NFM

Let A = (a_{ij}) be a square NFM and B = A_{ij} NFM whose elements are the cofactors of the corresponding elements in |A| then the transpose of B is called the adjoint or adjugate of A and its equal to A_{ji}. The adjoint of A is denoted by adj (A).

Property 4.5 (Reflection Property)

The value of the determinant remains unchanged when any two rows or columns are interchanged.

Proof: Let A = (a_{ij}) be a NFM of order n × n. If B = $(b_{ij})$of order n × n is obtained from A by interchanging the rth and sth row (r < s), then it is clear that bij = aij, i ≠ r, i ≠ s and $b_{rj} = a_{sj}$, $b_{sj} = a_{rj}$. Now,

$$|B| = \sum_{\sigma\in S_n} \operatorname{sgn} \sigma(b_{1\,\sigma(1)}, b_{2\,\sigma(2)}, \dots, b_{r\,\sigma(r)}, \dots, b_{s\,\sigma(s)}, \dots, b_{n\,\sigma(n)})$$

$$= \sum_{\sigma\in S_n} \operatorname{sgn} \sigma(a_{1\,\sigma(1)}, a_{2\,\sigma(2)}, \dots, a_{r\,\sigma(r)}, \dots, a_{s\,\sigma(s)}, \dots, a_{n\,\sigma(n)}$$

$$\sum_{\sigma\in S_n} sgn\ \sigma\big(\mu_{1\,\sigma(1)}(x), \sigma_{1\,\sigma(1)}(x), \nu_{1\,\sigma(1)}(x)\big), \dots, (\mu_{r\,\sigma(r)}(x), \sigma_{r\,\sigma(r)}(x), \nu_{r\,\sigma(r)}(x), \dots, (\mu_{s\,\sigma(s)}(x), \sigma_{s\,\sigma(s)}(x), \nu_{s\,\sigma(s)}(x), \dots, \big(\mu_{n\,\sigma(n)}(x), \sigma_{n\,\sigma(n)}(x), \nu_{n\,\sigma(n)}(x)\big).$$

Then, λ is a transposition interchanging r and s and Sgn = 1
$|B| = |A|$.

Property 4.6 (All zero Property)

If all the elements of a row (column) of A are (0, 0, 1) then |A| = (0, 0, 1).

Proof: Let A = (aij) be a NFM of order n × n where $(a_{ij}) = \mu_{ij}(x), \sigma_{ij}(x), \nu_{ij}(x)$

$|A| = \sum_{\sigma\in S_n} \operatorname{sgn} \sigma(a_{1\,\sigma(1)}, a_{2\,\sigma(2)}, \dots, a_{n\,\sigma(n)})$
= (0, 0, 1).

Property 4.7 (Scalar multiple Property)

If all the elements of a row (column) multiplied by a non zero constant, then the determinant gets multiplied by the same constant.

Proof : If k = (0, 0, 1) then the result is obviously true.

Let A = (a_{ij})be a NFM of order $n \times n$. If B = $(b_{ij})$of order n × n is obtained from A by multiplying its rth row by a scalar k ≠ 0.

Obviously $\mu_{ij}(x), \sigma_{ij}(x), \nu_{ij}(x) = \mu_{ij}'(x), \sigma_{ij}'(x), \nu_{ij}'(x)$ for all i ≠ 0
and $\mu_{ij}(x), \sigma_{ij}(x), \nu_{ij}(x) = k\,\mu_{ij}'(x), k\sigma_{ij}'(x), k\nu_{ij}'(x)$ Then by definition,

$$|B| = \sum_{\sigma\in S_n} \begin{matrix} \operatorname{sgn} \sigma\big(\mu_{1\,\sigma(1)}(x), \sigma_{1\,\sigma(1)}(x), \nu_{1\,\sigma(1)}(x)\big), \dots, \\ (\mu_{2\sigma(2)}(x), \sigma_{2\,\sigma(2)}(x), \nu_{2\,\sigma(2)}(x), \dots, \\ \dots\big(\mu_{r\,\sigma(r)}(x), \sigma_{r\,\sigma(r)}(x), \nu_{r\,\sigma(r)}(x)\big), \dots, \\ \big(\mu_{n\,\sigma(n)}(x), \sigma_{n\,\sigma(n)}(x), \nu_{n\,\sigma(n)}(x)\big). \end{matrix}$$

$$|B| = \sum_{\sigma\in S_n} \begin{matrix} \operatorname{sgn} \sigma\big(\mu'_{1\,\sigma(1)}(x), \sigma'_{1\,\sigma(1)}(x), \nu'_{1\,\sigma(1)}(x)\big), \dots, \\ (\mu'_{2\sigma(2)}(x), \sigma'_{2\,\sigma(2)}(x), \nu'_{2\,\sigma(2)}(x), \\ \dots\big(k\mu'_{r\,\sigma(r)}(x), k\sigma'_{r\,\sigma(r)}(x), k\nu'_{r\,\sigma(r)}(x)\big), \dots, \\ \big(\mu'_{n\,\sigma(n)}(x), \sigma'_{n\,\sigma(n)}(x), \nu'_{n\,\sigma(n)}(x)\big). \end{matrix}$$

$$= k \sum_{\sigma\in S_n} \begin{matrix} \operatorname{sgn} \sigma\big(\mu_{1\,\sigma(1)}(x), \sigma_{1\,\sigma(1)}(x), \nu_{1\,\sigma(1)}(x)\big) \dots \\ (\mu_{n\,\sigma(n)}(x), \sigma_{n\,\sigma(n)}(x), \nu_{n\,\sigma(n)}(x) \end{matrix}$$

= k |A|.

Property 4.3 (Triangle Property)

If A is triangular NFM then $|A| = \prod_{i=1}^{n} a_{i\sigma(i)}$ (i.e) the determinant will be equal to the product of the diagonal.

Proof: Let A = (a_{ij})be a NFM of order n × n for i < j, (a_{ij}) = (0, 0, 1). Put |A| = k.

$$K = \big(\mu_{1\,\sigma(1)}(x), \sigma_{1\,\sigma(1)}(x), \nu_{1\,\sigma(1)}(x)\big), (\mu_{2\sigma(2)}(x), \sigma_{2\,\sigma(2)}(x), \nu_{2\,\sigma(2)}(x), \dots, \big(\mu_{n\,\sigma(n)}(x), \sigma_{n\,\sigma(n)}(x), \nu_{n\,\sigma(n)}(x)\big).$$

Let σ(1) ≠ 1. (i.e) 1< σ(1) and so that $(\mu_{1\,\sigma(1)} =$ 0, σ1 σ1=0 ν1 σ1=1,
Consequently k = (0, 0, 1).
Similarly, proceeding like this,
We get,
$|A| = (\mu_{11}, \sigma_{11}, \nu_{11})\,(\mu_{22}, \sigma_{22}, \nu_{22}), \dots, (\mu_{nn}, \sigma_{nn}, \nu_{nn})$
$|A| = \prod_{i=1}^{n} (\mu_{ii}, \sigma_{ii}, \nu_{ii})$.

Property 4.4 (Product of determinant Property)

The determinant of a product equals the product of the determinant.

Proof: Case (i) If A is not invertible, then det (A) = 0
det (A) det (B) = 0 = det (AB).
Case (ii) If A is invertible, then A can be written as the product of elementary matrices
$AB = E_n.E_{n-1}.,\dots,.E_1$
$\det(AB) = \det(E_n.E_{n-1}.,\dots,.E_1.B)$
$= \det(E_n.)\det(E_{n-1}.,\dots,.E_1.B)$
$= \det(E_n.)\det(E_{n-1}).,\dots,.\det(E_1.B)$
$= \det(E_n.)\det(E_{n-1}).,\dots,.\det(E_1).\det(B)$
$= \det(E_n.E_{n-1}.,\dots,.E_1.)\det(B)$
$= \det(A)\det(B)$.

Property 4.5

The determinant of a NFM equals the determinant of the transpose of NFM.

Proof: Let $A = (a_{ij})$be a NFM of order $n \times n$ and $A' = B = (b_{ij})_{n\times n}$. Then,
$|B| = \sum_{\sigma\in S_n} \operatorname{sgn}\sigma(b_{1\,\sigma(1)}, b_{2\,\sigma(2)}, \dots, b_{n\,\sigma(n)})$
Let φ be the permutation of {1, 2, …, n} such t. hat $\varphi\sigma = I$, the identity permutation Therfore,

$$. |B| = \sum_{\sigma\in S_n} \begin{array}{l} \operatorname{sgn}\sigma(\mu_{1\,\sigma(1)}(x), \sigma_{1\,\sigma(1)}(x), \nu_{1\,\sigma(1)}(x)) \dots \\ (\mu_{n\,\sigma(n)}(x), \sigma_{n\,\sigma(n)}(x), \nu_{n\,\sigma(n)}(x) \end{array}$$

$$|B| = \sum_{\varphi\in S_n} \begin{array}{l} \operatorname{sgn}\varphi(\mu_{1\,\varphi(1)}(x), \sigma_{1\,\varphi(1)}(x), \nu_{1\,\varphi(1)}(x)) \dots \\ (\mu_{n\,\varphi(n)}(x), \sigma_{n\,\varphi(n)}(x), \nu_{n\,\varphi(n)}(x) \end{array}$$

$= |A|$.

5. Conclusion

In this paper, the expansions of determinant of Neutrosophic fuzzy matrices are discussed. Further some of the properties are studied and these are supported by some example. Most of the properties are found to be analogous to the properties of determinant of matrices in crisp cases. The determinant is used in many context specific ways. It is an indicator whether a system of linear equations has a unique solutions.

Conflict of Interests

The authors have declared that no Conflict of Interest exists.

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