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Deterministic and Probabilistic Engineering Cost Estimating Approaches for Complex Urban Drainage Infrastructure Capital Improvement (CIP) Programs

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Abstract: ***Accurate and reliable project cost estimates are fundamental to achieve successful municipal capital improvement (CIP) programs. Engineering cost estimates typically represent critical information for key decision makers to authorize and efficiently allocate the necessary funds for construction, budgeting, to generate a request for proposals, contract negotiations, scheduling, etc. for these reasons, cost estimators are using different estimating methods and approaches that allow for required levels of accuracy. As the project's scope becomes more detailed and the potential risks are identified and/or the project design stage progresses these cost estimates are revised and updated. In this paper, the most common project cost estimation methods and approaches were collected and categorized into two main groups of (1) probabilistic and (2) deterministic methods. Under these groups overall ten different methods were identified and discussed addressing their requirements, advantages, and shortcomings, including the potential risk that can positively or negatively affect the project's cost outcome. This paper will be a good resource for professionals who are in budget development and/or are seeking to a better understanding of different methods in determining an appropriate base cost margin and produce a meaningful and reliable project cost estimate.***

Keywords: Risk Management; Deterministic; Probabilistic; Engineering Cost Estimating; Uncertainty; Cost Estimating Methods; Urban Drainage Infrastructure; Capital Improvement (CIP) Programs

1. Introduction

The conventional deterministic cost estimation methods for capital improvement projects in most municipal agencies and the local governments are based on preparing a single-point-estimates. A single-point-base-estimate is based on typically on the level of a project's scope definition and the project design phase, available historical data, current contractor rates and preliminary quotes from sub-contractors and other vendors [7, 34]. Moreover, to adjust for inflation costs of labor, material, and equipment additional Consumer Price Index (CPI) is added to each cost item every year. This poses a challenge on the accuracy of the project cost estimate and/or budget(s) and may cause cost overruns [11, 50]. Accurately estimating the costs of complex infrastructure projects in the design, and construction phases have typically become a unique challenge for engineers, architects, owners, municipal agencies, and contractors [8]. Complex and technologically advanced projects are usually contained much uncertainty and related challenges than other projects. Therefore, engineering cost estimates must adequately address uncertainty at the preliminary stages of projects where neither the exact quantities nor specific costs or ultimate prices are known. However, dealing with risks and uncertainties are usually a problem [11, 54].

The sources of risks and uncertainties in a project are several. At the early stage of the project, the uncertainty in a cost estimate increases due to the available information quantity and quality. As the design progress, more and better information becomes available; the uncertainty in the cost estimate is gradually reduced [38, 42-43, 49].In the deterministic approach, information about uncertainties and their characteristics such as higher or lower values, ranges of quantities, and potential costs cannot easily be taken into consideration even though this information is generally available or can be estimated. However, the probabilistic approach used best fit probability distributions to model the uncertainties and risk in the cost estimate. The main advantages of the probabilistic cost estimating approaches are its ability to provide insight in the accuracy of the estimate and the impact of uncertainties and risks of cost overruns will be known [34, 65-67].

The aim of this paper is to provide a resource for professionals who are involved in budget development and/or are seeking to a better understanding of different methods in determining an appropriate base cost margin to produce a meaningful and reliable project cost estimate. It highlights the various methods of engineering cost estimating approaches advantages, and shortcomings, including the potential risk that can positively or negatively affect the project's cost outcome.

However, it should be recognized that any technical books, municipalities' guidelines, etc. cannot provide entirely complete and practical engineering cost estimation guidance applicable to all capital improvement projects and cannot replace experience and sound judgment. As such, deviations from those books or guidelines may, sometimes, be unavoidable or otherwise justifiable.

2. Accuracy of Cost Estimates

The overall purpose of an accurate cost estimate is its use in establishing the budget for a project and as a tool used for scheduling and monitoring and controlling of the project cost. The level of accuracy of engineering cost estimates increases as the project phase progresses and the potential risks are identified. The earlier the estimate in the life of the project the lower its accuracy consequently, assessments of conceptual estimate accuracy are quite low [2, 10, 19, 45]. Figure1 below shows the Characteristic curve of accuracy vs. time to make estimates.

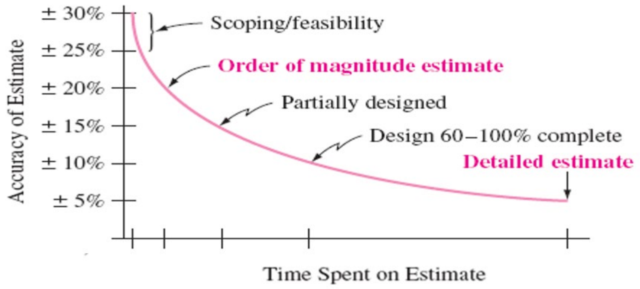

Figure 1: Characteristic Curve of Accuracy vs. Time to Make Estimates

The target cost estimate accuracy set calculated from programmatic data, prior to design generally assumed to be around +/- 30%. However, Experts assert that this variance allows conceptual estimates to be useful for determining feasibility but not for establishing a control budget. There are a different factor that affects the accuracy of conceptual engineering cost estimates, such as basic process design (23.2%), team experience and cost information (13.3%), time to estimate (12.1%), site requirements (11.5%), and bidding and labor climate (10.2%). In general, in schematic and/or preliminary stage (order-of-magnitude) cost estimates accuracy are in between ±20% of actual costs and in detailed estimates are in range of ±5% of actual costs [9, 14, 21, 24, 36].

3. Classifications of Cost Estimation Methods

3.1. Deterministic and Probabilistic Cost Estimating Methods

There are several different deterministic methods of preparing a cost estimate depending on the purpose, the level of planning, and/or design, as well as the project type, size, complexity, circumstances, schedule, and location. In general, regardless of whether the project technical scope is traditional (capital funded, construction, equipment purchases, etc.) or nontraditional (expense funded, research and development, operations, etc.). The levels of requirements and techniques used are the common characteristics of most project cost estimates [45, 56, 60-61, 67]. These includes (1) Status of Project life cycle, (2) the detail information available, (3) cost estimation methods (e.g., parametric vs. definitive), and/or (4) Constraints and other estimating variables such as time [50]. Preparing cost estimate also depending on the purpose, level of planning, and/or design, as well as the project type, size, complexity, circumstances, schedule, and location. These methods can fall into categories such as parametric, historical bid-based, unit cost/quantity based, range, and probabilistic risk-based estimates [15, 34, 43, 52-53]. Figure2 below shows, the two major Classifications of Cost Estimation approaches namely deterministic and probabilistic method.

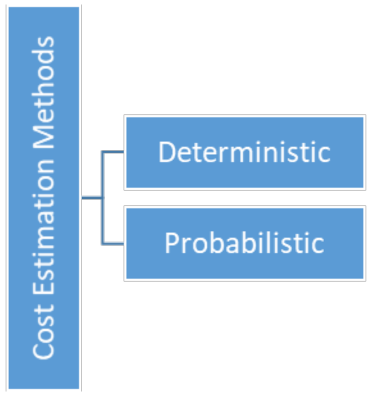

Figure 2: Classifications of Cost Estimation Methods

Generally, in the deterministic approach, information about uncertainties and their characteristics such as higher or lower values, ranges of quantities, and potential costs cannot easily be taken into consideration even though this information is generally available or can be estimated [1, 6, 10, 41, 46, 48]. However, the probabilistic approach used best-fit probability distributions to model the uncertainties and risk in the cost estimate [4, 12, 28, 38]. The main advantage of the probabilistic cost estimating approach is its ability to provide insight into the accuracy of the estimate and the impact of uncertainties and risks of cost overruns [18, 26, 42, 54, 56].

The fundamental difference between these two-cost estimation approaches (probabilistic and deterministic) is that by using the probabilistic cost estimation approach, we are enabling explicitly model the uncertainties and risk associated with it using appropriate statistical distributions [39, 59, 62, 66]. Moreover, the selection of these cost estimation techniques will be adopted for different projects, depending upon the aims and objectives sought by the client [28].

3.1.1. Deterministic Cost Estimating

Under this category, Parametric, Detailed, Comparative, (Unit, cost, and Power law and sizing method), and Factored Estimates methods have been discussed below:

I. Parametric Cost Estimating (Top-Down Estimating)

This method is generally used during the earliest stage of the project [48]. However, it also can be used to establish a baseline at any stage, where the comparison or validation of other estimating methods are needed or estimation of the use of resources required to perform for a new project [13, 29, 52-53]. This model has a mathematical representation of the cost estimating relationships (CERs) that able to predict and provides a logical correlation between the physical and functional characteristics of a project [5, 24].

A particular cost or price can be established and estimated using cost estimating relationships (CERs) with an independent variable. The cost estimating relationship (CER), mathematical ratio or equation can be developed using an independent variable that demonstrates a measurable relationship between contract cost and price. It usually derived from regression analysis of historical systems or subsystems. Equations (1) and (2) are the associated linear and nonlinear form of cost estimation relationships (CER). These equations are called cost estimating relationships (CER's) framework. The CER uses quantitative techniques to quantify a relationship between an independent variable and contract cost or price [6, 46, 49].

$$T_c = \sum_{i}^{n} P_{CR} P_I \quad (1)$$

Where: $\mathbf{T_C}$ = Total Cost; $\mathbf{P_{CR}}$ = parameter cost ratio; $\mathbf{P_I}$ = parameter of an interest.

Equation (2) for CER with associated nonlinear form cost estimation relationships

$$T_c = \sum_{i}^{n} P_{CR} P^{n_i} \quad (2)$$

Where: $\mathbf{P_I}$ = parameter of independent variable of interest; $\mathbf{n_i}$ = exponent used to transform $\mathbf{P_I}$

n= is the number of units (Suppose that a construction project is divided into "n" elements for cost estimation).

The exponents n_i(E.q-2) used to transform and normalize the temporal effects of cost including an inflation, rapid increases of material cost, and for an independent variable and other metrics. In general, Parametric cost estimating can also be incorporated with probabilistic estimating to form range estimating that predict uncertainties and potential risks [34, 39, 46, 52, 66]. Table-1 below shows the advantages and disadvantages of Parametric Cost Estimating Method.

Table 1: Parametric Cost Estimating Method

Advantage	Disadvantage	Requirement
Relatively quick and accurate way to estimate costs	Documentation of Cost Estimating Relationships (CERs) can be difficult	Sufficient historical data for statistical analyses
Reduced likelihood of serious cost overruns	Traceability of CERs can be difficult	Database with historical data that can be updated regularly
Reduced cost of preparing project proposals	Historical data must be available	Database
Multiple decision options for project managers	Improper use of CERs can lead to serious estimating errors	
Model can be used as basis for uncertainty andrisk analyses	Maintenance of database with historical data	
Easy to combine with other estimating methods	Periodically updated to capture the most current cost, technical, and programmatic data.	

II. Detailed Cost Estimating /Bottom-up/ Analytical Estimating Method

The detailed cost estimating requires is the most accurate estimating technique when, the project is decomposed into manageable tasks, or when works breakdown structure is available [17, 18]. A work breakdown structure is used to divides project deliverables into a series of work packages and each work package comprised of a series of tasks [22, 53, 58]. During detailed cost estimate, the project teams of cost estimators work with engineers, Architects etc. to complete each itemized task, work packages, and develop the total detailed cost estimate for the entire proposed project. The cost estimator's quantity estimates have to be validated by the professional engineers to make sure this cost estimation process is leads to a consistent and reproducible result. The general mathematical formula for detailed cost estimating method is given by Equation-3. However, this method is different for each project depends on complete work breakdown structure availability, [40, 57].

$$T_C = \sum_{i} q_i (M_i + W_i + L_i) + \sum_{j} I_j (UC_j) \quad (3)$$

Where: $\mathbf{T_C}$ = Total Cost; $\mathbf{q_i}$ = quantity of work; $\mathbf{M_i}$ = Unit material cost; $\mathbf{W_i}$ = Unit Wage rate; $\mathbf{L_i}$ = Unit Labor Rate;$\mathbf{I_j}$ = measure of work in indirect cost elements; $\mathbf{UC_j}$ = Unit cost of in indirect elements.

Certainly, the detailed cost estimating is the most accurate and provides insight into the major cost contributors, all cost components and make sure nothing can be overlooked [33, 20]. However, it can also be time-consuming, and requires a lot of effort to establish especially in large and complex projects with numerous work breakdown structure components [16, 22, 57]. Table-2 below shows the advantages and disadvantages of Detailed Cost-Estimating Method.

Table 2: Detailed Cost-Estimating Method

Advantage	Disadvantage	Requirement
A greater level of confidence Very high accuracy possible	More time needed to develop the estimate	Collaboration of the engineers that conduct the work
All cost components are taken into account	more costly to develop than relationship estimating	Work Breakdown Structure
Nothing can be overlooked	Historical data must be available	Additional 'sanity' check or benchmark
Parts of the estimates can be reused	Project's scope must be determined and understood considerably	
Actual cost data of ongoing project can be used as predictor for future	Confidence level difficult to determine	

III. Comparative Cost Estimating/Analogous Estimating Method

The comparative estimating method can be used to make a quick comparison when a new project is similar to another project recently completed. During this process the major cost components that were used on previous similar projects and direct and recent experience is needed [35]. Adjustment shall be made on the proposed cost estimate factoring the differences in project size and complexity, performance requirements, duration, location and available technology [9, 44]. This relation factors are not usually linear. Cost capacity factors and economies of scale are the main factors that determine the nonlinear form of cost estimation relationships (CER) [3, 15, 31]. Commonly used technique for preliminary design stage cost estimates are Unit Method, cost indexes, Cost-Capacity Equation or power law and sizing model, and Factored Estimates [39, 64]. The general mathematical Cost estimate equations are presented below.

I. Unit Method

$$\mathrm{T_c} = \sum_{i}^{n} U * N \quad (4)$$

Where: $\boldsymbol{T_C}$ = Total Cost; U= per unit cost; N= quantity of work, n= is the number of units (Suppose that a construction project is divided into "n" elements for cost estimation).

II. Cost Indexes

Cost Index (CI) is the ratio of cost to date versus cost in the past. The CI change in cost over time to account the impact of inflation and it is dimensionless [63]. The general mathematical formula used to calculate the total Cost estimate is:

$$T_C = \sum_{t} C_0 \left(\frac{I_t}{I_0}\right) \quad (5)$$

Where: $\boldsymbol{T_C}$ = Estimated total cost of present time; $\boldsymbol{C_0}$ = Cost at previous time; $\boldsymbol{I_t}$ = Index value at timet; $\boldsymbol{I_0}$ = Index value at base time 0.

III. Cost-Capacity Equation or Power Law and Sizing Model

The general mathematical formula used to calculate the total Cost estimate is:

$$C_2 = C_1 \left(\frac{Q_2}{Q_1}\right)^x \quad (6)$$

Where: $\boldsymbol{C_1}$ = Cost at Capacity$\boldsymbol{Q_1}$; $\boldsymbol{C_2}$ = Cost at Capacity$\boldsymbol{Q_2}$; x= Correlating Exponent.
Where: x = 1, relationship is linear; x< 1, economies of scale (larger capacity is less costly than linear); x> 1, diseconomies of scale.
Cost-Capacity Combined with Cost Index: Multiply the cost-capacity equation by a cost index $\left(\frac{I_t}{I_0}\right)$ to adjust for time differences and obtain estimates of current cost (in constant-value dollars)

$$C_2 = C_1 \left(\frac{Q_2}{Q_1}\right)^x \left(\frac{I_t}{I_0}\right) \quad (7)$$

Some of the advantages of this method are its ability to generate quick, easily, very accurate and understandable cost estimate for the proposed project, especially when the proposed project has minor deviations from an appropriate comparative similar past project that has been completed[3, 15, 37].The shortcomings of this method are its dependent on a single data point, its requirement of normalization in order to create baseline and ensure a good accuracy of the estimate, and the difficulties of finding an appropriate comparative data for similar past project and experts to make judgment to adjustment factors[25, 62-63].Table-3 below shows the advantages and disadvantages of Comparative Cost Estimating Method.

Table 3: Comparative Cost Estimating Method

Advantage	Disadvantage	Requirement
Easy to generate and estimate, provided historical data is available.	Uncertainty due to subjective evaluations made by estimator.	Requires analogous product and program data.
Provides better credibility than plain detailed estimating. Can be used early in project even if scope of the project is not complete	Difficult to apply for differences in scope of work, design, configuration and number of aircraft or aircraft programs.	Requires a detailed program and technical definition of the analogous system as well, as the system being estimated.
Quick and reasonable accuracy for similar systems, or end items. Estimate is easy to understand	Once the technical assessment has identified the analogous system, actual cost data on that system must be obtained.	Experience or data of a relevant comparative project
Good accuracy for similar systems if comparative and recent data is available	Accuracy is limited, Cost impacting factors have to be determined, and Normalization required	Comparison factors

IV. Ratio or Factored Estimates Method

In this method, scaling relationships used to forecast the cost of new project when historical and component data are available from similar project [19]. However, this scaling relationship does not include economical factor, location and the timing of the work. Generally, this method is used in estimating total plant cost in the processing industries. Both direct and indirect costs can be included [20, 24, 37, 41]. The general mathematical formula used to calculate the total Cost estimate can be expressed as:

$$T_C = C_E \left(\sum_{i} f_i * C_{E_i}\right) * (f_I + 1) \quad (8)$$

Where: $\boldsymbol{C_E}$ = The total cost of major equipment item; $\boldsymbol{f_i}$ = Overall cost factor which can be determined using two basis; Delivered equipment cost including purchase cost of major equipment Installation cost; $(\boldsymbol{f_I + 1})$ = the cost factor (commonly the sum of a direct cost component and an indirect cost component) for i = 1, 2… n components, including indirect costs.

3.1.2. Probabilistic Cost Estimating Method

The probabilistic cost estimating techniques focus on the risks and uncertainties involved in the project and attempt to quantify the project cost variability based on one or more parameters. It addresses the concerns regarding the chance of exceeding a particular cost in the range of possible costs, the possible amount of the cost overrun, and the different types of uncertainties and how they drive cost [4, 38, 42, 60, 66-67]. The probabilistic cost estimating techniques uses probability distribution to consider range estimation rather than point estimates to reflect the different outcomes [17, 26, 30]. The Expected value, Variance, Covariance and the Central Limit are some of the key aspects of the mathematical application of probabilistic cost estimating techniques.

I. Expected Value

The expected value of a cost parameter can be defined as the weighted average of all possible values. The term expected value in essence means the same as the often used term average [46]. The expected value equals:

$$E(X_i) = \mu_x = \int_{-\infty}^{\infty} x \ f(x)\, dx \qquad (9)$$

Where: $\boldsymbol{f(x)}$= The probability density functions of cost parameter $\boldsymbol{i}$. If all cost parameters of $\boldsymbol{i}$ are correlated such that $\boldsymbol{Y = x_1 + x_2}$, then

$$E(Y) = E(X_1) + E(X_2) \qquad (10)$$

The variance in this case is given by $\delta Y^2 = \delta_1^2 + \delta_2^2 + 2\delta_{1,2}$ in this formula $\delta_{1,2}$ is the covariance of random variables of$x_1 and x_2$. If the random variables are independent then $\delta_{1,2}$ is equal to zero. If the total cost is the product of independent, continuous, random variables, such that= $x_1 * x_2$, then

$$E(Y) = E(X_1) + E(X_2) \qquad (11)$$

$$\delta Y^2 = X_1^2\delta_1^2 + X_2^2\delta_2^2 + \delta_1^2\delta_2^2 \qquad (12)$$

II. Variance

In probability theory, variance gives a measure of how much the values of a function of a random variable x vary as we sample x from a probability distribution. When the variance is low, values of f(x) cluster around its expected value. The square root of the variance is known as the standard deviation and usually indicated with the symbol σ [38, 43].

III. Covariance

Covariance measures how two values are linearly related, as well as scale of variables. Calculating correlation is an important to analyze the correlation between two or more cost components that can have a large impact on the degree of risk associated with using the variance. If two random variables have no correlation with covariance equal to zero they are called independent [11]. The covariance can be high absolute, positive, zero or negative. High absolute values of covariance means values change very much and are both far from their mean. Positive value means both variables take relatively high values far from mean. Negative value means one variable takes on high values & another takes low values [59, 63].

The formula that can be used to calculate the covariance of two random variables X and Y, denoted by $\boldsymbol{Cov(X,Y)}$is defined as:

$$Cov(X,Y) = E(XY) - \mu_X\mu_Y \qquad (13)$$

Therefore, the Pearson's correlation coefficient between data sets X and Y can be calculated:

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}} \qquad (14)$$

Where: r = Pearson's correlation coefficient $\boldsymbol{\bar{X}}$ = Mean of data set X; $\boldsymbol{\bar{Y}}$ = Mean of data set Y.

IV. Central Limit Theorem

The Central Limit is the second fundamental theorem of the probability function that allows us to develop a process to estimate and test the mean of a population using a sample. The central limit theorem of statistics states that the sample mean X¯ follows approximately the normal distribution with mean μ and standard deviation √σ n, where μ and σ are the mean and standard deviation of the population from where the sample was selected. In order to be able to give lower and upper bounds on the total cost we use confidence limits. Confidence limits are the probability that the interval estimate will include the lower and upper bound of cost parameter [22-23, 55]. Table-4 below shows the Confidence Level Using the Standardized Normal Distribution.

$$LBTC = ETC - Z * \sigma \qquad (15)$$

$$UBTC = ETC + Z * \sigma \qquad (16)$$

Where: $\boldsymbol{LBTC}$ =Lower bound on Total Cost; $\boldsymbol{UBTC}$= Upper bound on Total Cost; $\boldsymbol{ETC}$ =Expected Total Cost; $\boldsymbol{\sigma}$ = Standard Deviation; Z= is determined by the confidence level using the standardized Normal distribution.

Table 4: Confidence Level Using the Standardized Normal Distribution

Confidence Level	Value of Z
90%	1.28
95%	1.65
98%	2.05
99.9%	3.09

3.1.3. Probability Distributions

Different cost parameters coupled with several simple probability distributions are useful in many engineering cost estimation modeling and risk analysis. Normal, Lognormal, Beta, Triangular and Weibull are typical probability distributions that are commonly used in the construction industry [4, 11, 17, 43, 56, 60, 65-66]. Below are summary of discussion together with the probability density function (PDF), the cumulative density function (CDF), the expected value (E(X)) and the variance (Var(X)) of each distributions. Table-5 below shows the advantages and disadvantages of Probabilistic Cost Estimating Method.

I. Uniform Distribution

The uniform distribution is a continuous probability distribution the assumption: the random event is equally likely in an interval. It is defined by two parameters, the minimum possible value (a) and the maximum possible value (b).A variable X is said to be uniformly distributed if the density function is:

$$f(x) = \begin{cases} \frac{1}{b-a} & for\ \ or -\infty < a \le x \le b < \infty \\ 0 & \end{cases} \quad (17)$$

Figure-3, below shows the sample of the uniform distribution curve graph

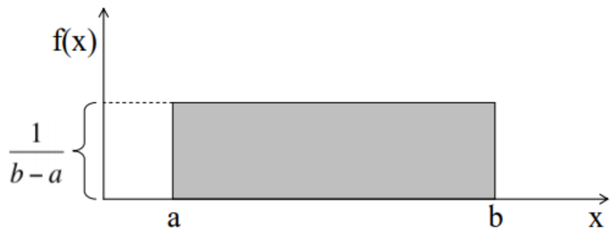

Figure 3: Sample of a Uniform Distribution Graph

The mean and variance of X following a uniform distribution is:

$$E(X) = \frac{(a+b)}{2} \quad (18)$$

$$V(X) = \frac{(b-a)^2}{12} \quad (19)$$

The standard uniform density has parameters a = 0 and b = 1, so the PDF for standard uniform density is given by:

$$f(x) = \begin{cases} 1, 0 \le x \le 1 \\ 0, otherwise \end{cases} \quad (20)$$

II. Triangular Distribution

In this method, it is assumed that a Triangular or Beta distribution can be used to describe each item T (a, m, b). This means that the user gives an optimistic estimate a, a most likely estimate m and finally a pessimistic estimate b [5, 32, 56]. Figure-4, below shows sample of triangular distribution graph.

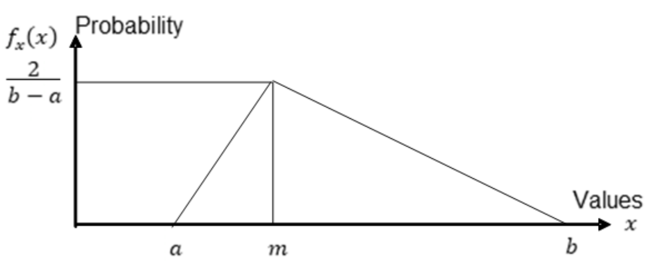

Figure 4: Sample of Triangular Distribution (a) =Lowest (b) = Highest, and (M) = Most Likely Values

The PDF of the triangular distribution is given by:

$$f_x(x) = \begin{cases} \frac{2(x-a)}{(b-a)(m-a)} & if\ \ a \le x \le m \\ \frac{2(b-x)}{(b-a)(m-a)} & if\ \ m \le x \le b \end{cases} \quad (21)$$

The cumulative probability distribution of the triangular distribution is given by

$$f_x(x) = \begin{cases} 0 & \\ \frac{(x-a)^2}{(b-a)(m-a)} & if\ \ a \le x < m \\ 1 - \frac{(b-x)^2}{(b-a)(b-m)} & if\ \ m \le x < b \\ 1 & \end{cases} \quad (22)$$

The expected value is given by:

$$E(X) = \frac{a+m+b}{3} \quad (23)$$

The variance is given by:

$$V(X) = \frac{a^2+m^2+b^2+ab+am+mb}{18} \quad (24)$$

The standard deviation is given by:

$$\delta = \sqrt{\frac{a^2+m^2+b^2-am-ab-mb}{18}} \quad (25)$$

III. Beta Distribution

One of its most common uses of this distribution is to model uncertainty and bounded continuous random variables based on expert's judgment. The Beta (α, β) distribution is a continuous probability that is defined by two shape parameters α and β [5, 27, 32]. The general formula for the probability density function of the beta distribution is:

$$f\ (x) = \begin{cases} \left(\frac{1}{H-L}\right)\frac{\Gamma(\alpha-\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{x-L}{H-L}\right)^{\alpha-1}\left(\frac{H-x}{H-L}\right)^{\beta-1} & L < x < H \\ = 0 & other \end{cases} \quad (26)$$

The shape parameters: $\boldsymbol{\alpha > 0, \beta > 0}$Figure-5, below shows sample of beta distribution graph.

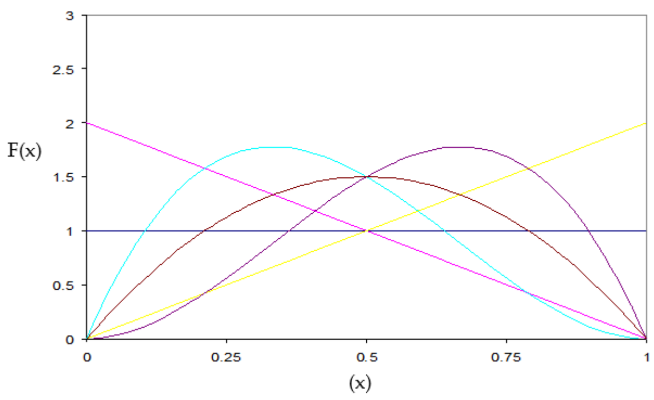

Figure 5: Sample of Beta Distribution

Most schedule or cost estimates follow right skewed pattern. The value of $\boldsymbol{\alpha and \beta}$ can be determined using Beta-PERT (L, H, M) Distribution using L, M, and H to calculate the expected value mean and standard deviation as [5, 27, 32]:

$$E(X) = \mu = \frac{(L + 4M + H)}{6} \qquad Var\ (X) = \frac{(H - L)^2}{36} \qquad \sigma = \frac{(H - L)}{6} \tag{27}$$

$$\alpha\ and\ \beta = \begin{cases} \alpha = \dfrac{(\mu - L)}{(H - L)} * \dfrac{(\mu - L)(H - \mu)}{\sigma^2} - 1 \\ \beta = \dfrac{(H - \mu)}{(\mu - L)} * \alpha \end{cases} \tag{28}$$

Or, from the expected value (μ) and the distribution P (L, M, H) the parameters α and β can be derived by

$$\alpha\ and\ \beta = \begin{cases} \alpha = \dfrac{(\mu - L)(2M - L - H)}{(H - L)(M - \mu)} \\ \beta = \dfrac{\alpha(H - \mu)}{(\mu - L)} \end{cases} \tag{29}$$

Where: $\boldsymbol{\alpha > 0}$, $\boldsymbol{\beta > 0}$; (L), lowest; (H) Highest and (M) Most likely values.

IV. Normal Distribution

The normal distribution is a continuous probability distribution and it has two parameters, μ and σ and is denoted N (μ, σ). Here μ is its mean, $\boldsymbol{\delta^2}$ its variance, and σ is its standard deviation [10, 12, 25]. The normal distribution is a continuous distribution with probability density function of:

$$f_x(x) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\pi}{\sigma}\right)^2} \tag{30}$$

The cumulative probability distribution of the normal distribution is given by:

$$f_x(x) = P(X \le x) = \int_{-\infty}^{x} \frac{1}{\delta\sqrt{2\pi}} e^{-\left[\frac{1}{2}\left(\frac{x-\pi}{\sigma}\right)^2\right]}\, dx \tag{31}$$

The expected value is given by

$$E(X) = \mu \tag{32}$$

The expected value is given by

$$Var(X) = \delta^2 \tag{33}$$

Figure-6, below shows sample of standard normal distribution graph.

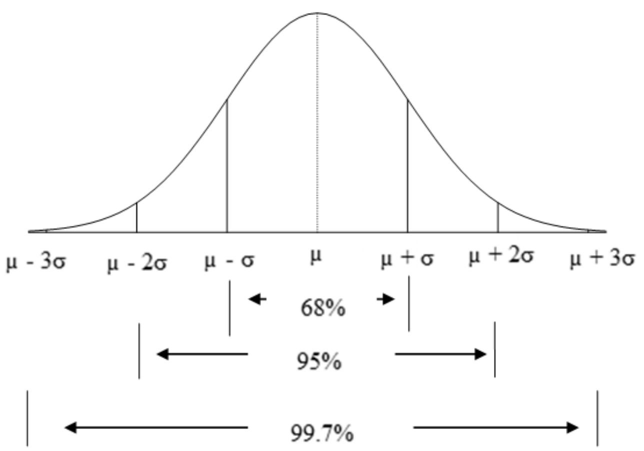

Figure 6: Sample of Standard Normal Distribution Graph

V. Lognormal Distribution

The lognormal distribution is the probability distribution where the natural log of the sample values has a normal distribution. The probability density function (pdf) is given by ln (N (μ, σ^2)) [5, 32, 43].

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}}\ e\ \left[-\frac{1}{2}\left(\frac{ln\, x - \alpha}{\beta}\right)^2\right] \tag{34}$$

Where: α is the mean of ln(x) and β is the standard deviation of ln(x). These are related to the mean and standard deviation of random variable x (μ and σ respectively) as follows:

$$\mu = \ e\ \left[\alpha + \frac{1}{2}\beta^2\right] \tag{35}$$

$$\delta = \sqrt{e[2\alpha + \beta^2](e[\beta^2] - 1)} \tag{36}$$

$$\alpha = ln\, \mu - \frac{1}{2} in\left[\left(\frac{\alpha}{\pi}\right)^2 + 1\right] \tag{37}$$

$$\beta = \sqrt{ln\left[\left(\frac{\alpha}{\pi}\right)^2 + 1\right]} \quad (38)$$

Figure-7, below shows sample of standard lognormal distributions graph

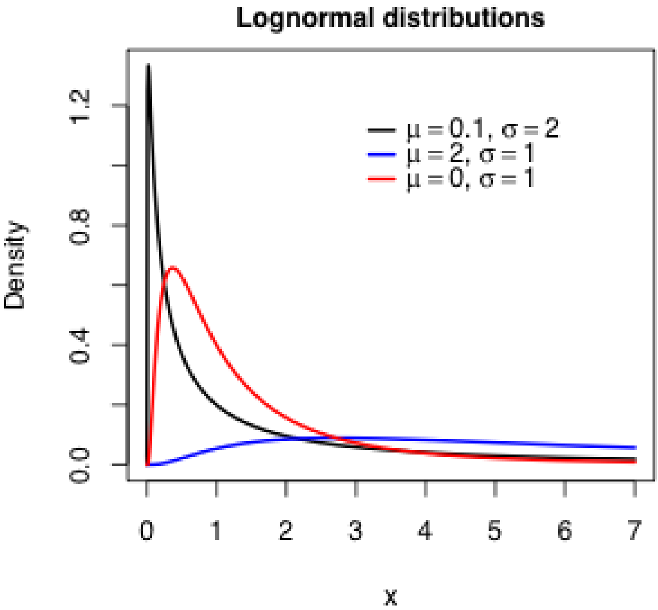

Figure 7: Sample of Lognormal Distributions Graph

P is the cumulative probability function (CDF) of the log normal distribution, is given by

$$P(x \le X) = \int_0^x \frac{1}{x\beta\sqrt{2\pi}} \quad e\left[-\frac{1}{2}\left(\frac{ln\,x - \alpha}{\beta}\right)^2\right] dx \quad (39)$$

Note that F is the cumulative probability function (CDF) for the standard normal probability distribution

$$P(x \le X) = F\left(\frac{ln\ X - \alpha}{\beta}\right) \quad (40)$$

The Expected value is given by:

$$E(X) = e^{\mu + \frac{\sigma^2}{2}} \quad (41)$$

The variance is given by:

$$Var(X) = \left(e^{\sigma^2} - 1\right)e^{2\mu + \sigma^2} \quad (42)$$

VI. Weibull distribution

Generally, the Weibull distribution is one of the most commonly used statistical model for project cost estimations and many other applications. The Weibull Distribution (1939) was first published to represent the probability of failure and has proven to be extremely useful for data analysis in many engineering applications such as aerospace, automotive, electric power, nuclear power, medical, dental, electronics and every industry [5, 18, 32, 36, 43].

A continuous function X is said to have a Weibull distribution with parameters δ> 0 and β> 0 if the PDF of X is:

$$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} e\left[-\left(\frac{x}{\delta}\right)^{\beta}\right], \quad for\ \ x > 0, and\ \ f(x) = 0, for\ \ x \le 0 \quad (43)$$

The Expected value is given by:

$$\mu = E(X) = \delta\Gamma\left(1 + \frac{1}{\beta}\right) \quad (44)$$

The Variance is given by:

$$\delta^2 = V(X) = \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \mu^2, \quad \text{where } X\sim \text{Weibull}(\delta, \beta) \quad (45)$$

The cumulative probability function (CDF) for the standard Weibull (δ,β) probability distribution is given by:

$$F(x) = 1 - e\left[-\left(\frac{x}{\delta}\right)^{\beta}\right], \quad F(x) = 0, \text{for } x \le 0 \quad (46)$$

Figure-8, below shows sample of standard sample of Weibull distributions graph

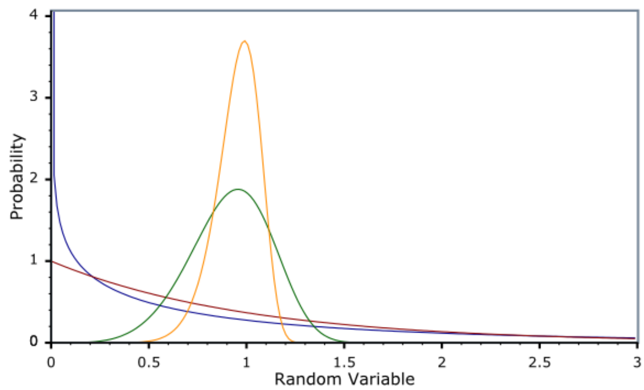

Figure 8: Sample of Weibull Distribution Graph

Table 5: Probabilistic Cost Estimating Method

Advantage	Disadvantage	Requirement
Probability of cost overrun is insightful and defined as opportunities or threats	Additional analysis that requires additional effort	Probability distribution of cost components based on historical data or experience
Improved reliability of the estimate	Determining probability distributions may be difficult	Software to run Monte Carlo Simulation
Range of outcomes is available		
Uncertainties and risks are mapped and quantified		

3.2. Sample Project to demonstrate and compare different Cost Estimating Methods for small urban drainage system improvement project.

VII. P50 and P90 cost estimation

From the director of the public work's point of view, the expected cost of a portfolio of projects is of more interest than the costs of projects cost separately. Individual projects are considered at the mean of simulated cost distribution, typically the P50 estimate. The P50 cost value is an estimate of the project cost based on a 50% probability that the cost will not be exceeded. The P90 value is an estimate of the project cost based on a 90% probability that the cost will not be exceeded. Project proponents (and their management) often prefer to have less commercial (and political) exposure in respect of capital budgets and often look for a P90 figure (or equivalent if done deterministically), meaning the contingency allowance on top of the base estimate is sufficient to ensure that there is a 90% chance that the amount will not be exceeded [69].

4. Validation through Case Studies

An example of deterministic approach, range based approach and Risk Based – Probabilistic Approach have been prepared as an accompaniment to this journal with all formulae intact. Practitioners are welcome to utilize this model for training or as a template, modified as appropriate for their own circumstances.

The example uses typical capital improvement project Cost Breakdown (PCB) template structure as the basis for aggregating inputs. However, aggregating inputs that reflect the type of risk exposure or other logical model structures such as aggregation based on geographically discrete work packages may be more suitable. Costs representing the most likely, the best case and worst case in this example are hypothetical only.

The selected typical capital improvement project demonstrates the validity, usefulness, and benefit of the different deterministic and risk-based probabilistic cost estimation approach. It compares the actual costs with traditional deterministic methods of cost estimation (such as single-point base-case estimates inclusive of contingency) and provide valuable insights that can aid management in evaluating alternatives and make informed decisions when estimating and allocating budgets to a portfolio of Urban drainage system and associated drainage infrastructure rehabilitation projects.

Table 6: Drainage Rehabilitation Capital Improvement Project (Base Cost Only) case study-1

		Quantity					Unit Price				Amount
Item	Description	Distribution	min	ml	max	unit	Distribution	min	ml	max	Q (ml) * UP (ml)
1	Remove existing storm drainage structures	Triangular	6	6	6	ea.	Triangular	$438.75	$450.00	$465.75	$2,700.00
2	Remove surface materials - asphalt	Triangular	87.75	90	92.97	lf.	Triangular	$8.53	$8.75	$9.06	$813.49
3	Construct single grate inlet catch basin box	Triangular	4	4	4	ea.	Triangular	$2,656.88	$2,725.00	$2,820.38	$10,900.00
4	Construct 15" diameter rcp storm drain pipe	Triangular	87.75	90	92.97	lf.	Triangular	$38.27	$39.25	$40.62	$3,649.07
5	Construct 4' diameter cleanout manhole	Triangular	2	2	2	ea.	Triangular	$2,486.25	$2,550.00	$2,639.25	$5,100.00
6	Furnish trench backfill materials	Triangular	48.75	50	51.65	tons	Triangular	$4.39	$4.50	$4.66	$232.43
7	Furnish bedding materials	Triangular	29.25	30	31.5	tons	Triangular	$10.24	$10.50	$10.87	$330.75
8	Construct concrete curb and gutter	Triangular	117	120	124	lf.	Triangular	$19.01	$19.50	$20.18	$2,417.22
9	Construct double hooded inlet catch basin box	Triangular	2	2	2	ea.	Triangular	$3,607.50	$3,700.00	$3,829.50	$7,400.00
10	Roadway patching	Triangular	98.48	101	104.3	sy.	Triangular	$19.01	$19.50	$20.18	$2,034.44
11	Traffic control and flagging	Triangular	13.65	14	14.46	days	Triangular	$146.25	$150.00	$155.25	$2,169.00
12	Landscaping & surface restoration	Triangular	95.55	98	101.2	sy.	Triangular	$195.00	$200.00	$207.00	$20,246.00
											$57,992.39

A. Deterministic Approach

The deterministic approach delivers a single figure that is the sum of the products of the most likely quantity multiplied by the most likely price as shown below:

$$= \sum \textbf{Most likely quantity x most likely price}]$$
$$= \$\ 57{,}992.39\ \textbf{USD}$$

B. Range approach with Minimum, Most Likely, and Maximum Cost.

Range approach with Minimum, Most Likely, and Maximum Cost Approach delivers three results as the sums of the following elements:

$= \sum \textbf{Min. Q} * \textbf{Min. UP} = \$54{,}792.91\ \textbf{USD}$

$= \sum \textbf{Most Likely (ml)Q} * \textbf{most likely. UP} = \$57{,}992.39\ \textbf{USD}$

$=\sum\ \textbf{Max. Q} * \textbf{Max. UP} = \$60{,}022.12\ \textbf{USD}$

A. Square root Approach

The Square Root approach delivers one single figure which is the sum of all base costs plus the square root of the sum of the squares of the risk contingencies as shown below

$$\textbf{Cost} = \sqrt{\sum \textbf{Cost Item } (\textbf{max} - \textbf{Min})^2} + \sum \textbf{Min} \quad = 57{,}345.91\ \textbf{USD}$$

B. Risk Based – Probabilistic Approach

Risk Based - Probabilistic Approach include base cost uncertainty, the monetary impact of discrete risks as defined in the risk register plus escalation, as well as the monetary impact of schedule delays through extended overhead and additional escalation caused by schedule delays. The non-escalated base-cost estimate for this contract has been determined through the estimating process to be $57,992.36 USD. Assuming no risk or uncertainty on this value and incorporating projected escalation provides an escalated base-cost estimate of $1,077 USD. Further incorporating risk associated with: (i) the base cost estimate, (ii) specific event risks and (iii) potential schedule delay provides a risk-based estimate of total Project costs. As shown in

Figure 9, there is an 80% probability that this cost will lie between $1,075 million (10th percentile) and $1,235 million (90th percentile). At the 70th percentile, the risk-based cost estimate is $1,188 million.

Figure 9: Graphical Representation of Cost-Risk Analysis Results

To compare this to a traditional cost estimating approach (in which allowance and/or contingency are set at fixed proportions of the base cost estimate), this outcome suggests the Project should budget a 10.3 percent allowance/contingency over the escalated base cost estimate of $1,077 million (to ensure a level of confidence of 70 percent).Figure -9 provides a graphical representation of cost-risk results.

Table 7: Drainage Rehabilitation Capital Improvement Project (Base Cost Only) case study-2

Drainage Projects	Initially Allocated Budget ($)	Design Engineer's Preliminary Estimate ($) Single-Point Base Estimate Inclusive of 10-20% Contingency (Max. Value)	Design Engineer's Risk-based Preliminary Estimate ($)		Actual Final Cost ($)
			P50	P90	
PR-1	$ 295,800.00	$ 300,000.00	$ 297,900.00	$ 300,588.00	$ 305.075.00
PR-2	$ 355,000.00	$ 406,048.00	$ 380,524.00	$ 413,195.00	$ 421,400.00
PR-3	$ 520,000.00	$ 595,000.00	$ 557,500.00	$ 605,500.00	$ 596,418.00
PR-4	$ 450,000.00	$ 495,958.00	$ 472,979.00	$ 502,391.00	$ 509,927.00

Table 7 illustrates the difference between the P50 and P90 cost estimation and the importance of adopting a P90 estimate. The P50 value predicted at the preliminary stage (when uncertainty is highest), considering only the rates of the allocated contractor, predicted the closest value for budgeting purposes when compared with the actual final cost. The incorporation of risk should be a necessary step in project cost estimation at all stages of a project, by applying the P90, rather than the P50, value. The application of the P90 value will assist the municipalities/ cost estimator professionals in their understanding and appraisal of project proposals, and improve the preparation of cost estimates that form part of the documentation to make them more transparent, reliable and consistent.

5. Conclusions

In this paper, the most common project cost estimation methods and approaches were collected and classified into two main categories of: (1) deterministic and (2) probabilistic methods. Then, the two main categories were further divided into four and six sub categories respectively: (1) Parametric cost estimating method, (2) Detailed cost estimating methods, (3) Comparative cost estimating methods, (4) Ratio or Factored estimates method, and (1) Uniform distribution methods, (2) Triangular distribution methods, (3) Beta distribution method, (4) Normal Distribution methods, (5) Lognormal distribution methods, and (6) Weibull distribution method. Overall, ten different methods were identified and discussed. Table 8 demonstrate the advantages and shortcomings of these methods including the potential risk that can positively or negatively affect the project's cost outcome. This paper will be a good resource for professionals who are in budget development and/or seeking to maximize and produce a meaningful and reliable project cost estimate for their projects. Summarized.

Table 8: Pros and Cons of Deterministic and Probabilistic Cost Estimations Methods

Deterministic		Probabilistic	
Pros	**Cons**	**Pros**	**Cons**
Varity of techniques may be used including engineering judgement, factor of safety, etc .	Does not determined residual risk. Unknown risk, can be inconsistent between sites. For areal sources, selection of deterministic event is uncertain	known risk, handles areal sources in a consistent way	more complex, still wide-spread misunderstanding
simple to use , Doesn't rely on statistics, Maintains dependencies	away from measured or interpreted data, Not statistical,	Uses impartial statistical rules, Exhaustive cases can be run,	Needs probabilistic thinking & understanding Needs software support
One single figure Well-known & accepted Quick Can be performed "manually"	No probability information of single value No Value at Risk information More often than not on the unsafe side (high, unknown probability of cost overruns)	All potential risks are included, best estimation assumptions, and follow well established methodology.	Time consuming
Good accuracy for similar systems if comparative and recent data is available	Accuracy is limited, Cost impacting factors have to be determined, and Normalization required	Multiple and common cause of failures can be easily assessed and addressed at the early stage	Comparison factors

It should be emphasized that a forecast is only good as the probabilistic model and the quality of data that is fed to the model. A probabilistic analysis does not necessarily imply precluding the use of a deterministic analysis. In fact, deterministic analysis is often required to provide input to probabilistic analysis. The key point is that deterministic analyses alone can have significant disadvantages and in such cases, should be complemented by probabilistic analyses as is seen through the fiscal results of the case studies.

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