

Computation of Lethal and Non-Lethal Angles of a Spherical Robotic Arm Manipulator from Localized Object Using Homogenous Transformation Matrix Method

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Abstract: *The effectiveness of a robotic arm is for joint angles to align so that the end-effector will position itself with reference to the base frame. This can be used in many applications like car wash system, car spray system, vehicle loading and off-loading systems, since the position of the target object is localized. In this research, the homogenous transformation matrix is effectively used to accurately compute the rotation of each of the joint angles from the base frame to the end-effector frame. Given the joint lengths and the object position, the angles which will perfectly locate and handle the object without harming the object and the end-effector with the ones which can harm both the object and the end-effector can be derived. The product of all the transformation matrices and all the frames from the base frame to the last frame is evaluated. Direct kinematics formulates how these joint angles can be determined from the position vector and given joint lengths while the excel application was used to demonstrate the angle variations and the corresponding effect on the end-effector.*

Keywords: direct kinematics, end-effector, homogenous transformation matrix, joint angles, position vector, robotic arm

1.Introduction

Most human activities are predetermined, so that it becomes very clear that such operations can be executed with a high level of precision using a well programmed robot. Also more complex actions are executed based upon sensor processing. If the object orientation and position is unknown, arms are often combined with machine vision and artificial intelligence to identify the object as well as controlling the arm using inverse kinematics (Oridate, 2008). While in direct kinematics an articulated robotic arm which comprises of measured links that connect a given number of rotary joints to determine the angles of the robotic arm relative to the end-effector handling the object is applied. (Denavit and Hartenberg 1955) and (Shah, et al., 2013).

The number of joints of a robotic arm represents the number of Degrees of Freedom (DOF) of the arm. The joints are usually actuated using servo-motors, which provide the necessary torque or force to rotate the connected links. Microcontroller is programmed to drive the electrical pulses necessary for controlling the angular motion of the servo-motor shafts. (Madiha et al., 2018). Ya-Fu et al, 2009 designed a multi-directional spherical robot using fuzzy controller. The robot was able to navigate without restriction in any direction. Chao et al, 2010 designed a spherical 5R parallel manipulator whose kinematics was analyzed using indices defined by a screw theory for performance study. Shaoping Bai, 2010 designed a spherical parallel manipulator for a defined workspace using numerical approach to find efficient design parameters. Guanglei et al, 2018 design an A3-

RRR spherical parallel manipulator reconfigured with four-bar linkages in each limb to alter one geometric parameter for different performances.

Contemporary applications of the robotic arm range from doing an accurate and reliable job of spray-painting an automobile on an assembly line, to robotic surgery. The da Vinci surgical robot uses robotic arms equipped with scalpels and other instruments to precisely target surgical objectives, allowing doctors to use smaller, less invasive incisions. (Oridate, 2008).

Ramish et al., 2016 developed a robotic arm using the CAD (Computer Aided Design) software. The analysis of the dynamic characteristics of the robotic arm was also presented by Euler-Lagrange method which has been adopted to derive the complex equation of motion of the robotic arm. The analytical results were compared with the simulations done on RoboAnalyzer© software. Madiha et al., 2018 describes the design of the three degrees of freedom of robotic arm, which carried and placed lightweight objects based on a colour sorting mechanism. This is made of three joints, a gripper, two rectangular shaped links, a rotary table and a rectangular platform. The angular rotation of each joint is powered by a servo-motor. Although, there are several studies found in open literature that consider the kinematic analysis of robotic arms. (Madiha et al., 2018).

This research focuses on inputting angles in homogenous transformation matrix derived equations of a spherical robotic arm manipulator to simulate lethal and non-lethal angles of an object in a known position.

2.Methodology

The homogeneous transformation matrix is used to solve for the joint angles of the spherical manipulator handling an object at a known position in the end effector frame (x3, y3, z3) as shown in figure 1 below.

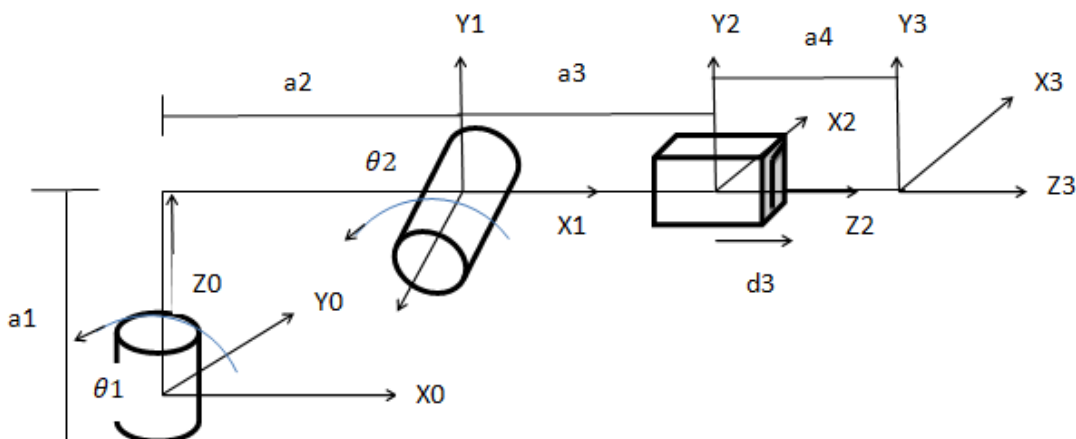


Figure 1: Spherical Manipulator

Figure 1 shows that the spherical manipulator has four frames. The first frame (base) has coordinates x0, y0 and z0. The angle of the rotation of the spherical joint is given by θ_1 . The base frame is rotated along the z0 axis. The height of the frame from the ground to the joint angle is given by a1. The second frame has coordinates x1, y1 and z1 with an angle of the rotation of the spherical joint given by θ_2 which is rotated along the z1 axis. The distance between the first frame and second frame is computed using sine and cosine trigonometry ratios. The rotation of frame one with respect to frame zero is computed using a rotation matrix. The third frame has coordinates x2, y2 and z2. The frame has a prismatic joint, that does not rotate, rather it moves out or in, in the z2 axis. The distance from the third frame to the second frame is also computed using trigonometry ratios and then inputted to the homogenous transformation matrix. The fourth frame (end-effector) has coordinates x3, y3 and z3. There is no rotation between the fourth frame and the third frame because all the coordinates align. So, the rotation matrix between the two frames is an identity matrix, which depicts no rotation. The distance between frame three and four is given by the sum of d3 and a4.

The homogenous transformation matrix for the robotic arm in figure 1 is given by:

$$H_n^0 = \begin{bmatrix} R_n^0 & d_n^0 \\ 000 & 1 \end{bmatrix}$$

Where,

- H_n^0 = homogenous transformation matrix from base frame 0 to the end effector frame n
- R_n^0 = rotation from base frame to end effector frame n
- d_n^0 = displacement from the base frame to the end effector frame n
- n = 3

The three zeros under the rotation matrix and one (1) under the displacement matrix (d_n^0) are added so as to obtain a square homogenous transformation matrix (H_n^0).

The rotation of the joint angles of the spherical manipulator are defined in the x, y and z plane as follows:

The rotation along the x axis is given as,

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

The rotation along the y axis is given as,

$$R_y = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

The rotation along the z axis is given as,

$$R_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While the rotation of each frame with respect to the previous frame is given by the matrices below:

Rotation of frame one (1) relative to frame zero (0),

$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Rotation of frame two (2) relative to frame one (1),

$$R_2^1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

Rotation of frame three (3) relative to frame two (two) is an identity matrix which means no rotation.

$$R_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Each of the rotation of each frame relative to the previous frame is then multiplied with the corresponding axis of rotation in x, y and z axis. The homogenous transformation matrix is now computed by performing matrix multiplication as shown:

$$H_1^0 = R_z * R_1^0$$

$$H_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & a_2\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & a_2\sin\theta_1 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = R_z * R_2^1$$

$$H_2^1 = \begin{bmatrix} 0 & -\sin\theta_2 & \cos\theta_2 & a_3\cos\theta_2 \\ 0 & \cos\theta_2 & \sin\theta_2 & a_3\sin\theta_2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_4 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation matrix is computed by multiplying all the homogenous matrices from all the frames.

$$H_3^0 = H_1^0 * H_2^1 * H_3^2$$

$$\begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & a_2\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & a_2\sin\theta_1 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -\sin\theta_2 & \cos\theta_2 & a_3\cos\theta_2 \\ 0 & \cos\theta_2 & \sin\theta_2 & a_3\sin\theta_2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_4 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The given homogenous transformation for figure 1 is given as;

$$H_3^0 = \begin{bmatrix} -\sin\theta_1 & -\sin\theta_2\cos\theta_1 & \cos\theta_2\cos\theta_1 & (a_4 + d_3)(\cos\theta_2\cos\theta_1) + a_3\cos\theta_2\cos\theta_1 + a_2\cos\theta_1 \\ \cos\theta_1 & -\sin\theta_2\sin\theta_1 & \cos\theta_2\sin\theta_1 & (a_4 + d_3)(\cos\theta_2\sin\theta_1) + a_3\cos\theta_2\sin\theta_1 + a_2\sin\theta_1 \\ 0 & 0 & 0 & \sin\theta_1 (a_4 + d_3)\sin\theta_2 + a_3\sin\theta_2 + a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The variations of the angles, and inputted values for a1, a2 and d3 in the homogenous transformation matrix equation is simulated in an excel application, the corresponding positions of the end effector with respect to the position

vector (object) is determine. This will help the robot programmer to program the spherical manipulator effectively by being able to identify angles that will harm both the object and the end effector.

3.Result and Discussion

The result for the homogenous transformation matrix on the spherical manipulator is discussed accordingly.

Table 1 shows result for theta1=90 and theta2 = 45

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		theta1	90	1.57079633	
a2	3		theta2	45	0.78539816	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	-4.33155E-17	4.33E-17	5.30297E-16		X-POSITION	5.30297E-16
6.12574E-17	-0.707106781	0.707107	8.656854249		Y-POSITION	8.656854249
0	0	1	10.65685425		Z-POSITION	10.65685425
0	0	0	1			

From the table 1 the values for the joint lengths and the value d3 for the prismatic joint are clearly represented and

the corresponding joint angles theta1 and theta2 is also represented. The values of theta1 and theta2 are converted

from degree to radian because the excel application works with radians. The result of the parameters on the homogenous transformation matrix and the displacement of the object with respect to base frame is computed by the application and represented. The x-position, y-position and z-position are positions use to represent a point in the end effector handling the object. From the results the end effector is 10.65685425 inches above the ground if theta1 is given as 90 degrees and theta2 is given as 45 degrees. The y-position which is the height of the object is

8.656854249 inches. So the end effector will not smash the ground or harm the object. The end-effector can handle the object at a point since the end-effector is 10.65685425 inches above the ground and the y-position is 8.656854249 inches also above the ground. The position vector which describes the orientation of the object is accurately calculated as the displacement of the object from the end-effector frame using the homogenous transformation matrix.

Table 2 shows result for theta1=90 and theta2 = 20

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		theta1	90	1.57079633	
a2	3		theta2	20	0.34906585	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	-2.09513E-17	5.76E-17	6.44277E-16		X-POSITION	6.44277E-16
6.12574E-17	-0.342020143	0.939693	10.51754097		Y-POSITION	10.51754097
0	0	1	7.736161147		Z-POSITION	7.736161147
0	0	0	1			

In Table 2 the value of theta2 is reduced by 15 degrees and the value of the end-effector from the ground is reduced significantly. The value of the end-effector above the

ground from table 1.2 is 7.736161147 inches. Compare to Table 1.1 which is 10.65685425 inches.

Table 3 shows result for theta1=90 and theta2 = 10

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		theta1	90	1.57079633	
a2	3		theta2	10	0.17453293	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	-1.06372E-17	6.03E-17	6.66387E-16		X-POSITION	6.66387E-16
6.12574E-17	-0.173648178	0.984808	10.87846202		Y-POSITION	10.87846202
0	0	1	6.389185421		Z-POSITION	6.389185421
0	0	0	1			

In Table 3 the value of theta2 is reduced by 10 degrees and the value of the end-effector from the ground also reduced significantly. The value of the end-effector above the

ground from table 1.3 is now 6.389185421 inches. Compare to Table 1.2 which is 7.736161147 inches.

Table 4 shows result for theta1=90 and theta2 = 0

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		theta1	90	1.57079633	
a2	3		theta2	0	0	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	0	6.13E-17	6.73832E-16		X-POSITION	6.73832E-16
6.12574E-17	0	1	11		Y-POSITION	11
0	0	1	5		Z-POSITION	5
0	0	0	1			

In Table 4 the value of theta2 is reduced by a 10 degrees and the value of the end-effector from the ground also reduced significantly. The value of the end-effector above

the ground from table 4 is now 5 inches. Compare to Table 3 which is 6.389185421 inches.

Table 5 shows result for theta1=90 and theta2 = -10

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		theta1	90	1.57079633	
a2	3		theta2	-10	-0.17453293	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	1.06372E-17	6.03E-17	6.66387E-16		X-POSITION	6.66387E-16
6.12574E-17	0.173648178	0.984808	10.87846202		Y-POSITION	10.87846202
0	0	1	3.610814579		Z-POSITION	3.610814579
0	0	0	1			

In Table 5 the value of theta2 is reduced by 10 degrees and the value of the end-effector from the ground also reduced significantly. The value of the end-effector above the

ground from table 1.5 is now 3.610814579 inches. Compare to Table 1.4 which is 5 inches.

Table 6 shows result for theta1=90 and theta2 = -20

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		theta1	90	1.57079633	
a2	3		theta2	-20	-0.34906585	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	2.09513E-17	5.76E-17	6.44277E-16		X-POSITION	6.44277E-16
6.12574E-17	0.342020143	0.939693	10.51754097		Y-POSITION	10.51754097
0	0	1	2.263838853		Z-POSITION	2.263838853
0	0	0	1			

In Table.6 the value of theta2 is reduced by a 10 degrees and the value of the end-effector from the ground also reduced significantly. The value of the end-effector above

the ground from table 6 is now 2.263838853 inches. Compare to Table 5 which is 3.610814579 inches.

Table 7 shows result for theta1=90 and theta2 = -30

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		theta1	90	1.57079633	
a2	3		theta2	-30	-0.52359878	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	3.06287E-17	5.31E-17	6.08176E-16		X-POSITION	6.08176E-16
6.12574E-17	0.5	0.866025	9.92820323		Y-POSITION	9.92820323
0	0	1	1		Z-POSITION	1
0	0	0	1			

In Table 7 the value of theta2 is reduced by a 10 degrees and the value of the end-effector from the ground also reduced significantly. The value of the end-effector above

the ground from table 7 is now 1 inch. Compare to Table 6 which is 2.263838853 inches.

Table 8 shows result for $\theta_1=90$ and $\theta_2 = -40$

INPUT PARAMETERS FOR SPHERICAL MANIPULATOR						
JOINT LENGTHS	VALUES		ANGLES	DEGREES	RADIANS	
a1	5		θ_1	90	1.57079633	
a2	3		θ_2	-40	-0.6981317	
a3	3					
a4	3					
d3	2					
HOMOGENOUS TRANSFORMATION MATRIX IMPLEMENTATION						
-1	3.93755E-17	4.69E-17	5.5918E-16		X-POSITION	5.5918E-16
6.12574E-17	0.64278761	0.766044	9.128355545		Y-POSITION	9.128355545
0	0	1	-0.14230088		Z-POSITION	-0.142300877
0	0	0	1			

In Table 8 the value of θ_2 is reduced by a 10 degrees and the value of the end-effector from the ground also reduced significantly. The end-effector hits the ground. From table 8 the end-effector is now -0.142300877 inches below the ground. Compare to Table 7 which is 1inch.

When the end-effector hits the ground, this can cause significant damage to the end-effector and the object that the robot might be holding. This will assist the robotic engineer to be able to identify the angles which might cause damage to the end-effector and avoid coding instructions for robots using such joint angles.

4. Conclusion

The homogenous transformation matrix approach is an efficient and easy method to describe the solution of the rotation and orientation of any degree of freedom robotic arm.

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