International Journal of Scientific Engineering and Research (IJSER)
ISSN (Online): 2347-3878
Impact Factor (2020): 6.733

Effect of Industrialization and Greenhouse Gases on Human Population: A Modeling Study

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Abstract: ***The purpose of this paper is to describe the impact of greenhouse gasses and Industrialization on human population. In this Model, we have used the system of non-linear differential equations to formulate the problem and we have considered four variables namely; susceptible population, Infected population, industrialization and the greenhouse gasses. We can find the criteria for the local stability of a system. Furthermore, we have also illustrated the results numerically.***

Keywords: Mathematical Model, Equilibria and Susceptible and Infected populations, Industrialization and Greenhouse gases

1. Introduction

This paper investigates the intricate dynamics between greenhouse gases, industrialization, and their collective impact on human populations. Employing a modelling approach, non-linear differential equations form the basis of our analysis, incorporating key variables such as susceptible population, infected population, industrialization, and greenhouse gas levels. The utilization of this system allows us to explore the interplay of these factors and derive criteria for local stability within the context of the human-environment interaction

Through numerical illustrations, this study provides a quantitative lens to comprehend the evolving patterns and consequences of the complex relationships examined. The integration of mathematical modelling not only enhances our understanding of the interconnectedness of these variables but also offers a platform for forecasting potential scenarios. By elucidating the quantitative implications of greenhouse gas emissions and industrialization on susceptible and infected populations, this research contributes valuable insights to the ongoing discourse on sustainable development and human well-being. Industrialization and urbanization are two important factors for our environment. Basically, urbanization is the process by which large number of people become permanently concentrated in relatively small areas forming cities. When large number of people congregate in cities, many problems occur. The main problem that our society is faces today is the depletion of forestry resources. It is well known that forestry resources play an important role in our life. But it being depleted by industrialization due to urbanization [8],[15]. Rapid urbanization leads to the exploitation of the natural resources for the construction industries, transport and consumption. Urbanization is the chief agent of increasing the industrialization [14],[16],[17]. Although industrialization is necessary for the growth of our country but it also have many negative impacts. Disease accounted for many deaths in industrial cities during industrial revolution. With a chronic lack of hygiene, little knowledge of sanitary, disease such as cholera, typhoid, typhes etc. occurs [18]. Many researchers has been developed the nonlinear mathematical model to investigate the effect of industrialization due to urbanization. A. K. Misra et.al. (2014) proposed a mathematical model for the depletion of forestry resources due to population pressure augmented industrialization. In this paper, they found that as the population pressure increase, the level of the industrialization increases leading to decrease in the umulative biomass density of forestry biomass. They also found that if the population pressure is controlled by using some economic efforts, the decrease in cumulative biomass density of forestry resources can be made much less than the case when no control is applied. A.K. Misra et.al. (2013) also have taken a mathematical model to see the effects of population and population pressure on the forest resources and their conservation. From the model analysis they shows that as the density of population or population pressure increases, the cumulative density of forestry resources decreases and the resource may become extinct, if the population pressure become too large. They also noted that by controlling the population pressure using some economic incentives, the density of forest resources can be maintained at an equlibrium level, which is population density dependent. B.Dubey et.al.(2010) studied the effects of industrialization, population and pollution on a renewable resources. They obtained that if the densities of industrialization, population and pollution increase, then the density of the resource biomass decreases and it is settles down at the equilibrium level whose magnitude is lower than its original carrying capacity. J.B. Shukla et.al. (1997) proposed a mathematical model for the depletion and conservation of forestry resources. They conclude that the resource biomass can be maintained at a desired level by conserving the forestry resources and by controlling the growth of population and the emission rate of pollutant in the habitat. Abhinav Tandon et. al. (2016) have proposed a mathematical model to investigate the effects of environmental pollution intensified by urbanization. They shown that the growth of population is responsible for the growing urbanization, but for very large increase of urbanization the population may not survive in the long run. Manju Agarwal et. al. (2009) presented a mathematical model for the depletion of forestry resources biomass due to industrialization pressure. In this paper, a model for interaction between forestry biomass, wild life population and industrialization pressure is proposed and the effects of forestry biomass depletion in a forested habitat caused by industrialization pressure on the survival of the forestry biomass dependent wilidlife speciess is studied. Niharika Verma et.al. (2017) have taken a mathematical model for the increase in global warming due to CO2 intensified by industrialization. In their analysis they shown that the reforestation is a key factor for controlling the global warming into environment. Niharika Verma et.al.(2018) also have proposed a mathematical model to study the effect of environmental tax for CO2 on the forestry biomass depleted by industrialization. O.P.Misra et. al. (2012) studied the modeling and analysis of a single species population with viral infection in polluted environment. In their paper, they have shown that when the effect of pollution is not considered then the susceptible population never vanish and on the other hand if the effect of environmental pollution has been considering the susceptible population can vanish.The incidence rate as well as the treatment rate has been recognized as a valuable tool in controlling or decreasing the spread of diseases such as measles, flu and tuberculosis (see [19, 20, 24]). The incidence rate is defined as the number of infected individuals per unit time. Several authors such as [21, 22, 23] have discussed the bilinear incidence rate i.e., αS I, where α is infection rate, S and I are susceptible and infected population, respectively. But this bilinear incidence rate is not realistic because if the number of susceptible populations increases, the incidence rate increases, i.e., the number of infected individuals per unit time increases. So, there is a need to modify this unreasonable bilinear incidence. rate Due to this non-realistic situation in bilinear incidence rate, several researchers have suggested different types of nonlinear incidence.rate. Anderson and May [2] and May and Anderson [41] introduced saturated incidence rate $\frac{\alpha SI}{1+\beta S}$, where β is saturation factor which controls the epidemiological spread of infection. Furthermore, several authors comprised to study this Beddington-DeAngelis-type incidence rate [31, 32, 33]. [26] introduced a model using Crowley–Martin and Holling type III responses to describe the epidemiological situation. Moreover, various epidemiological models have been studied using different treatment rates [27,28,29,30]. It might be noticed that different epidemic models with different removal rates have been studied.

2. Mathematical Model

Industrialization is the cause of increase in bacterial population and when bacterial population increases, human population suffered from many diseases.In view of above, the model governing by the following ordinary differential equations as:

$$\frac{dS}{dt} = A - \beta SI - \lambda SG - \lambda_1 S + vT \qquad (2.1)$$

$$\frac{dI}{dt} = A - \beta SI - \lambda SG - vT - \lambda_2 I \qquad (2.2)$$

$$\frac{dI_n}{dt} = rI_n - \frac{r_0 I_n^2}{K} + \beta_1 N \qquad (2.3)$$

$$\frac{dG}{dt} = G_0 - \mu_0 G + \beta I_n \qquad (2.4)$$

with the initial conditions and $S(t) \geq 0$, $I(t) \geq 0$, $I_n(t) \geq 0$, $G(t) \geq 0$.

In the model, system given by equation (2.1) to (2.4), S(t) and I(t) are presenting the densities of susceptible and infected population at time t. At time t, I_n and G are presenting the industrialization Greenhouse gasses and N(t) is the total population at time t. In the model, constants are given as;

A= constant immigration rate of human population from outside the region under the consideration.

β= it is presenting the rate of infection from susceptible population.

λ= it is the rate at which susceptible population decreases due to bacteria.

λ_1= natural death rate of susceptible population.

ν= increase in susceptible population through treatment.

λ_2 = natural death rate of infected population.

β_1= Increase in industrialization due to total population (susceptible and infected population).

β_0= it is the rate at whichindustrialization is controlled due to some government agencies.

α=natural death rate of bacterial.

3. Boundedness of the System

Lemma 3.1 The solution of the system given by equation (2.1) to (2.4) is bounded within the following region:

$$w = \left\{(S, I, I_n, G): 0 < N \leq \frac{A}{\lambda_1}, 0 < I_n \leq \frac{R}{\delta},\ \ 0 < G \leq \frac{G_0\delta+\beta R}{\mu\delta}\right\}$$

Proof: From the equation (2.1) & (2.2)

$$\frac{dN}{dt} \leq A - \lambda_1 S - \lambda_2 I$$

$$\leq A - \lambda_1(S + I)$$

$$\frac{dN}{dt} \leq A - \lambda_1 N$$

by Comparision Theorem as $t \to \infty$

$N_{max} = \frac{A}{\lambda_1}$, Provided$\lambda_1 = \lambda_2$

From the equation (2.3)

$$\frac{dI_n}{dt} \leq rI_n - \frac{r_0 I_n^2}{k} + \beta_1 N_{max}.$$

$$\leq rI_n - \frac{r_0 I_n^2}{k} + \beta_1.\frac{A}{\lambda_1}$$

$$\frac{dI_n}{dt} + \delta I_n \leq \frac{K.r^2}{2r_0} - \frac{K^2r^2}{4r_0} + \frac{\beta A}{\lambda_1}.$$

where δ is a very small quantity.

by ***Comparision Theorem*** as $t \to \infty$

$$(I_n)_{max} = \frac{R}{\delta}$$

where $R = \frac{K.r^2}{2r_0} - \frac{K^2r^2}{4r_0} + \frac{\beta A}{\lambda_1}$.

$$\frac{dG}{dt} \leq G_0 - \mu G + \beta (I_n)_{max}$$

$$\frac{dG}{dt} \leq G_0 - \mu G + \beta\frac{R}{\delta}$$

by ***Comparision Theorem*** as $t \to \infty$

$$G_{max} = \frac{G_0\delta+\beta R}{\delta\mu}$$

This completes the proof of lemma (3.1)

4. Existence of the Equilibrium points

After analysis of the model, we found the system has only one-non-negative equilibrium point E (S^*, I^*, I_n^* G^*).

The value of S^*, I^*, I_n^* G^* is given by

$$A - \beta S^*I^* - \lambda S^*G^* - \lambda_1 S^* + vT = 0 \qquad (4.1)$$

$$\beta S^*I^* + \lambda S^*G^* - vT - \lambda_2 I^* = 0 \qquad (4.2)$$

$$rI_n^* - \frac{r_0 I_n^2}{K} + \beta_1(S^* + I^*) = 0 \quad (4.3)$$

$$G_0 - \mu_0 G^* + \beta I_n^* = 0 \quad (4.4)$$

By adding equation (4.1) & (4.2),

$-\lambda_1 S^* - \lambda_2 I^* + A = 0$

$A = \lambda_1(S^* + I^*)$

From the equation (4.3)

$rI_n^* - \frac{r_0 I_n^{*2}}{K} + \beta_1 N = 0$

$rI_n^* - \frac{r_0 I_n^{*2}}{K} + \beta_1(S^* + I^*) = 0$

$rI_n^* - \frac{r_0 I_n^{*2}}{K} + \beta_1\frac{A}{\lambda_1} = 0$

$\frac{r_0}{K} I_n^{*2} - rI_n^* - \frac{\beta_1 A}{\lambda_1} = 0$

$I_n^* = \frac{(r+P)K}{2r_0}$

Where $P = \sqrt{r^2 + \frac{4\beta_1 r_0 A}{K\lambda_1}}$

From the equation (4.4)

$G_0 - \mu_0 G^* + \beta I_n^* = 0$

$G_0 - \mu_0 G^* + \beta\frac{(r+P)K}{2r_0} = 0$

$\mu_0 G^* = G_0 + \frac{\beta K(r+P)}{2r_0}$

$\mu_0 G^* = \frac{2r_0 G_0 + \beta K(r+P)}{2r_0}$

$G^* = \frac{2G_0 r_0 + \beta K(r+P)}{2r_0\mu_0}$

$G^* = \frac{1}{\mu_0}\left(G_0 + \frac{\beta K(r+P)}{2r_0}\right)$

Now from equation (4.2)

$$\frac{\beta S^*(A - \lambda_1 S^*)}{\lambda_1} + \frac{\lambda S^*}{\mu_0}\frac{\beta K(r+P)}{2r_0} - vT - \lambda_2\left(\frac{A - \lambda_1 S^*}{\lambda_1}\right) = 0$$

$S^* = \frac{D_1 + \sqrt{D}}{2\lambda_1\beta} > 0$

Provided; $D > 0$

where $D_1 = \frac{\lambda\lambda_1}{\mu_0}\left(G_0 + \frac{K\beta(r+P)}{2r_0}\right) + \beta A + \lambda_1$

& $D = \left\{\frac{\lambda\lambda_1}{\mu_0}\left(G_0 + \frac{K\beta(r+P)}{2r_0}\right) + \beta A + \lambda_1\right\}^2 - 4\lambda_1\beta(\lambda_1 vT\lambda_2 A)$

Now from equation (4.1)

$A - \beta\left(\frac{D_1+\sqrt{D}}{2\lambda_1\beta}\right)I^* - \lambda\left(\frac{D_1+\sqrt{D}}{2\lambda_1\beta}\right)\frac{1}{\mu_0}\left(G_0 + \frac{K\beta(r+P)}{2r_0}\right) - \lambda_1\left(\frac{D_1+\sqrt{D}}{2\lambda_1\beta}\right) + vT = 0$

$I^* = \frac{2\lambda_1(A+vT)}{(D_1+\sqrt{D})} - \left[\frac{\lambda_1}{\mu_0\beta}\left(G_0 + \frac{K\beta(r+P)}{2r_0}\right) + \frac{\lambda_1}{\beta}\right] > 0$

Provided; $\frac{2\lambda_1(A+vT)}{(D_1+\sqrt{D})} > \frac{\lambda_1}{\mu_0\beta}\left(G_0 + \frac{K\beta(r+P)}{2r_0}\right) + \frac{\lambda_1}{\beta}$

Theorem 5.1

The interior equilibrium point is E (S^*, I^*, I_n^* G^*) non-linearly asymptotically stable within the region of attraction given by R, provided following inequality is satisfied

$$K_{30} < \frac{4}{9} K_{20}\frac{\mu_0}{\beta^2}\left(-r + \frac{2r_0}{K}I_n^*\right)$$

Where $K_{30} = \max\left\{\frac{3}{2}\frac{K_1(\lambda S^*)^2}{\mu_0(-\beta S^* + \lambda_2)}, \frac{3}{2}\frac{(-\lambda S^*)^2}{\mu_0(\beta I^* + \lambda G^* + \lambda_1)}\right\}$

$K_{20} = \min\{\frac{2}{3}\frac{(-r + \frac{2r_0}{k}I_n^*)(\beta I^* + \lambda G^* + \lambda_1)}{\beta^2}, \frac{2}{3}\frac{K_1(-\beta s^* + \lambda_1)(-r + \frac{2r_0 I_n^*}{K})}{\beta^2}\}$

Where $K_1 = \frac{\beta S^*}{\beta I^* + \lambda G^*}$

Proof:

Let us consider S^*, I^*, I_n^* and G^* are the small perturbation around the equilibrium point E (S^*, I^*, I_n^* G^*)

So, we first linearize the model by assuming $S = S^* + S_1$, $I = I^* + I_1$, $I_n = I_n^* + I_{n_1}$, $G = G^* + G_1$, After linearization The model is given by

$\frac{dS_1}{dt} = -\beta S^* I_1 - \beta S_1 I^* - \lambda S^* G_1 - \lambda S_1 G^* - \lambda_1 S_1$

$\frac{dI_1}{dt} = \beta S^* I_1 + \beta S_1 I^* + \lambda S^* G_1 + \lambda S_1 G^* - \lambda_2 I$

$\frac{dI_n}{dt} = rI_{n_1} + \frac{2r_0}{K}I_n^* I_{n_1} + \beta_1(S_1 + I_1)$

$\frac{dG}{dt} = -\mu_0 G_1 + \beta I_{n_1}$

Let us consider a positive definite function

$$V = \frac{1}{2}S_1^2 + \frac{1}{2}K_1 I_1^2 + \frac{1}{2}K^2 I_{n_1}^2 + \frac{1}{2}K_3 G_1^2$$

Where K_1, K_2, K_3 are Positive Constants taken to be suitably. After differentiating Vwith respect to t we get

$$\frac{dv}{dt} = S_1\frac{dS_1}{dt} + K_1 I_1\frac{dI_1}{dt} + K_2 I_{n_1}\frac{dI_{n_1}}{dt} + K_3 G_1\frac{dG_1}{dt}$$

$$\begin{aligned}\frac{dv}{dt} = & -S_1^2(\beta I^* + \lambda G^* + \lambda_1) - I_1^2(-K_1\beta S^* + k_1\lambda_2) \\ & - I_{n_1}^2\left(-r K_2 + \frac{2r_0}{K}K_2 I_n^*\right) - G_1^2(K_3\mu_0) \\ & + S_1 I_1(-\beta S^* + K_1\beta I^* + K_1\lambda G^*) \\ & + S_1 G_1(-\lambda S^*) + G_1 I_1(K_1\lambda S^*) \\ & + S_1 I_{n_1}(K_2\beta_1) + I_1 I_{n_1}(K_2\beta_1) \\ & + I_{n_1} G_1(K_3\beta)\end{aligned}$$

now choosing $K_1 = \frac{\beta S^*}{\beta I^* + \lambda G^*}$, we found that $\frac{dv}{dt}$ will be negative if

$$a_{14}^2 < \frac{2}{3}a_{11}a_{44} \quad (5.1)$$

$$a_{13}^2 < \frac{2}{3}a_{11}a_{33} \quad (5.2)$$

$$a_{23}^2 < \frac{2}{3}a_{22}a_{44} \quad (5.3)$$

$$a_{34}^2 < \frac{4}{9}a_{33}a_{44} \quad (5.4)$$

$$a_{24}^2 < \frac{2}{3}a_{22}a_{44} \quad (5.5)$$

Where $a_{11} = \beta I^* + \lambda G^* + \lambda_1$

$a_{14} = (-\lambda S^*)$

$a_{44} = (-\lambda S^*)$

$a_{13} = (K_3\mu_0)$

$a_{13} = K_2\beta$

$a_{33} = (-rK_2 + \frac{2r_0}{K}K_2 I_n^*)$

$a_{23} = (K_2\beta)$

$a_{22} = (-K\beta S^* + K_1\lambda_2)$

$a_{34} = (K_3\beta)$

$a_{24} = (K_1\lambda S^*)$

After combining the inequalties we get the condition for the local stability:

$$K_{30} < \frac{4}{9} K_{20}\frac{\mu_0}{\beta^2}\left(-r + \frac{2r_0}{K}I_n^*\right)$$

From the equations (5.2) and (5.3)

$K_{20} = \min\{\frac{2}{3}\frac{(-r + \frac{2r_0}{k}I_n^*)(\beta I^* + \lambda G^* + \lambda_1)}{\beta^2}, \frac{2}{3}\frac{K_1(-\beta s^* + \lambda_1)(-r + \frac{2r_0 I_n^*}{K})}{\beta^2}\}$

Where $K_1 = \frac{\beta S^*}{\beta I^* + \lambda G^*}$

From the equations (5.1) and (5.5)

And K_{30}= max $\left\{\frac{3}{2}\frac{K_1(\lambda S^*)^2}{\mu_0(-\beta S^*+\lambda_2)}, \frac{3}{2}\frac{(-\lambda S^*)^2}{\mu_0(\beta I^*+\lambda G^*+\lambda_1)}\right\}$

Hence theorem is proved.

5. Numerical Simulation

In this section, we introduce numerical simulation to explain the applicability of the result discussed. above.

We choose the following set of parameters in the model given by equation (2.1) to (2.4)

$A = 0.001$, $\beta = 0.01$, $\lambda = 0.003$, $\lambda_2 = 0.002$

$\lambda_1 = 0.002$, $v = 0.001$, $T = 0.02$, $\beta_1 = 0.2$

$\beta = 0.02$, $r_0 = 1$, $r = 1$, $\mu_0 = 0.1$, $G_0 = 0.8$

For this value of the parameters the value of interior equilibrium point E is given by

$S^* = 0.1004$, $I^* = 0.099$, $I_n^* = 0.2033$, $G^* = 0.507722$

6. Conclusion

A non-linear mathematical model is developed to see the effect of greenhouse gasses on human population generated by Industrialization. Analysis of model reveals that the obtained model system exhibits only one non-negative equilibrium points.

E (S^*, I^*, I_n^*, G^*) and it is given by

$S = 0.02$, $I = 0.0999$, $I_n = 0.2033$, $G = 0.507722$.

The condition for the stability of equilibrium points is obtained using the Stability Theory of differential Equations. Numerical analysis has been done to illustrated the feasibility of the obtained, results.

The result of model, qualitatively and numerically Show that growth of Industrialization is responsible for growing greenhouses gasses.

If the growth is Continues, the human population is not survived in the long run. The Industrialization is responsible for increase of Green House gasses and increase in green Rouse gases is responsible for many diseases due to which human population.

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