Control Chart using Area Biased Quasi Aradhana Distribution with Application

S. Muthusamy[1], D. Venkatesan[2]

Department of statistics, Annamalai University, India
E-mail: [1]*muthustat1997[at]gmail.com*,
[2]*sit_san[at]hotmail.com*

Abstract: **A** ***new model of Quasi Aradhana distribution known as Area biased Quasi Aradhana distribution is introduced in this paper. The proposed distribution's structural attributes, such as Moment Generating Function and Characteristic Function, mean and variance, have been presented and examined. Lastly, an application to a real-world data set is provided to demonstrate the superiority of a newly developed model.***

Keywords: Statistical Quality control, Control charts, Quasi Aradhana distribution

1. Introduction

One of the most important applications of statistical techniques in industry is statistical quality control. These techniques, which are based on probability theory and sampling, are widely employed in practically all industries, including weapons, aviation, automobiles, textiles, electrical equipment, plastic, rubber, electronics, chemicals, petroleum, transportation, medical, and so on. There is no industrial field where statistical quality control is not applied. The key word in the phrase "Statistical Quality Control" is quality. By quality, we mean a feature of a product that indicates its suitability for use. The range of these properties is quite broad-physical, chemical, artistic, and so on.A product may have numerous quality features as well as an overall quality that is greater than the sum of its distinct quality aspects - a property called technically as synergy. Quality refers to a product's level or standard, which is determined by numerous other elements such as materials, personnel, machines, and management. Quality control is a potent productivity strategy for identifying and correcting flaws in materials, processes, machines, or end products. It is critical that the end products have the attributes that consumers expect, because the success of the sector is dependent on successful product marketing. Quality control ensures this by insisting on quality criteria throughout the process, from the arrival of materials through the final delivery of items. Section 2 assesses the relevant literature. Section 3 describes the Area Biased Quasi Aradhana distribution and measurements. Section 4 discusses the statistical features of the moment generating function and the characteristic function. Section 5 discusses the limitations of control. The section explains the procedure of identifying out of control using actual data. Section 6 contains a summary.

2. Review of Literature

The concept of weighted distribution is used in fields such as quality and reliability to construct appropriate statistical models. Weighted distributions have established a new standard for effective statistical data modelling and prediction when standard distributions are inadequate. Weighted distributions provide a way for fitting the model to the unknown weight function when samples are obtained from both the original and produced distributions. Fisher (1934) was the first to propose weighted distribution theory as a means to model ascertainment bias. Rao (1965) proposed and formulated a unified approach of applying standard distributions when the previous method was shown to be unsuccessful.

In this case, weighted distributions were developed in order to record the observation based on some weight function. Weighted distributions give an adequate way for dealing with model building and data interpretation. Weighted distributions are used to pick applicable models for observed data, especially when samples are created without a suitable frame. When samples from both the original and constructed distributions are available, weighted distributions provide a way for fitting models to the undetermined weight function. Weighted distributions are utilized in a range of research domains, including dependability, biomedicine, ecology, family data analysis, meta-analysis, intervention data analysis, and others, to develop acceptable statistical models. Weighted distributions are significantly used in the study and modelling of lifetime data in numerous applications, including engineering, medical, behavioral sciences, finance, and insurance. The weighted distributions influence the likelihood of events as observed and recorded. Fisher (1934) proposed weighted distributions to describe ascertainment bias. Patil and Rao (1978) defined size biased sampling and weighted distributions by identifying specific scenarios in which the underlying model preserves its shape. When the weight function solely considers the length of the units of interest, the weighted distribution becomes length biased. Within the framework of the renewal theory, Cox (1962) first proposed the concept of a length biased distribution. A distribution is said to be size biased when the sampling method selects units with a probability proportional to the unit size. A specific example of size biased distributions is weighted distributions, a more broad variant. Ganaie and Rajagopalan investigated the length biased weighted quasi gamma distribution with properties and applications (2020). Oluwafemi and Olalekan (2017) proposed a length and area biased Exponentiated Weibull distribution based on forest inventories. Perveen (2016) investigated the Area biased weighted Weibull distribution in terms of applicability. Osowole et al. (2020) demonstrated an Area biased Quasi-Transmuted uniform distribution. Ahmed et al. investigated size-biased first-kind generalized beta distribution (2013). Fazal (2018) investigated the area-biased poisson exponential distribution with applications. Ade et al., 2021, recently presented the characterization and calculation of the Area biased Quasi-Akash distribution.

3. Description of the distribution

Definition: The probability density function of Quasi Aradhana distribution is given by

$$f(x,\theta,\alpha)=\frac{\theta}{\alpha^2+2\theta+2}\left(\alpha+\theta x^2\right)e^{-\theta x};\quad x>0,\theta>0,\alpha^2+2\theta+2>0 \quad \textbf{(1)}$$

The density function given in (1) is a combination of exponential and Quasi Aradhana distribution with parameter θ respectively.

The cumulative distribution function (c.d.f) Quasi Aradhana distribution can be given as:

$$F(x;\theta;\alpha)=1-\left[1+\frac{\theta x(\theta x+2\theta+2\alpha)}{\alpha^2+2\alpha+2}\right]e^{-\theta x};\qquad x,\theta>0,\alpha^2+2\alpha+2>0 \quad \textbf{(2)}$$

Let X be a random variable following non-negative condition with probability density function $f(x)$ .Let $w(x)$ be the weight function which is a non-negative function, and then the probability density function of the weighted random variable X_w is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))},\quad x>0.$$

Where *w(x)* be a non-negative weight function and

$$E(W(X))=\int w(x)f(x)dx<\infty.$$

Depending upon the choice of the weight function *w(x)*, we have different models. Clearly when *w(x) = x*, the resulting distribution is called length biased or size biased. In this paper, we have to obtain the area biased version of Quasi Aradhana distribution, so consequently the weight function at $w(x) = x^2$ to obtain the area biased Quasi Aradhana model.

Definition 2: The probability density function of area biased distribution is given by

$$f_a(x)=\frac{x^2 f(x)}{E(x^2)} \quad \textbf{(3)}$$

Where $E(x^2)=\int_0^\infty x^2 f(x)dx$

$$=\int_0^\infty x^2\frac{\theta}{\alpha^2+2\alpha+2}(\alpha+\theta x)^2 e^{-\theta x}dx$$

$$E(x^2)=\frac{2\alpha^2+24+12\alpha}{\theta^2(\alpha^2+2\alpha+2)} \quad \textbf{(4)}$$

Substitute equation (1) and (4) in equation (3), we will obtain probability density function of Area biased Quasi Aradhana distribution as

$$f_a(x)=\frac{\dfrac{\theta^3}{\alpha^2+2\alpha+2}\left(\alpha+x+x^2\right)e^{-\theta x}}{\dfrac{2\alpha^2+24+12\alpha}{\theta^2\left(\alpha^2+2\alpha+2\right)}}$$

$$f_a(x)=\frac{x^2\theta^3}{2\alpha^2+12\alpha+24}(\alpha+\theta x)^2 e^{-\theta x}dx \quad \textbf{(5)}$$

and the cumulative distribution function of area biased Area Biased Quasi Aradhana distribution can be obtained as

$$F_a(x)=\int_0^x f_a(x)dx$$

After the simplification of equation (6), we will obtain the cumulative distribution function of area biased distribution as

$$F_a(x)=\frac{1}{2\alpha^2+12\alpha+24}\left[\alpha^2\gamma(3,\theta x)+\gamma(5,\theta x)+2\alpha\gamma(4,\theta x)\right] \quad \textbf{(6)}$$

4. Statistical Properties

In this section, we will discuss the various statistical properties of area biased Quasi Aradhana Distribution including its moments, Moment Generating function (MGF) and Characteristic Function

4.1 Moment Generating Function

Let X be a random variable following area biased Quasi Aradhana distribution with parameters θ, then the MGF of X can be obtained as

The moment generating function

$$M_X(t)=\int_0^\infty e^{tx}f(x)dx$$

One gets after simplifications

$$M_X(t)=\frac{1}{2\alpha^2+12\alpha+24}\sum_{j=0}^{\infty}\frac{t^J}{J!\theta^J}\left[\alpha^2\Gamma J+3+\Gamma J+5+2\alpha\theta\Gamma J+4\right]$$

4.2 Characteristic Function

Similarly the characteristic function of Area biased Quasi Aradhana Distribution obtained

$$\phi_X(t) = M_X(t)$$

$$\phi_X(t) = \frac{1}{2\alpha^2+12\alpha+24}\sum_{j=0}^{\infty}\frac{it^J}{J!\theta^J}\left[\alpha^2\Gamma J+3+\Gamma J+5+2\alpha\theta\Gamma J+5\right]$$

5. Performance measures of the Area biased Quasi Aradhana distribution

From the above obtained Area biased Quasi Aradhana distribution density function (pdf), the r^{th} moment $E(X^r)$ of the Area biased distribution can be calculated. First Let X be the random variable of Area biased distribution with parameter θ.

$$E(X^r) = \mu_r^1 = \int_0^\infty x^r\, f_a(x)dx$$

$$= \int_0^\infty x^r \frac{x^2\theta^3}{2\alpha^2+12\alpha+24}(\alpha+\theta x)^2 e^{-\theta x}dx$$

After simplifications, one gets

$$E(X^r) = \mu_r^1 = \frac{\alpha^2\Gamma r+3+\Gamma r+5+2\alpha\theta\Gamma r+4}{\theta^r(2\alpha^2+12\alpha+24)} \quad \textbf{(7)}$$

Putting r =1 and 2 in equation (7), we get mean and second moment of Area biased Quasi Aradhana distribution as

$$E(X) = \mu_1' = \frac{6\alpha^2+120+4\theta^2\alpha}{\theta(2\alpha^2+12\alpha+24)} \quad (8)$$

$$E(X^2) = \mu_2' = \frac{24\alpha^2+720\theta+240\alpha\theta}{\theta^2(2\alpha^2+12\alpha+24)} \quad \textbf{(9)}$$

Using the above equation (8) and (9), the variance can be obtained as

$$V(X) = E(X^2) - [E(X)]^2$$

$$= \frac{24\alpha^2+720+240\alpha\theta}{\theta^2(2\alpha^2+12\alpha+24)} - \left[\frac{6\alpha^2+120+4\theta^2\alpha}{\theta(2\alpha^2+12\alpha+24)}\right]^2$$

And after simplifications, one gets the

$$V(X) = \frac{3\alpha^2+60+2\alpha\theta^2}{\theta(\alpha^2+6\alpha+12)}\left[(4\alpha\theta^2+200+120)(\alpha\theta^2+3\theta+12)-3(\alpha\theta^2+3\theta+12)^2\right]$$

(10)

6. Control limits

Three sigma UCL and LCL are obtained as specified by Montgomery (2012) and one can get the control limits for Area biased Quasi Aradhana distribution using (8) and (10) and are given by

$$UCL = \frac{3\alpha^2+4\theta+2\alpha\theta^2}{\theta(\alpha^2+6\alpha+12)} + 3\sqrt{\left[\frac{(12\alpha^2+360+120\alpha\theta^2)(\alpha^2+6\alpha+12)-3(\alpha^2+60+2\alpha 0^2)^2}{\theta(\alpha^2+6\alpha+12)}\right]}$$

$$CL = \frac{3\alpha^2+4\theta+2\alpha\theta^2}{\theta(\alpha^2+6\alpha+12)}$$

$$LCL = \frac{3\alpha^2+4\theta+2\alpha\theta^2}{\theta(\alpha^2+6\alpha+12)} - 3\sqrt{\left[\frac{(12\alpha^2+360+120\alpha\theta^2)(\alpha^2+6\alpha+12)-3(\alpha^2+60+2\alpha 0^2)^2}{\theta(\alpha^2+6\alpha+12)}\right]}$$

7. Numerical Illustration

An example regarding the construction of control limits is considered for illustrating the applications of the proposed method. The control limits of the Area Biased Quasi Aradhana distribution are obtained using simulated data set for parameters θ. All the generated samples are reported in Table 1.

Data Set:
A production manager for a pen company has inspected the number of defective pens in five random samples with 500 pens in each sample. The table below shows the number of defective pens in each sample of 500 pens.

35,	38,	40,	42,	45,	48,	55,	60,	25,	35,	25,	35,
45,	17,	56,	42,	52,	60,	35,	50,	38,	26,	20,	12,
9,	35,	25,	71,	52,	36,	45,	27,	34,	70,	15,	32,
67,	29,	12,	25,	31,	54,	45,	31,	12.			

Table 1: Control limits Using Area biased Quasi Aradhana distribution

α	θ	UCL	CL	LCL
1	1	52.68	3.42	-45.84
	3	35.48	1.42	-34.06
	5	25.15	4.91	-18.33
	7	63.79	1.21	-160.16
3	1	76.25	2.38	-71.49
	3	57.01	1.21	-54.59
	5	39.28	1.22	-37.38
	7	31.08	1.08	-29.64
5	1	94.44	2.16	-90.12
	3	72.63	1.12	-70.39
	5	52.20	1.15	-49.90
	7	36.65	1.33	-33.99
7	1	110.90	2.15	-106.60
	3	124.49	1.08	-122.33
	5	63.08	1.09	-60.92
	7	35.52	1.24	-33.04

It is observed from the Table1, that for the fixed value of the parameter 'α', the deviation of the control limits decrease whenever the parameter 'θ' increases. As well as for the increasing of parameter 'α', the deviation of control limits increases whenever the parameter 'θ' is fixed. The Area Biased Quasi Aradhana Control chart is shown in Fig.1 for $\alpha = 5, \theta = 3$

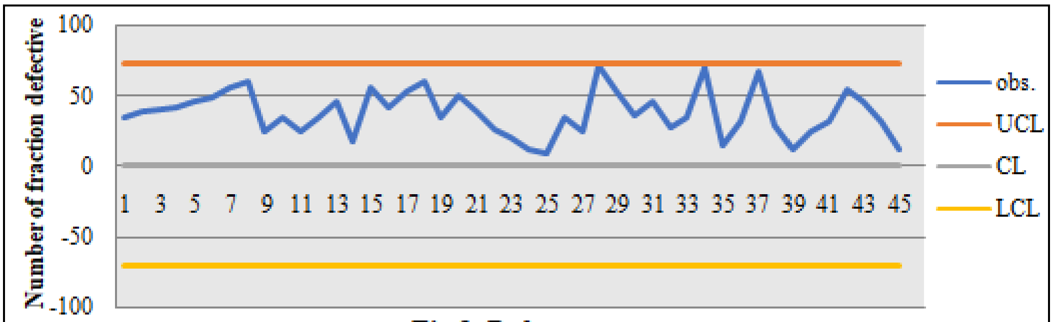

Fig.1. P chart

It is also pointed out that, the observation of the process control must be θ>0, $\alpha \geq 0$ otherwise the process will be out of control. That is, depends on the manufacturing products, the manufacturing engineers should fix the parameter α and θ based on what type of data they are working with. As we can see on Table 1, the more parameters value increase, the higher control limits.

8. Conclusions

In this paper, the process control has been developed using a Area biased Quasi Aradhana distribution. It is given for sentencing the process while manufacturing. The control limits are given using the Area biased Quasi Aradhana distribution with different values of parameter 'α' and 'θ'. Table 1 is constructed to help to the selection of the parameters based on the type of data the manufacturing engineer is faced with. The control chart is drawn by considering the parameter $\alpha = 5$ and $\theta = 3$ where all observations are showing to be in control. Hence it is advisable to keep than the observed process data.

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