Turbulent Flow Analysis, An Oscillatory Approach

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Abstract: ***Purpose**: Turbulent flows play an important role in industrial equipment. Therefore the study and investigation of flow of turbulent nature gains an attention of researchers. **Methodology**: As a special case of study on oscillatory model is considered for different wave lengths and frequencies and need to be interpreted graphically. **Findings**: The turbulent energy spectrum is investigated and typical experimental values are shown for Prandtl Eddie's, energy containing Eddie's and energy dissipating. **Originality/ value**: In this paper the author highlighted harmonic of basic frequency and the corresponding fractional length that often arise, taking into account the presence of those waves and an undulatory model for turbulent flow is put forward.*

Keywords: Flow velocity, Frequency, Strouhal number Turbulent Flow, Boundary layer, Longitudinal and Transverse waves, Reynolds number.

Nomenclature:

U = Flow velocity
S = Strouhal number
d = Nozzle radius
λ = Wave length
f = Basic frequency
$\bar{f}$ = Puffing Frequency
E = Specific mechanical energy
d^* = Displacement thickness
d_w = Wall layer thickness
u_t = Friction velocity
v = Kinematic energy
λ_p = Particle-path wave length
u = Local velocity
C_w = Wave celerity
f_n = Frequency Eddies
K = Energy quanta
N = Total number of Eddies
E_m = Mean energy

1. Introduction

Nature offers many examples of restrained fluid layers that an outer flow of velocity U forces to oscillate with a basic frequency $f = U/2\pi d$ approximately, d being the layer thickness.when the oscillations are convected by the current, waves of length $2\pi d$ are formed. Harmonics of the basic frequency and the corresponding fractional-length waves often arise. Taking into account the presence of those waves, an undulatory model for the turbulent flow is propounded. As an example of its application, a turbulent-energy spectrum is obtained through elementary wave- mechanics considerations.

2. A Universal Strouhal Law

Roshko [32] analysed the frequency of vortex shedding from cylindrical bodies taking into account the width d of the wake (i.e., the spacing of the free streamlines delimiting it) and the velocity U at the point in which these streamlines separate from the body, instead of the traditional parameters: body width and approach velocity. So the Strouhal number fd/U resulted to be independent of the body shape and flow Reynolds number; its average value was found to be

$$S = \frac{fd}{U} = 0.16 \qquad (2.1)$$

It was subsequently proved that the S value remains practically the same when the flow is constrained by a central splitter plate dividing the wake [32, 2], by parallel walls confining the flow [12, 13] or by forcing the cylinder to vibrate, in order to change artificially the shedding frequency [7].

Now, this value 0.16 for the Strouhal number is not peculiar to the wake vertices. In fact, it is not uncommon to find it, or a very near value, associated with other modes of fluid oscillations, as well as the value

$$\lambda = \frac{U}{f} = 6.2d \qquad (2.2)$$

for the length λ of travelling waves resulting from the convection of those oscillations by the main flow.

So for instance Crow and Champagne [4], Observing the response of a round turbulent jet to a periodic surging imposed to its exit in the form of puffs emitted downstream, found that $\bar{f}d/U = 0.15$, f being the puffing frequency, d the nozzle radius and U the exit speed of the jet. Similarly, from Cervantes and Goldschmidth data [3],one infers that a plane jet flaps according to the formula $\bar{f}d/U = 0.154$, f being the flapping frequency, d the jet width and U the center line mean velocity at a given section.

Measuring the frequency f of intermittent erect vertices that form upstream from a weir set across a rectangular water channel, U being the approach mean speed, Levi [16] found $\bar{f}d/U = 0.154$. Here d is the upstream water depth. For the frequency of orifice vertices upstream from a screen crossing the channel, he found $\bar{f}d/U = 0.176$, d being the screen submergency.

The length λ of wind waves produced with minor wind speeds U and fetches x appears to satisfy eq.2, d representing the thickness of the wind laminar boundary layer. From Sen's laboratory measurements [26] one gets $\lambda/d = 6.21$ for U=5.12 m/s, x=54cm, and $\lambda/d = 6.63$ for U=6.52 m/s, x=49cm. From Sudolskiy's field measurements [27], one gets $\lambda/d = 5.89$, 6.99 and 6.22 for U=5 m/s and fetches of 1, 2 and 5km respectively.

Yalin [31] suggests that the length of dunes formed in a loose-bed river of depth d is on average equal to $2\pi d$. A similar result can be inferred from Thorpe's measurements of the increasing with time length of waves formed at a density interface between miscible fluids, provided that the thickness of the mixing layer is taken as d [28].

3. An Oscillatory Model

Birkhoff attempted to justify the value of the wake vortex-shedding Strouhal number through an analysis of the wake mechanism [2]. However, the validity of eq. 1 for so many different flow modes suggests that we are in presence of a very general physical law, independent of the specific features of each single mode. The following simple reasoning [17] will lead us to corroborate this assumption and formulate the law.

Let us suppose that a restrained fluid layer of width d is forced to osillate with the frequency f by the presence of a continuous free flow of speed U, and that this frequency is the same that would correspond to an elementary Oscillator of length d. The specific mechanical energy of the latter is

$$E = \frac{1}{2}(2\pi f d)^2 \quad (3.1)$$

while the available kinetic energy is $U^2/2$. Equating both, one gets

$$S = \frac{fd}{U} = \frac{1}{2\pi} \quad (3.2)$$

that is, 0.159, which agrees with eq, 1. Oscillations governed by this law, if convected by the flow, will look to a stationary observer as undulatory perturbances of wave length

$$\lambda = \frac{U}{f} = 2\pi d \quad (3.3)$$

that agrees with eq. 2. By the way, eq. 4 suggests the expedience of preferring the number $S' = 2\pi S$ to the usual Strouhal number S, in order that the value 1 should correspond to critical conditions, as it occurs for instance for Mach and Froude numbers.

4. Evidence of Strouhal-law Validity in Turbulent Flows

4.1 Boundary-layer transition.

Three Succesive stages characterize the trasition from laminar to turbulent flow [13]: at first a procession of longitudinal waves appears, then corss waves, and finally the resulting doubly-periodical waves shatter into "hairpin eddies" preluding to turbulence. Now, all these stages appear to obey the Strouhal law.

The correlation between the length of longitudinal waves and the boundary layer thickness d can be deduced from an old Tollmien's result [29]. In fact, he showed that, provided that the flow Reynolds number exceeds a certain critical value, the minimum wave length of an oscillatory disturbance able to compromise the stability of a flat-plate laminar boundary layer is equal to $(2\pi/0.36)d^*$, d^* being the displacement thickness. Now, this is about 6d, taking as usual $d = 2.9d^*$.

The transversal periodicity is usually visualized through the furrows grooved by the current into a fresh wall coating. Data from a relevant NACA technical note [10] give, on an average, a furrow spacing of 3.09 d, which agrees with eq. 5, because the furrows appear to be the result of an accumulation of paint at the nodes of standing transversal cross waves, and the node spacing is half the wavelength.

Klebanoff, Tidstorm and Sargent [11], measuring the frequency f of hairpineddy production obtained that $\bar{f}d^*/U = 0.13$, U being the free-flow velocity. Since in their case $d/d^* = 2.55$, it results that $\bar{f}d/U = 0.33 = 2X0.165$. This is double the value given by eq. 4, suggesting the presence of a first harmonic.

Wall layer: Longitudinal and transversal waves of the same length λ_w appear to coexist also within the viscous sublayer, but they are much smaller then the transition waves, because they scale with the wall-layer thickness d_w.

Evidence of longitudinal waves can be found in a paper by Fage and Townend [6]. when observing by ultramicroscope the motion of paricles dragged by a turbulent current, they recorded regular oscollations of the particle paths inside a layer very near the wall, whose non dimensional thickness was about $y^+ = yu_\tau/v = 0.4$, u_τ being the fricion velocity and v the kinematic viscosity. Now, if λ_p is the particle-path wavelength, it should be to the local velocity u as λ_w is to the wave celerity c_w. Since from Fage data one infers that $\lambda_p^+ = 4.43$ and Morrison [18] finds that $c_w^+ = 8.2$, taking $y^+ = 0.2$ as a mean position for the observed path, we get $u^+ = 0.2$, and the $\lambda_w^+ = 182$. On the other hand, since $d_w^+ = 30(12)$, eq. 5 gives the theoretical value $\lambda_w^+ = 2\pi d_w^+ = 188$.

Coming now to the low-speed viscous-sublayer longitudinal striations, let us suppose that, as the transition ones, they correspond to nodes of transversal standing waves. Their spacing λ_w' should then be equal to $\lambda_w/2$, the theoretical value of $\lambda_w' +$ being thus 94. In fact an experimental average for it is about 97 [22].

The other typical feature of wall layer is its bursting activity. Narahari Rao discovered that the burst frequency f scales with outer parameters, i.e., the boundary layer overall thickness d and the free-flow velocity U. His measurements [19] give for fd/U values between 0.14 and 0.33. More precise results are now available. For instance from the measurement of wall pressure fluctuations (that are closely related to bursting activity) by Schewe [25], one obtains $\bar{f}d/U = 0.172$.

5. Fully developed turbulent flow

Nychas, Hershey and Brodkey [21] pointed out the alternation of low-speed and high-speed fluid bodies in the region of fully-developed turbulent flow. Wallace, Brodkey and Eckelmann [30], working in a channel of $d = 22$ cm width, with a centerline velocity $U = 21$ cm/s, measured the time of passage T of a characteristic pattern of the fluctuation of the stream wise velocity component, that is likely to correspond to the passage of one of those bodies. A typical graph in their paper gives T=3.3 s; therefore 2TU/d=6.30, 2TU being the streamwise width of a low-speed-high-speed pair. Comparing with eq. 5, we get that $2TU = \lambda$, λ being the length of a fundamental wave. This suggests that the speed alternation ensues from the passage of the wave, the low speed corresponding to the wave outward half-length, the high speed to the wallaward half-length (see Fig.1).

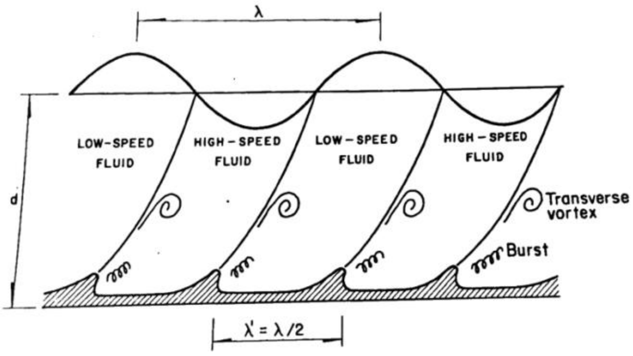

Figure 5.1: Aletrnation of low-speed and high speed fluid bodies and burst-inducing mechanism.

Turbulent structures display a near-periodicity. Badri Narayanan and Marvin [32], auto correlating velocity and pressure fluctuations across the boundary layer at a wide range of Reynolds and Mach numbers, found out that , f being the fluctuation frequency, d the boundary-layer thickness and U the free-flow velocity. From recent measurement by Hofbauer [9] one gets that $\bar{f}d/U = 0.152$.

Finally, let us assume [5] that the charaacheristic length 1_0 of large eddies in a pipe flow be such that

$$1_0 = \frac{\tilde{u}}{f} \tag{5.1}$$

f being their frequency and $\tilde{u}$ the turbulent intensity at the pipe axis. If those eddies are envisaged as oscillators of length $\pi 1_0$ and their energy is equated to the one given by eq. 3, one gets that $\pi 1_0 = d$, that is,

$$\frac{1_0}{d} = \frac{1}{\pi} = 0.32 \tag{5.2}$$

Therefore, taking into account eqs. 6,7 and 4, one gets

$$\frac{\tilde{u}}{U} = \frac{fd}{U}\frac{1_0}{d} = \frac{1}{2\pi^2} = 0.050$$

U being the mean velocity at center line. Experimental results by Laufer [14] give $\tilde{u}/U$=0.047.

6. A New Turbulence Mode

Let us accept that, as the foregoing results suggest, within a turbulent boundary layer of thickness d associated with an outer free flow of speed U, oscillations of frequency $f = U2\pi d$ and wave length $\lambda = 2\pi d$ occur, that mainfest themselves in the alternation of low-and high-speed fluid bodies of width $\lambda/2$.

According to Nychas [21], in the shear layers between these bodies transverse vertices arise. They usually move outward, and this motion seemingly rouses low-velocity tongues up from the viscous-sub layer streaks. As shown else where [15], there are good reasons for assuming that the bursts are the wakes formed behind those tongues by the circumventing faster flow.

Bursts, possessing a velocity component normal to the wall inherited by the parent uprising tongue, leave the wall layer and spread into the region of fully developed turbulence, creating there structures endowed with verticity.

Now, the travelling waves of length $\lambda = 2\pi d$ are not alone. They coexist with shorter waves of length $\lambda/2, \lambda/3$,...., carrying the oscillations that correspond to the harmonics $f_2 = 2f$, $f_3 = 3f$,...of the basic frequency $f_1 = f$. A progressive wave forces fluid particles to turn with the wave frequency, following oval orbits whose size diminishes as the wall is approached. It is thus reasonable to expect that, through this timing-and-shaping activity, the travelling waves control the coherent structures arised from ejected bursts, creating eddies of various frequencies (Fig.2). Travelling waves should also control cascade processes, shaping into higher-frequency eddies the pieces into which a coherent structure would ecentually disrupt.

On these premises, it seems reasonable to try to build an oscillatory theory of turbulence, that could use the analytic tools of wave mechanics. As an advance, we will solve the problem of obtaining a turbulent-energy spectrum by deterministic means [17].

7. The Turbulent Energy Spectrum

Let us admit that turbulent eddies of frequencies $f_n = nf(1,2,3,.)$ are able o recieve or emit energy only through quanta ϵ_n. At a certain state of flow, d and U being given, one may expect by eqs, 3 and 4 that

$$\epsilon_n = \alpha S_n^2 \tag{7.1}$$

α representing an energetic factor, function of the free-flow Reynolds number, and $S_n = nS = n/2\pi$. Now, let us observe that S_n represents also the ration of the energy $(f_n dU)/2$ associated with the frequency f_n and the total kinetic energy $U^2/2$. In view of the considerable quantity of eddies that are present, this fact suggests that the probability of finding an eddy of frequency f_n endowed with a quantum of energy has to be proportional to $e^{-}S_n$, the probability of finding such an eddy endowed with two energy quanta has to be proportional to e^{-2S_n}, and so on. Therefore, the number of eddies with frequency f_n and k energy quanta (k=1,2,3,..) can be written as

$$N_k = ce^{-kS_n}$$

c being a numberical constant. The total number of f_n-frequency eddies will then be

$$N = N_1 + N_2 + N_3 + \ldots = c(e^{S_n} + e^{2S_n} + \ldots)$$
$$= ce^{S_n}(1 - e^{S_n})^{-1}$$

As a consequence

$$c = Ne^{S_n}(1 - e^{S_n}) \qquad (7.2)$$

The total energy E_t corresponding to all the f_n-frequency eddies will be

$$E_t = N_1\epsilon_n + N_2(2\epsilon_n) + \ldots$$
$$= ce^{S_n}\epsilon_n(1 + 2e^{S_n} + 3e^{2S_n} + \ldots)$$
$$= \frac{ce^{S_n}\epsilon_n}{(1 - e^{S_n})^2}$$

that is by eq. 7.2

$$E_t = \frac{N\epsilon_n}{(e^{S_n} - 1)} \qquad (7.3)$$

Introducing now eq. 8 into eq. 10 and dividing by N, the following expression results for the mean energy $E_m = E_t/N$ of the whole of f_n-frequency eddies:

$$E_m = \frac{\alpha S_n^2}{(e_n^S - 1)}$$

that is, since $S_n = n/2\pi$,

$$\frac{E_m}{\alpha} = \left(\frac{n}{2\pi}\right)^2 \left(e^{n/2\pi} - 1\right)^{-1} \qquad (7.4)$$

Eq. 11 has been plotted in Fig. 7.1, showing $4\pi^2 E_m/\alpha$ as a function of the frequency number n. The resulting curve agrees qualitatively with energy spectrum deduced on dimensional grounds [8].

To show its quantitative validity, three points have been marked on the n-axis, pointing out the typical values that, according to Davies[5], correspond, for medium Reynolds numbers, to (a) Prandtl eddies (i.e., those whose characteristic dimension is the Prandtl mixing length), (b) energy -containing eddies, and (c) energy-dissipating eddies. Their position has been ascertained according to the following consideration. Nikuradse [20], experimenting

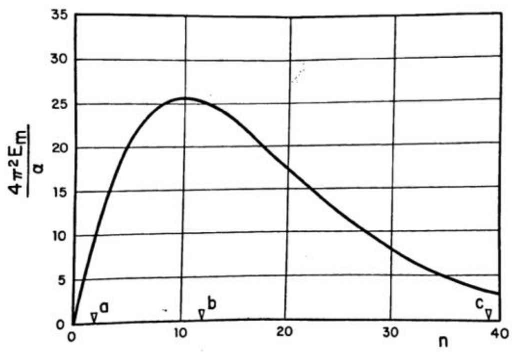

Figure 7.1: Energy Spectrum as a function of the frequency number n.Typical experimental values of n are shown for (a) Prandtl eddies, (b) energy-containing eddies and (c) energy-dissipating eddies.

With smooth circular pipes, was able to determine the mixing length 1_m as a function of the distance from the pipe wall, for different Reynolds number. At values of 10^5 or more he found that, at the pipe axis,$1_m/R = 0.16$, R being the pipe radius. Now if, as suggested before for axisymmetrical flows, we take d=R and compare with eq. 7, we find that $1_m = 1_0/2$, that is, that the Prandtl eddies correspond to n=2. Having thus found the location of Prandtl eddies, a simple proportion applied to Davies values give n=12 for energy-containing eddies and n=39 for energy-dissipating eddies. These are the abscissas marked as a, b, c in Fig.7.1. Their position with respect to the energy curve agrees with accepted beliefs [9].

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