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Topology Optimisation of Passive Constrained Layer Damping - A Comprehensive Review

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Abstract: ***The developments made in the passive constrained layer damping (PCLD) treatment which is intended to control structural vibration are discussed in this paper. The paper begins with a brief review of the literature on passive damping methods employing viscoelastic materials on PCLD. Viscoelastic material is sandwiched in the CLD, which is located between the upper constraining layer and base layer. Cutting the constrained and constraining layer known to be partialization can improve the outcomes of the CLD's damping efficiency. Complete coverage of the constrained damping layer will add extra mass, causing wastage of materials, and would not be able to significantly enhance the damping performance of the composite structure. In order to get the best distribution of the constrained damping layer, topology optimization strategies are suggested in this article. Various optimisation methods proposed by many researchers to improve the damping efficiency in PCLD are presented.***

Keywords: layer damping, optimization, partialization, optimal solution

1. Introduction

A fundamental aspect of effective mechanical design that is frequently overlooked is the damping of structural parts and materials. Endless variety of constructions has experienced many mechanical breakdowns as a result of the absence of damping in structural components. Many studies and attempts have been made in this field to account for the damping effects in order to prevent mechanical failures and vibration. As the damping performance of structure is said to be improved by using damping material in PCLD, there are many researchers started to work on it either to change existing material or to go for new material in order to improve structural damping of components in which damping layer is applied. Viscoelastic materials are common damping materials. Materials that are viscoelastic naturally dissipate energy during deformations and because of this characteristic, the viscoelastic materials are used to reduce structural vibrations. These substances are often applied as a layer that is constrained between a constraining layer and the hosting structure. Sandwiched between the top and lower layers, the damping material creates shear motion [1],[2]. Several factors, such as changing the viscoelastic material's thickness and cut, influence how well such sandwiched structures perform. The long molecule chain's ability to relax causes the energy to be converted into heat. Cutting this whole damping treatment might result in further improvement [21], [22]. As a result, the volume and quantity of high-shear zones grow, which raises the damping rate. In addition, structural topology optimization is a powerful technique for enhancing damping treatment. Topology optimization for PCLD aims to determine the optimal layout and distribution of these passive control elements within the structure to effectively dampen vibration while minimizing weight and maintaining structural integrity. It has been proven that PCLD treatment is effective with optimisation.

2. Passive Constrained Layer Damping

Efforts have been made to provide PCLD treatments for vibrating structures that are as effective as possible in order to enhance the vibration damping of structures with the optimal amount of viscoelastic material. Many researchers have been motivated by the increasing usage of these structures to focus more intently on vibration and acoustic performance research as well as sandwich damped structure design. A schematic diagram of the PCLD treatment is shown in Figure 1 where viscoelastic layer is constrained between base beam and constraining layer. Recently, this type of structure has emerged as an effective alternative. Thus, the works below describe the fundamentals of passive constrained layer damping.

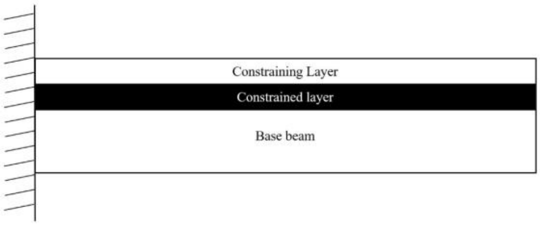


Figure 1: Schematic diagram of a beam with full CLD

Swallow [1] was the one who initially developed the PCLD/CLD treatment. Application of the damping material is done in a sandwiched form. The host structure is bonded to the viscoelastic layer which is constrained by a rigid layer. The damping layer is mostly deformed in shear in this structure. Kerwin et al.[2] developed a theory of damped thin structures with a constrained viscoelastic layer and they were the first one to present this theory. Using viscoelastic material in composite structures, they proposed a theory to compute the modal loss factor. They claimed that the shear motion of the constrained core is what causes the energy dissipation process. They also added that the loss of vibrational energy in heat is due to the shear motion of a constrained layer. Ross et al.[3] performed the research in this area by using a three-layer model to find damping in plates having constrained layer damping treatments. They developed a model for predicting the effects of the constrained layer treatment. The theoretical method of dampening this structure using a constrained viscoelastic layer was initially introduced by them. They reported that the shear motion is responsible for the energy dissipation process in the constrained core and demonstrated the first study of a simply supported beam using a viscoelastic core represented by a complex modulus. They examined how to define loss factor in terms of energy, particularly for composite structures with high damping.

Ungar et al.[4] re-evaluated the concept of loss factor which is applied to the viscoelastic systems. Their key finding is that the energy storage and dissipation mechanisms must be known in order to determine the stored energy. Yet, as these data are often neither well known nor particularly important, the idea of an equivalent elastic spring offers a straightforward computation method that is consistent with theories relevant to lightly damped systems.

DiTaranto [5] used an analytical model to calculate the loss factor of a freely vibrating layered beam with any boundary conditions. He made a new equation that takes the effect of the viscoelastic layers into consideration. One may solve static and dynamic bending problems for layered beams in the same way as for homogeneous beams by using this equation in conjunction with the conventional bending equations used for homogeneous beams. DiTaranto et al.[6] extended Kerwin's work by calculating the loss factors (η) for three and five layered beams as a function of frequency by taking into account the extensional deformations in the viscoelastic layer.

$$\eta = \frac{R_1 K_1 \beta \delta^2 {\lambda_0}^3}{\rho\omega^2[(SR_1 + \lambda_0)^2 + (SR_1\beta)^2]} \qquad (1)$$

Mead and Markus [7] developed a mathematical equation (2) for the transverse displacement of a three-layered composite beam with a viscoelastic core. They derived sixth order differential equation. Several boundary conditions, such as no transverse displacement, no rotation, no bending moment, or no shear force, were assumed to exist at one end of the beam. Mead [9] further investigated the different theories of the three-layer sandwich beam given by Yan and Dowell [8]. Mead then validated models of DiTaranto, Mead and Marcus, Yan and Dowell and compared with differential equation accounting for rotational inertia and shearing in skins and lateral displacement field.

$$w(x,t) = \sum_{n=1}^{\infty} \left(\frac{W_n(x) \int_0^l Q(x) W_n^* \, dx \, e^{i\omega t}}{\mathrm{m} \int_0^l |\, W_n^2 \,|\, dx \, [\omega_n^2 (1 + i\eta_n) - \omega^2]} \right) \qquad (2)$$

Yu and Yuan [10] included the effect of longitudinal displacement and rotational inertia in their study for the symmetrical plates. They introduced damping parameters concept which determines the effectiveness and importance of different types of viscoelastic damping. Durocher and Solecki [13] ensured the shear stress and displacement continuity at the interfaces by including shearing effects in skins in the modelled symmetrical plate as the authors such as Yu, Rao and Nakra [11],[12] neglected the effect of shearing in skin of symmetrical and unsymmetrical plates. Kung and R. Singh [14] proposed an analytical method considering rotational, longitudinal, flexural and shear deformations in sandwich beam's layers with the number of constrained layer patches as shown in Figure 2. They verified the method by comparing the results gathered for patches in literatures.

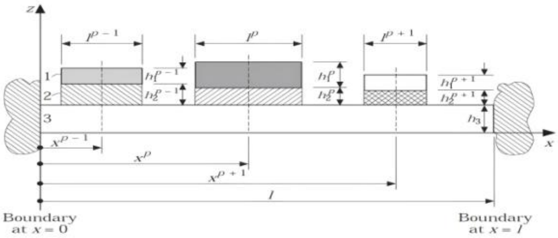


Figure 2: Beam with constrained layer damping patches

Notably for door panels, floor panels, dash panels, brake covers, skin, stringers, frames of fuselage parts in commercial aircraft, and flutter suppressing of aeronautical panels, this damping treatment is proven to be helpful in automotive and aerospace engineering application. The damping (PCLD) treatment is also heavily used in the manufacturing of machine tools, spinning disc compression blades, outlet guide vanes, computer hardware, tram squeal noise and railway wheels [15]-[20].

3. Partial constrained layer damping

The majority of early research focused on full coverage passive constrained layer damping techniques, which are generally not intended for practical use. For instance, the weight constraint may restrict full PCLD coverage in treated applications for the automotive and aviation sectors since any weight increase caused by modifications to the design for vibration and noise reduction must always be kept to a minimum. In order to satisfy the PCLD treatment standards, a partial coverage treatment as shown in Fig.3 is a best option. It has been demonstrated that this type of damping treatments enables manufacturers to reduce weight and expense while maintaining performance in terms of noise, vibration, and harshness. It becomes vital to optimise for finding out improved damping properties. Thus, the basic works on passive constrained layer damping optimisation is given below.

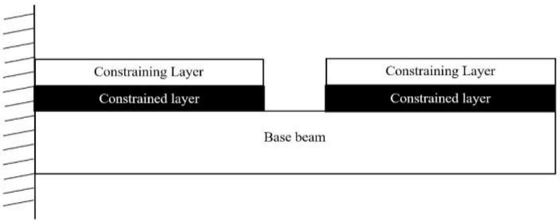


Figure 3: Schematic diagrams of a beam with partial CLD

Nokes and Nelson [21] were one of the first researchers to present an answer to the sandwich beam problem with a partial cover. For symmetric boundary conditions, the modal strain energy technique was applied to compute the treated beams modal loss factors. Plunkett and Lee [22] discussed a method for increasing the damping by cutting the constraining layer into appropriate lengths. Their research included experiments and the development of a formula for the optimal distance of their arrangement of equidistant cuts. They found that for optimum element length (L_0), primarily the energy dissipation depends upon constraining layer's stiffness and viscoelastic materials loss coefficient.

$$L_0 = 2\left(t_1 t_2 \frac{E_2}{G_1}\right)^{1/2} \qquad (3)$$

Kress and Gerald [23] simulated segmented constrained layer treatment by solving a shear lag model which comparable to the inquiry of Plunkett and Lee, employing a transfer-matrix approach. For the whole length of the beam, they gave the shear stress distribution illustration. Measurements and simulations were found to be in good agreement. They also derived a formula which differs from the formula of Plunkett and Lee for optimum spacing of cuts arrangement. Only for the first bending mode of cantilever beam, the effects of cuts were investigated. Torvik et al. [24] investigated a base plate with a multi-layer damping treatment with unanchored constrained layers bonded to one side. They reported the effectiveness of added damping with an experimental investigation. There is increase in damping of the system with few percent added weights. Lall et al. [25], [26] conducted a more in-depth analytical analysis for partially covered plane structures. A notable conclusion from the research was that, for well-chosen parameters and with the same added weight of full PCLD treatments, larger values of the modal damping factor could be achieved for a partially covered beam than for a fully covered one. This motivates a lot of researchers to look into the best layout for CLD treatment.

Mantena et al. [27] also investigated at the optimum side length of a material which is used for dampening constrained layers. They took into account several geometric configurations of a load-bearing structure in terms of the clamping condition, with special attention given to the damping material. Their research was limited primarily by the fact that they only looked at one mode and one segment. Kodiyalam and Molnar [28] introduced a novel way to account for variations in viscoelastic material properties with frequency and enhanced the modal strain energy (MSE) method. They proposed an efficient methodology for visco elastically damped honeycomb sandwich structures to optimise its design for passive vibration control. Alam and Asmani [29] explored the optimal damping treatment design by taking the parameter thickness of the damping material into consideration. They evaluated the damping effectiveness in terms of system loss factor for several layered plates and investigated the geometrical and material property parameters. Kung and Singh [30] introduced an energy-based strategy using several constrained layer dampening patches. Only the individual effects of constrained layer damping patches were examined at several modes. Lumsdaine and Arnold [31] identified the optimal design for a constrained damping layer on a beam using topology optimization in a symmetric finite element model as shown in Figure 4. The goal of optimization is to increase the system loss factor for the beam's initial mode. They used ABAQUS finite element code to model the structure with two dimensional elements.

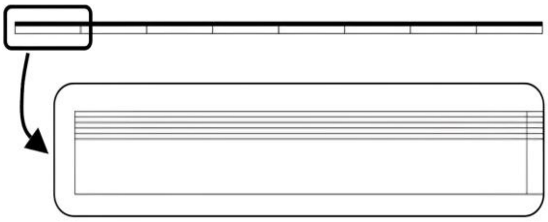


Figure 4: Symmetric finite element model used for Topology Optimization

Wang and Chen [32] studied the damping using a discrete layer annular finite element for annular plates with fully and partially CLD treatment as shown in Figure 5. They solved complex eigen valued problem and extracted the modal loss factors and frequencies of the composite plate. They also presented the effects of stiffness and thickness of layers on natural frequencies and modal loss factors.

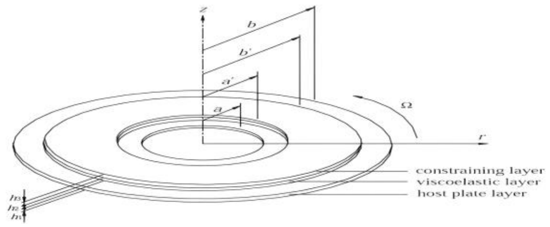
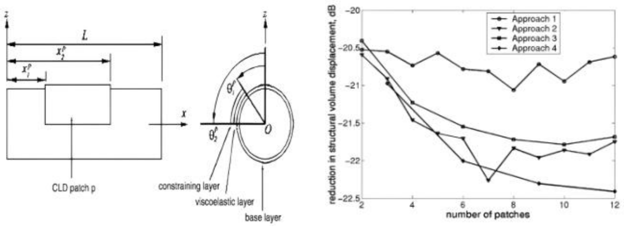


Figure 5: The rotating annular plate with partial CLD

Zheng et al. [33] investigated the optimal location for rectangular damping patches to reduce the structural displacement of cylindrical shells using the genetic algorithm as shown in Figure 6. They studied the effects due to number of patches, total weight of added PCLD and their aspect ratios. Moreira and Rodrigues [34] located passive constrained viscoelastic damping layers on structures using the MSE approach. By comparing their findings with those of experimental testing, they further validated their study. They used MSE method in the analysis of extension effects and location of partial constrained viscoelastic treatments on the damping of thin plates.



(a) (b)

Figure 6: Optimization of rectangular damping patches in cylindrical shells (a) A cylinder with one partial CLD patch (b) number of patches versus SVD

Li and Liang [35] examined and optimized the vibro acoustic characteristics of the damping structure using the response surface method (RSM). They considered the thickness and material parameters of damping panel as the design variable. Bourisli et al. [36] optimised the length of segments. They looked at only one optimization approach and could determine a distribution of segments for one mode as shown in Figure 7. The efficiency of the chosen algorithm in determining the optimal solution was not evaluated in comparison to other optimization techniques.

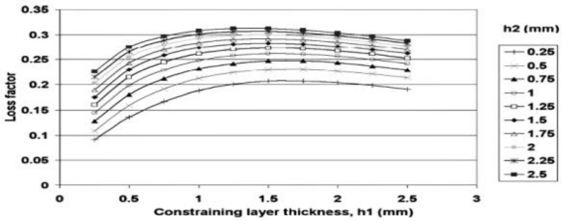


Figure 7: Loss factor vs. constraining layer thickness for each viscoelastic layer

A L Aroujo et al. [37] looked at the significance of using the right finite element model and gradient base optimizer to maximise damping. Dynamic problems are solved using the frequency domain technique, while the viscoelastic behaviour of materials is studied using the complex modulus approach. They use their own model and gradient-based optimization to maximise the modal loss factor. Chia et al. [38] optimises the design of the CLD-treated plate using cellular automata in addition with finite element analysis as shown in Figure 8. The efficiency of the optimization approach is validated by theoretical calculation and testing. They compared the effectiveness of several sets of local rules governing cellular automata with a plate with optimum coverage.

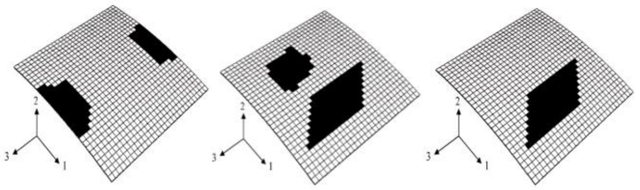


Figure 8: Treatment coverage obtained using cellular automata for curved panel

Zheng et al. [39] developed a dynamic model based on the energy technique for the partially covered CLD beam structure. They then used a genetic algorithm to optimise the location and length of the CLD patch as well as the shear modulus of the viscoelastic layer for various cases where the VE of the beam throughout a frequency range including the first four resonant modes and that at a particular resonant mode is to be minimised, both without and with inclusion of the constraint of minimum damping material utilised, optimal solutions are shown and explored. Alvelid and Magnus [40] described a strategy for designing the optimal placement of attached constrained damping layers and provided examples to illustrate its use. In the context of finite elements, a modified gradient approach is applied to incrementally add sections of constrained damping layers at the elemental locations exhibiting the target function's steepest gradient as the treatment result. Ling et al. [41] researched topology optimization for CLD plate layouts to reduce vibration and sound emission. Reduction of structural modal damping ratios using the method of moving asymptotes (MMA) is the objective of their study. They assigned modal damping ratio as the objective function in optimization approach. The effectiveness of proposed optimisation approach is demonstrated using numerical examples. Andreassen et al. [42] presented MATLAB code of 88 line for topology optimization. It provides a useful tool that might aid individuals starting to learn the subject of topology optimization in easing their learning curve. The article also addresses straightforward upgrades of the fundamental code to incorporate modern PDE-based and monochrome projection filtering techniques.

Shu et al. [43] suggests level set-based optimization of structural topology to reduce frequency response. With a preset excitation frequency or predefined frequency range and a volume constraint, the general objective function is defined as the frequency response reduction at the specified points or surfaces. Ansari et al. [44] investigated the optimal quantity and placement of CLD sheets on the flat structure surface using an improved gradient approach with the aim of maximising the system's loss factor. The damping patches of optimum damping patch shapes is placed in the best possible locations on two separate plate constructions. The correctness of the suggested technique is verified from one example comparing the numerical findings with those from the experimental testing.

Zhang et al. [45] solved the topology optimization problem of CLD plates using the parametrized level set method (PLSM) in this research, where the viscoelastic material with frequency and temperature dependent properties is used as the damping core. The iteratively solved single/weighted modal loss factors are said to be objective functions. Kim et al. [46] compared the modal loss factors obtained by the mode shape (MSO approach) and topology optimization to the conventional strain energy distribution (SED method). It has been observed that topology optimization using the RAMP model and the optimality criteria (OC) approach may offer modal loss factors up to 61.14 percent greater than SED and MSO methods. Fang et al. [47] used evolutionary structural optimisation ESO approach and developed a topology optimization model for the plates using sensitivity analysis as shown in Figure 9 and that reduced the square of the vibration response peak under the excitation of specified frequency band. They developed improved sensitivity analysis method to improve computational accuracy by considering the modal damping ratio derivative. Xie et al. [48] proposed a topology optimization design method based on Moving Morphable Component for CLD-treated structures by taking the viscoelastic material's frequency dependency into account. The objective function is the kinetic energy of mechanical basis structures over a specified frequency range, and the constraint condition is the coverage area of the constrained damping layer.

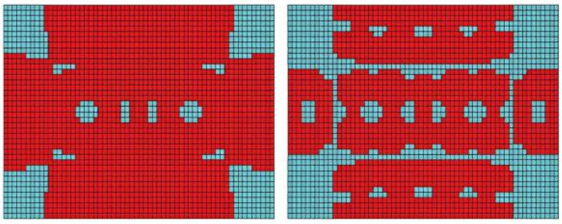


Figure 9: The rotating annular plate with partial CLD

Khalfi et al. [49] obtained the transient and harmonic responses with the analytical modelling of plate with PCLD. They performed two validations such as harmonic motion natural frequencies comparison with literature results and transient response displacement comparison with experimental setup. Khalfi et al. [50] also presented a parametric study of PCLD characteristics on both transient and harmonic response in a rectangular plate. They obtained several conclusions such as identifying different layer thickness proportion for optimal damping, defined area of plate for maximum shear in the viscoelastic layer and the relationship between loss factor and deformation mode.

4. Conclusion

Topology optimization is a valuable tool for optimizing the placement and thickness of the viscoelastic layer in constrained layer damping (CLD) to achieve optimal vibration and noise reduction in structures. Full constrained layer damping involves adding the viscoelastic layer to the entire structure surface, while partial constrained layer damping is used in specific regions of high vibration and noise. The optimal approach is determined by the requirements of the application, and each has its own advantages and disadvantages. Topology optimization of CLD involves defining performance objectives, selecting materials and modelling techniques, and optimizing the design to meet constraints. The process can be computationally intensive, and the resulting designs may pose challenges in manufacturing. However, topology optimization of CLD has been successful in a range of industries including aerospace, automotive, and civil engineering, leading to reduced weight, improved vibration and noise performance, and increased efficiency.

In order to fully understand the most recent findings in the broad field of structural vibration control employing PCLD layout, a literature review is presented. A significant amount of research papers on the passive damping of structural vibration employing viscoelastic materials in layer form via PCLD arrangement are revealed by this literature review. In order to assess the damping characteristics of viscoelastic materials and optimise them, a series of papers, standards, and thesis are used to get an understanding of the various methods. The damping capacity of CLD treatment can be improved not only by partializing the layer but also can be improved by using standoff spacer layer and viscoelastic composite layers. Recent advancements in the field of constrained layer damping have led to several exciting developments. One such development is the use of hybrid constrained layer damping, which combines traditional viscoelastic materials with other types of damping materials, such as magneto-rheological or electro-rheological fluids. Another promising approach is the use of PCLD with irregularly shaped damping patches, which can be optimized using topology optimization to achieve better vibration and noise reduction performance with less material. Additionally, the use of multi-layer constrained layer damping has been explored as a means of achieving improved damping performance in complex geometries. Finally, advancements in modelling and simulation techniques have made it possible to quickly and accurately optimize the placement and thickness of the damping layer using finite element analysis (FEA) combined with optimization algorithms.

There are many exciting opportunities for research and development in topology optimization of constrained layer damping. One promising area for further exploration is the integration of CLD topology optimization with other techniques, such as topology optimization of composite structures. This could result in even more optimal and efficient designs with enhanced vibration and noise reduction capabilities. Another area for potential advancement is the development of real-time control strategies that adjust the damping properties of CLD structures in response to changing conditions, which could further improve performance. Such advancements have the potential to significantly impact a range of industries and applications, including aerospace, automotive, and civil engineering. The application of topology optimization in constrained layer damping has shown great potential in designing high-performance structures with improved vibration and noise performance. As this field continues to advance, it has the potential to revolutionize the design and manufacturing of lightweight and high-performance structures across a wide range of applications. Overall, the use of topology optimization in constrained layer damping is a promising technique that has created new opportunities for the design and manufacturing of high-performance structures with improved vibration and noise performance, across a wide range of applications.

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