

Dynamic Analysis of Prefabricated Box Culvert Formwork Filled with Fresh Concrete Using Finite Element Modeling

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Abstract: *Dynamic analysis is becoming increasingly important in evaluating structural systems. Dynamic analysis is mandatory for structures other than low-rise and small-sized buildings. Especially in parallel with the advancement in computer technology, the analysis of large structures with thousands of degrees of freedom can be done in a short time with advanced software programs such as SAP2000. This study aims to identify key dynamic parameters of the structural element under consideration. Parameters such as free vibration frequencies and mass participation ratios are important in dynamic analysis. Furthermore, Ritz vectors and Eigenvectors can be used alternatively in the Mode Combination Method. This study was executed using Finite Element Method employed in SAP2000 software.*

Keywords: Prefabricated box culvert formwork, External vibrator, Vibration, Ritz vector, Eigenvector

1. Introduction

External vibrators are widely used in manufacture of prefabricated concrete members. The vibration generated by the external vibrator is transmitted to the concrete formwork.

Wenzel [1] noted that external vibrator vibrations employed for concrete compression in precast structural element generation generally cannot reach a penetration depth greater than 20 cm. Therefore, for sections greater than this, the vibrators must be attached to both sides. It was observed that applying a cyclic frequency of 50 Hz to the mold caused "segregation" due to the large amplitude. This led to the view that higher frequencies should be exposed to the concrete to achieve a good compression impact. Optimal frequencies for this goal were specified to be between 75 and 200 Hz.

Wilson et al. [2] used Ritz vectors and Eigenvectors in Dynamic Mode Combination analysis and applied their comparisons on some example structures.

This study is essentially a theoretical dynamics study. The aim is to find some important dynamic parameters. This theoretical analysis was conducted using SAP2000 [3] computer software.

This study is significant as it addresses the gap in understanding the dynamic behavior of prefabricated formworks under fresh concrete load. The findings can guide structural engineers in optimizing formwork designs to prevent resonance and ensure structural integrity during the casting process.

2. Modeling of empty box culvert formwork

The specifications of the external vibrators used in production are given in Table 1.

Table 1: Clamp-on vibrator's properties

Mechanical Properties				Electrical Properties	
his	Centrifugal force		Weight	Max. input power	
Vibr./min	kg	kN	kg	W	Max. current A
6000 (200 Hz)	1157	11.34	25	1200	42V 250V
					23 -

The mold is made of steel plate 5 mm in thickness. Steel sections in diverse dimension and section are attached to mold in horizontal, vertical and diagonal ways to reinforce the structural strength of the system.

Mold's plan view and the coordinates of the points where the measurements were taken are illustrated for box element in Figure 1.

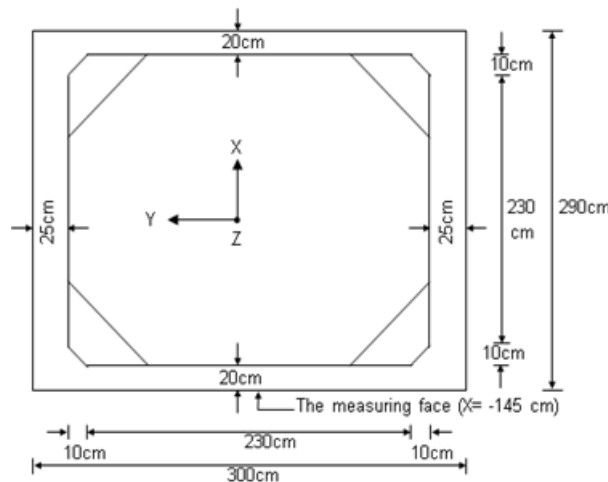


Figure 1: Plan view of the box culvert formwork (h = 97 cm)

3. Modeling of empty box culvert formwork

The equation of motion of a structural system with N degrees of freedom modeled with a finite number of elements can be written in terms of nodal displacements as;

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{P}(s, t) \quad (1)$$

Where \mathbf{M} , \mathbf{C} and \mathbf{K} $N \times N$ dimensional mass, damping and stiffness matrices of the system, $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$ and \mathbf{u} and $\mathbf{P}(s, t)$ are the $N \times 1$ -dimensional time-dependent acceleration, velocity and displacement vectors of the system, respectively, and $\mathbf{P}(s, t)$ is the $N \times 1$ -dimensional load vector that varies with location and time.

The system equation of motion is solved in the time domain using the mode combination method. The mode

vectors used can be either eigenvectors or load-dependent Ritz vectors.

Mold bodies for the box culvert prefabricated structural element are manufactured from 5 mm thick sheet steel. Additionally, steel sections of different sizes and cross-sections are attached horizontally, vertically, and diagonally to reinforce the mold. Therefore, the molds are constructed from shell and bar elements. Depending on the geometry, shell finite elements are constructed from four-node square, rectangular, trapezoidal, and three-node triangular elements with dimensions of approximately 10x10 cm. The bar elements are obtained by using common nodal points with the shell elements.

The three-dimensional finite element mesh of the box culvert formwork is presented in Figure 2.

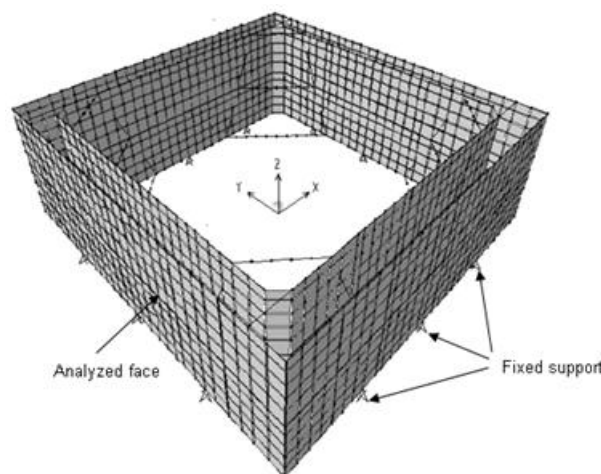


Figure 2: 3D view of box culvert form

3.1. Load applied by external vibrators to box culvert formwork

External vibrators, attached to the mold surface

$$P(t) = P_o \sin(\omega t) \quad (2)$$

This applies a sinusoidal dynamic load perpendicular to the surface.

Where, P_o is amplitude of load, ω is forced frequency, t is time

The period (T) and cyclic frequency (f) of the load are expressed in terms of angular frequency.

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \text{ (Hz)} \quad (3)$$

can be written. The vibrators are attached to a 20x25cm rigid plate and fixed to the mold surface by this plate. In the analyses conducted on the empty mold in this study, it is assumed that the vibrator load acts as a uniformly distributed compressive load on the shell element faces in touch with the plate to which vibrator is connected.

4. Modeling of box culvert formwork when filled with fresh concrete

Fresh concrete poured into the mold significantly alters the mold behavior compared to the empty state, and the fresh concrete-mold interaction must be considered in patterning. For this goal:

- Fresh concrete enforces a pressure force on mold surface, the intensity of which varies depending on time and location,
- The concrete mass can be defined as a large number of point masses concentrated at the nodes on the formwork surface; this modeling assumption is adopted.

Modeling of the mold and vibrator load is the same as for the empty mold.

4.1. Compressive force exerted by fresh concrete to formwork

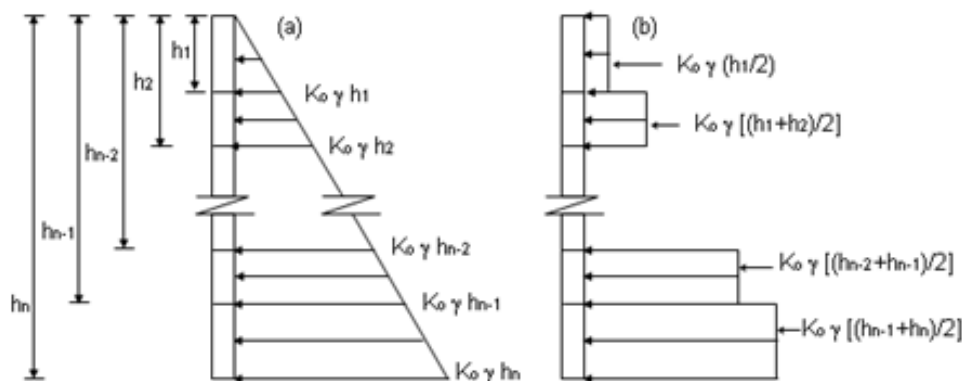


Figure 3: (a) Actual circulation of lateral pressure, (b) Simulation of lateral pressure

4.1.2. Time-dependent function w(t)

In order to determine the change in pressure enforced by fresh concrete on formwork surface over time during the vibration process, the experimental measurement results performed while the mold was empty and full were

Compressive load applied to the mold by fresh concrete, in the shape of equation (4) is given below.

$$P(s, t) = b(s).w(t) \quad (4)$$

It is expressed as the product of two functions that depend on location [b(s)] and time [w(t)]. These functions are defined below.

4.1.1. Function b(s) dependent on location

The function b(s) displays the lateral static pressure enforced by the non-solid fresh concrete on the formwork,

$$b(s) = K_o \gamma h \quad (5)$$

$$K_o = 1 - \sin \phi \quad (5a)$$

$$K_o = \nu / (1 - \nu) \quad (5b)$$

is defined by the expressions. Where,

b: Static lateral pressure (force/area),

K_o: Lateral pressure coefficient,

γ: Specific weight of the material,

h: Hight,

φ: Internal friction angle of the material,

ν: Poisson Rate of the Material,

K_o is a coefficient varying between 0 and 1 and is expressed in two various ways: depending on the internal friction angle of the material as seen in equation (5a) or depending on the Poisson Ratio of the material as in equation (5b).

The actual circulation of static lateral pressure along the mold height is shown in Figure 5a. Assuming that the mold is divided into n shell elements along the height, the uniform pressure load values to be applied to each element are calculated as explained in Figure 3b.

evaluated. The w(t) function, which expresses the change in the reaction force created by the fresh concrete on the surfaces modeled in this way, was chosen as a periodic function as seen in Figure 4. θ shows the phase difference between the vibration load and the concrete response.

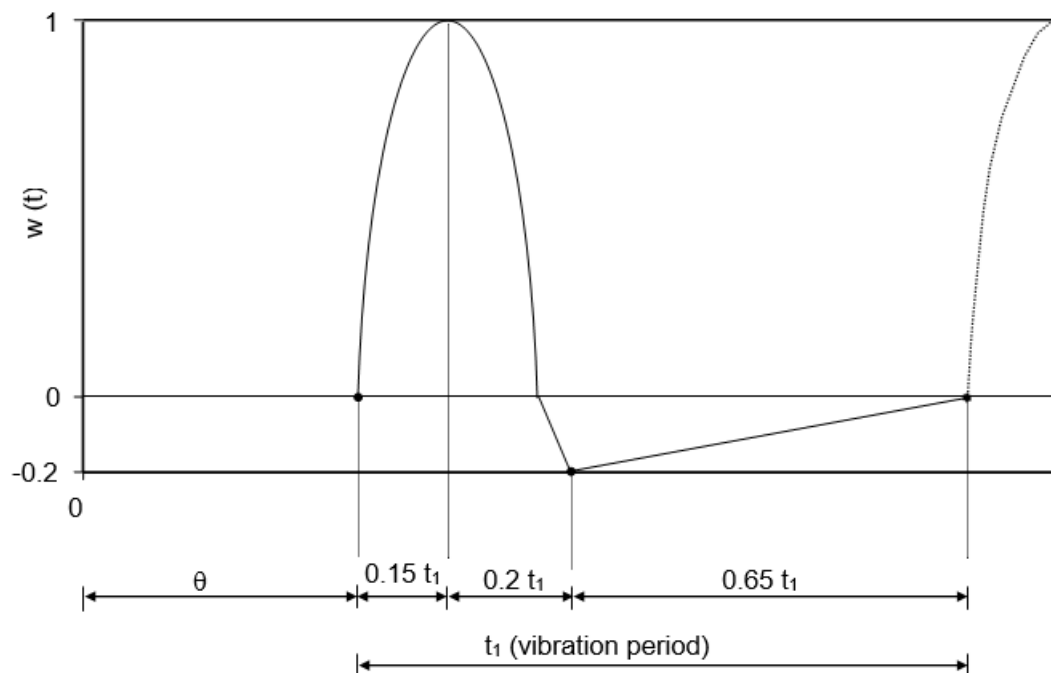


Figure 4: Variation of the function $w(t)$ with time

5. Implementations

In this section, theoretical analyses were performed using the SAP2000 software. The number of vibration modes in the analyses was determined based on the cumulative mass participation ratios of construction in the global X, Y, and Z directions exceeding 90%.

Some parameter values of the steel and concrete from which the molds are produced are selected as follows.

Specific weight of steel: $7.682 \times 10^{-5} \text{ N/mm}^3$
 Elasticity module of steel: 199948 N/mm^2
 Poisson ratio of steel: 0.30
 Specific weight of fresh concrete: 2.45 t/m^3
 Internal friction angle of fresh concrete (ϕ): 18°
 Poisson ratio of fresh concrete: 0.40
 Lateral pressure coefficient of fresh concrete (K_0): 0.75
 Phase difference between vibration load and concrete response: 4.5 msn

5.1. Implementation 1

In this application, free vibration analysis was performed for box culvert formwork. The cyclic frequency values for the first six vibration modes of the formwork are shown in Table 2.

Table 2: Free vibration frequencies of the box culvert formwork when filled with fresh concrete

Mod No	Box culvert form Frequency (Hz)
1	18.49
2	21.77
3	21.82
4	25.48
5	25.94
6	27.98

As can be seen from the examination of the frequency values, the most effective frequency values for the mold are much lower than the vibrator frequency of 100 Hz. Therefore, applying vibration to the mold using 100 Hz vibrators will not cause resonance or impair mold stability.

In this application, the contribution of Eigenvectors and Ritz vectors, which can be used as alternatives in the Mode Superposition Method, to the cumulative mass participation ratio is investigated. The directional variation of the mass participation ratios based on the number of modes is shown in Table 3 for the box culvert form.

As can be seen from the examination of the tables, the mass participation rates obtained using Ritz vectors are much higher than those obtained with Eigenvectors.

Table 3: Mass participation ratios of the box culvert formwork when filled with fresh concrete

Mode numbers	Vector type	Mass participation rates (%)		
		X way	Y way	Z way
50	Eigenvector	29.08	31.03	0.005
	Ritz vector	88.64	87.15	98.15
100	Eigenvector	34.78	34.52	0.01
	Ritz vector	96.05	95.87	99.17
150	Eigenvector	35.54	37.18	0.05
	Ritz vector	97.91	97.82	99.65

5.2. Implementation 2

In this implementation, a theoretical vibration analysis was conducted for the box culvert formwork.

The Time History analysis of the box culvert formwork was conducted using ninety-five Ritz vectors. The initial Ritz vectors were the pressure load enforced by the vibrator to formwork, pressure load applied by the fresh

concrete, and the acceleration vectors on the global axis. The formwork vibration parameters are shown in Table 4.

Simulated surface of box culvert formwork is displayed in Figure 5.

Table 4: Vibration parameters of the box culvert formwork when filled with fresh concrete

Number of dynamic degrees of freedom	Cumulative mass participation rates (%) (Ritz vector number = 95)		
	X way	Y way	Z way
7353	95.67	95.44	99.11

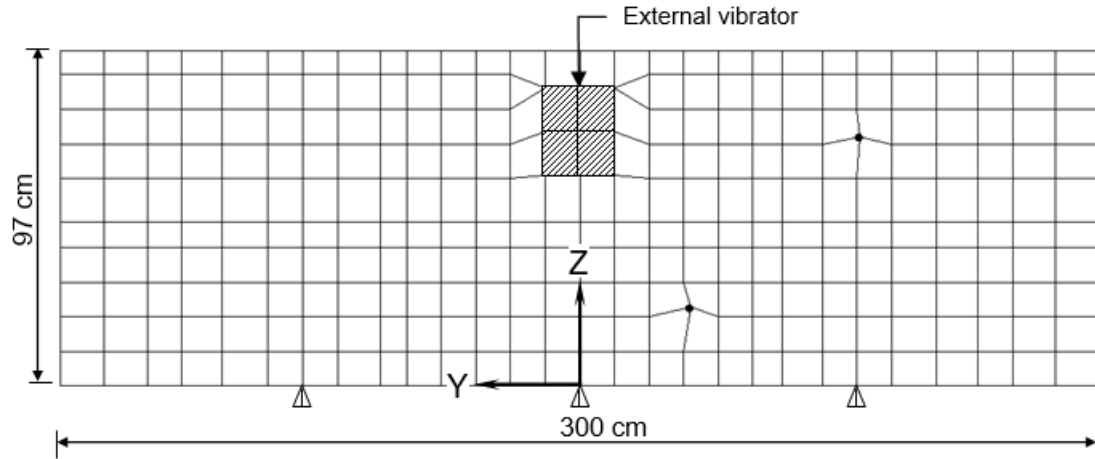


Figure 5: Simulated surface of box culvert formwork (Y-Z plane, X = -145 cm)

6. Results and Discussion

In this study, the computational result is obtained for full of fresh concrete mold. In theoretical analysis, using of Ritz vectors in any number of modes not only reduces computing time significantly but also provides the greater participating mass ratios when compared to eigenvectors, provided that the same number of modes is used. Hence, for the complex structural systems, which have the mass degrees of freedom of several thousands, it is recommended to be used Ritz vectors in mode superposition method of dynamic analysis.

References

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