

Creep Analysis in Whisker Reinforced Composite Disc with Linear Thickness

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Abstract: The objective of the present study is to investigate the effect of creep behavior of the rotating functionally graded material disc with linear thickness in presence of thermal gradients. It has been concluded that the thermal gradients effect the distributions of the stresses and strain rates of the anisotropic rotating disc. Thus, the presence of thermal gradients in rotating disc plays a significant role in developing the creep response.

Keywords: Modeling, Composites, Creep, functionally graded material, Thermal gradients

1. Introduction

Rotating disc is a vital component that uses in many engineering applications in the jet engine, turbine rotors, automotive breaks, flywheels, compressors and computer's discs etc. In most of engineering applications, the rotating disc operates at higher angular velocity and high thermal gradient. Because of the high degree of influence on the properties of disc, creep analysis has gained the attention of the researchers. Keeping this in mind, the study has carried out the analysis of creep behavior for the disc in presence/absence of thermal gradients.

2. Analysis

The constitutive equations for creep are described under multiaxial stress, are expressed,

$$\dot{\epsilon}_r = \frac{\dot{\epsilon} \times \left[x(r) - \frac{(H/F)}{G/F + H/F} \right] \left(\frac{G}{F} + \frac{H}{F} \right)}{\left[x(r)^2 - 2 \frac{(H/F)}{G/F + H/F} x(r) + 1 \right]^{1/2}} = \frac{d\dot{u}_r}{dr} \quad (4.1)$$

$$\dot{\epsilon}_\theta = \frac{\dot{\epsilon} \times \left[\left(1 + \frac{H}{F} \right) - \frac{H}{F} x(r) \right]}{\left[x^2 - 2 \frac{(H/F)}{G/F + H/F} x(r) + \frac{(1 + H/F)}{G/F + H/F} \right]^{1/2}} = \frac{\dot{u}_r}{r} \quad (4.2)$$

$$\dot{\epsilon}_z = \frac{\dot{\epsilon}}{2\bar{\sigma}} \{ -G\sigma_r - F\sigma_\theta \} \quad (4.3)$$

The effective strain rate of disc is assumed, as described by following Sherby's law [1977],

$$\dot{\epsilon} = [M(\bar{\sigma} - \sigma_0)]^n \quad (4.4)$$

where $\dot{\epsilon}$, M , $\bar{\sigma}$, σ_0 , n are the effective strain rate, creep parameter, effective stress, threshold stress, the stress exponent.

Dividing Eq. (4.1) by Eq. (4.2) and integrating the resulting equation by taking limit a to r on both sides,

$$\frac{\dot{u}_r}{\dot{u}_{r_i}} = \exp \int_a^r \frac{\phi(r)}{r} dr \quad (4.5)$$

where, $x(r) = \frac{\sigma_r}{\sigma_\theta}$, is the ratio of radial and tangential

stresses at any radius and $\dot{u}_r = du/dt$, is the radial deformation rate and \dot{u}_{r_i} , is the radial deformation rate at the inner radius.

The equilibrium force in the radials direction is,

$$\frac{d}{dr}(r h \sigma_r) + \rho \omega^2 r^2 h = h \sigma_\theta \quad (4.6)$$

Boundary Conditions are $\sigma_r(a) = 0 = \sigma_r(b)$ (4.7)

Dividing Eq. (4.5) by r and equated to Eq. (4.2),

$$\frac{\bar{\sigma} - \sigma_0}{\psi(r)} = \frac{(\dot{u}_{r_i})^{1/n}}{M} \quad (4.8)$$

where,

$$\psi(r) = \left\{ \frac{\left[x(r)^2 - 2 \frac{(H/F)}{G/F + H/F} x(r) + \frac{1 + H/F}{G/F + H/F} \right]^{1/2}}{\frac{1 + H/F}{\sqrt{G/F + H/F}} - \frac{H/F}{\sqrt{G/F + H/F}} x(r)} \cdot \frac{\dot{u}_r}{r \cdot \dot{u}_{r_i}} \right\}^{1/n} \quad (4.9)$$

and

$$M(r) = e^{-35.38} P^{0.2077} T(r)^{4.98} V^{-0.622} \quad (4.10)$$

$$\sigma_0(r) = -0.03507 P + 0.01057 T(r) + 1.00536 - 2.11916 \quad (4.11)$$

Simplify Eq. (4.8) into Eq. (4.1) to get the tangential stress (σ_θ),

$$\sigma_\theta = \frac{\bar{\sigma} - \sigma_0}{\psi(r)} \psi_1(r) + \psi_2(r) \quad (4.12)$$

Where,

$$\psi_1(r) = \frac{\psi(r)}{\left[x(r)^2 - 2 \frac{H/F}{G/F + H/F} x(r) + \frac{1 + H/F}{G/F + H/F} \right]^{1/2}} \quad (4.13)$$

$$\psi_2(r) = \frac{\sigma_0}{\left[x(r)^2 - 2 \frac{H/F}{G/F + H/F} x(r) + \frac{1 + H/F}{G/F + H/F} \right]^{1/2}} \quad (4.14)$$

$$\sigma_{\theta_{avg}} = \frac{1}{b-a} \int_a^b \sigma_{\theta} dr \quad (4.15)$$

where, $\sigma_{\theta_{avg}}$ is the average tangential stress.

Now radial stress can be expressed by integrating Eq. (4.6),

$$\sigma_r(r) = \frac{1}{r \cdot h} \left[\int_a^r \sigma_{\theta} dr - \frac{\omega^2 \rho (r^3 - a^3)}{3} \right] \quad (4.16)$$

For a disc, the thermal conductivity can be calculated as,

$$K(r) = \frac{[100 - V(r)]K_m + V(r)K_d}{100} \quad (4.17)$$

Where $K_m = 247 \text{ W/mK}$ is matrix conductivity and $K_d = 100 \text{ W/mK}$ is dispersoid conductivity.

$$T(r) = 619.69 + 0.6083r - 0.0208r^2 + 3.27 \times 10^{-4}r^3 - 1.96 \times 10^{-6}r^4 + 4.43 \times 10^{-9}r^5 \quad (4.18)$$

where $T(r)$ is the temperature taken from Gupta et al.⁴.

3. Results and Discussion

The analysis has been carried out for anisotropic FGM discs to find the effect of thermal gradients on plastic stress distributions and creep deformation and computer program has been made for this analysis. This study has been done for the anisotropic FGM discs containing silicon carbide whisker in a matrix of pure aluminum in presence of thermal gradients and obtained results are compared for both the discs in absence of thermal gradients to analyze the effect of thermal gradients. For the analysis of creep behavior, Some parameters has been taken for disc as shown in Table 1. The distribution of tangential stresses in anisotropic whisker reinforced discs for presence and absence of thermal gradients has been shown in figure 1. An anisotropic disc with thermal gradients has little higher the tangential near the inner radius and slightly lowers near the outer radius in disc with absence of thermal gradients. The variation of radial in anisotropic whisker reinforced discs for presence and absence of thermal gradients has been shown in figure 2. The change in the magnitude of radial stress distribution is very small in the anisotropic disc with/without thermal gradients. The distribution of tangential strain rate in anisotropic rotating discs along the radius in the disc for presence and absence of thermal gradients has been shown in figure 3. The magnitude due to thermal gradients is smaller compared to disc without thermal gradients. But, the trend of variation of tensile strain rate in tangential direction remains the same in the anisotropic discs with/without thermal gradients. In figure 4, the radial strain rate has been obtained in the anisotropic whisker reinforced discs with thermal gradients

and the results have been compared with those obtained for an anisotropic whisker reinforced discs without thermal gradients. By employing thermal gradients in anisotropic discs, the magnitude of radial strain rate can be reduced as compared to anisotropic discs without thermal gradients.

4. Conclusion

The effect of thermal gradients significantly affects the creep behavior in an anisotropic whisker reinforced rotating disc although the effect of thermal gradients on stresses is relatively small. So, this aspect which is requiring for safe designing of a rotating disc should be taken care of.

Table 1

| |
|---|
| Density of disc material $\rho = 2812.4 \text{ kg/m}^3$ |
| Angular velocity, $\omega = 15,000 \text{ rpm}$ |
| Inner radius of disc, $a = 31.75 \text{ mm}$ |
| Outer radius of disc, $b = 152.4 \text{ mm}$ |
| Particle size, $P = 1.7 \text{ }\mu\text{m}$ |
| Particle content, $V = 20\%$ |
| Operating temperature, $T = 623 \text{ K}$ |
| Creep parameters for whisker reinforced disc: $m = 53.0 \times 10^{-4} \text{ s}^{-1/8} / \text{MPa}$ and $\sigma_0 = 52.83 \text{ MPa}$ |
| The ratio of anisotropic constants, $G/F = 1.34$, $H/F = 1.64$ |

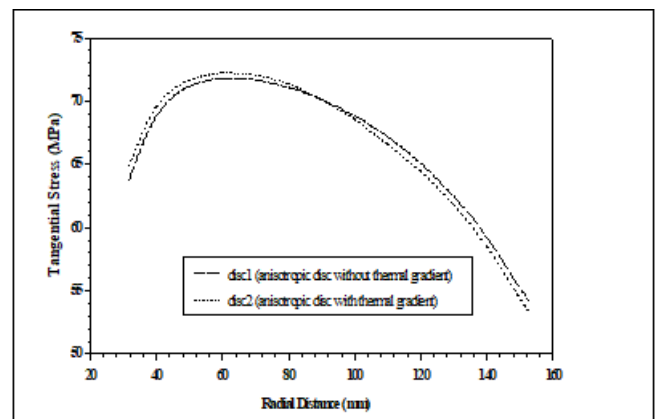


Figure 1: Variation of tangential stresses in anisotropic disc with/without thermal gradients.

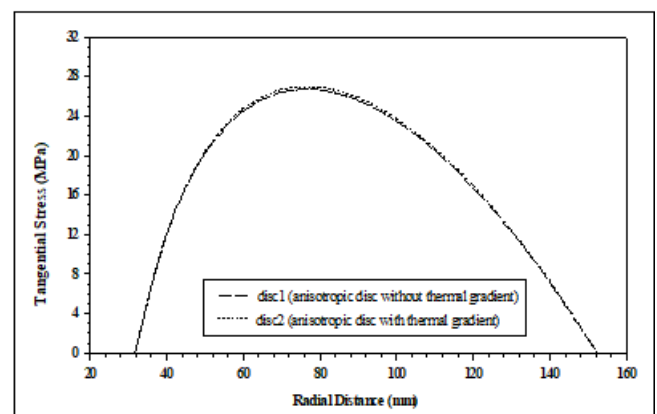


Figure 2: Variation of radial stresses in anisotropic disc with/without thermal gradients.

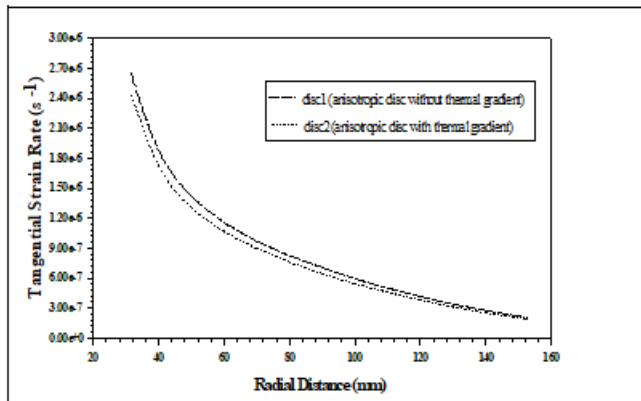


Figure 3: Variation of tangential strain rates in anisotropic disc with/without thermal gradients

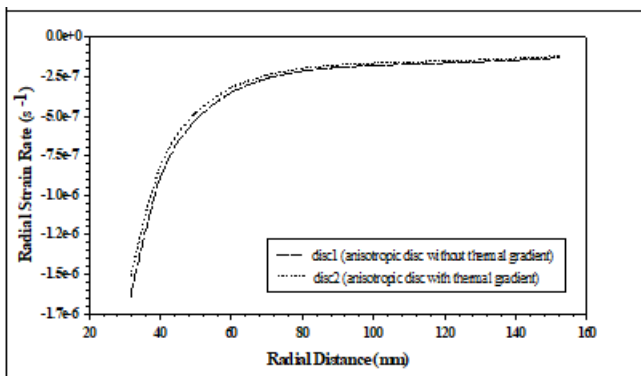


Figure 4: Variation of radial strain rates in anisotropic disc with/without thermal gradients.

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