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Preliminary Static Balancing of Rotating Masses Using MS Excel Solver

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Abstract: ***The balancing of rotating elements is an important condition for reducing the reactions that arose in their supports thus ensuring stable and long-term operation of the construction. In the case of massive composite rotating structures, it is prudent to pre-arrange the individual components before their welding or assembling in order to achieve maximum static balancing. This could facilitate the subsequent final bench balancing. The purpose of this study is to demonstrate how MS Excel Solver can be applied to achieve preliminary static balancing of rotating masses, specifically for arranging the blades of a Francis turbine runner, to minimize eccentricity.***

Keywords: static balancing, center of mass, Francis turbine, Excel Solver, assignment problem

1. Introduction

The goal of static balancing is to have the Center of Mass (CoM) of a rotating body (or material system) lain on the rotation axis. Theoretically, this would not allow an occurrence of a centrifugal inertial force during the rotation. In fact, if the CoM before the correction and the mass added for balancing are in different planes which are perpendicular to the rotation axis, then a torque of couple would arise during the rotation. The latter could be avoided using a dynamic balancing. Nevertheless, if in the beginning the CoM of the system is as close to the rotation axis as possible, then following balancing could be done more convenient.

The consideration here is about a problem which arises from the production of Francis turbine runners (Figure 1). The used blades have a complex shape and after their manufacturing it turns out that their masses are not quite equal. In addition, before welding (or other type of fixation), the blades are partially polished because of some areas on their surface become difficult to access (finally, the entire runner going to be polished). Ultimately, it could be found that some difference in the masses of the blades occurs. Therefore, it is reasonable, before their fixation, to arrange them in a circle in such a way that the static imbalance to be minimal. This requires no additional resources or effort, but would facilitate the subsequent additional balancing which could be carried out on a stand. One preliminary arrangement is done by the staff, but the author's observation shows that usually the worker tries to arrange the blades at his own estimation, without making any serious calculations. Of course, the worker's actions possess their logic and prevent a large imbalance, but they usually do not lead to an optimal distribution. The method proposed here solves this problem.

Figure 1: Runner of a small water turbine [1]

2. Methods

About the CoM of one material system, the following equation is known [2]–[4]:

$$M\vec{r}_C = \sum_{i=1}^{n} m_i \vec{r}_i; \quad M = \sum_{i=1}^{n} m_i \tag{1}$$

where: M – mass of the material system; $\vec{r}_C$ – radius-vector of the CoM of the same system, n - number of the pointlike masses (bodies) in the system; m_i – mass of a single point from the system; $\vec{r}_i$ – radius-vector of the same point. For convenience, it could be assumed that the origin of the mentioned vectors coincides with the origin of the coordinate system.

Like any vector equation, the left-hand expression (1) could be written relative to the coordinate system *Oxy* (assuming a plane case):

$$Mx_C = \sum_{i=1}^{n} m_i x_i; \quad My_C = \sum_{i=1}^{n} m_i y_i \qquad (2)$$

where: x_C and y_C are the Cartesian coordinates of the CoM; x_i and y_i are the Cartesian coordinates of a single pointlike mass from the system.

Let the both sides of the equations (2) being squared and then added together. The following expression is obtained:

$$M^2\left(x_C^2 + y_C^2\right) = \left(\sum_{i=1}^{n} m_i x_i\right)^2 + \left(\sum_{i=1}^{n} m_i y_i\right)^2 \qquad (3)$$

The right-hand side of this expression is entered into the Solver module and its minimum value is calculated (this is deep explained in the next section).

On the other hand, the numerical value of each vector (r_C here) is connected to its projections by the equation:

$$r_C^2 = x_C^2 + y_C^2 \qquad (4)$$

After one substitution of (4) into (3), an expression for finding the magnitude of the eccentricity (the numerical value of the radius-vector $\vec{r}_C$) is obtained:

$$r_C = \sqrt{\frac{\left(\sum_{i=1}^{n} m_i x_i\right)^2 + \left(\sum_{i=1}^{n} m_i y_i\right)^2}{M^2}} \qquad (5)$$

Since the numerator of the fraction has a minimum value acquired by Solver, the resulting eccentricity r_C is also minimized. The arrangement of the blades in this case is visualized in one corresponding array which is shown in the next section.

The above expressions are sufficient to solve the given problem, i.e. - for getting of an optimal arrangement of the blades in order to obtain minimum eccentricity r_C. However, one could additionally find the coordinates x_C and y_C of the specific CoM position by transformation of the expressions (2).

3. Calculations

The gist of the problem is how to arrange n number of masses (m_1, m_2 ... m_n) in the same number of positions which are distributed evenly in a circle. The goal is the CoM of such a system to be as close as possible to the center of this circle. To address this problem, it is convenient to use MS Excel Solver which is an add-ins for MS Excel. This is an affordable software with great capabilities [5]. Despite its specificity, the presented task could be classified as an "assignment problem" type [6]–[7].

One sample scheme of the studied material system is shown in Figure 2. A distribution of 6 masses ($n = 6$) is considered here because of more visualization convenience. The radius of the circle is assumed to be 1, since its value and unit of measurement have no impact on the optimal distribution of the masses.

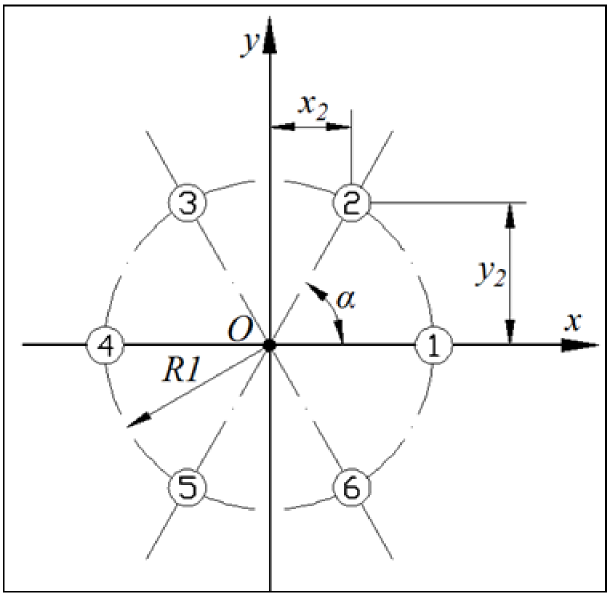

Figure 2: Scheme of the studied material system

First, some preliminary calculations are made (Figure 3). The number of rotating masses (in this case 6) was entered (by the user) in cell A2. The angle between them is calculated: B2 = 2*PI()/A2 (the angle α in Figure 2). For convenience only, in cell B3 = 360/A2 this angle is reckoned in degrees. In cells C2:C7, each of the mass positions (1 to 6) is numbered. In cells D2:E7, the x_i and y_i coordinates of each of these positions are computed: D2 = COS((C2-1)*$B$2) and E2 = SIN((C2-1)*$B$2) (in Figure 2 the x_2 and y_2 are depicted). In cells G2:G7, the values for each of the rotaing masses were entered (by the user). For the demonstration, it was assumed that the masses are in the range 10 ± 1 (the measurement unit of the masses is not relevant for the following calculations). The sum of the masses is calculated in cell H2 = SUM(G2:G7), this is the right-hand expression (1).

	A	B	C	D	E	F	G	H
1	Number of blades	Angle	Position №	X-axis	Y-axis		Masses	Total mass
2	6	1.05 rad	1	1.000	0.000		11.00	61.50
3		60.00°	2	0.500	0.866		9.00	
4			3	-0.500	0.866		10.00	
5			4	-1.000	0.000		10.25	
6			5	-0.500	-0.866		10.50	
7			6	0.500	-0.866		10.75	

Figure 3: Preliminary calculations

Next, a table is constructed in which each x_i coordinate is multiplied by each of the available masses m_i (Figure 4). For better visualization, the x_i coordinates are placed in cells B14:B19 (B14 = D2) and the masses m_i in cells C13:H13 (C13 = G2), respectively. The mentioned multiplication is performed in the array of cells C14:H19, where C14 = B14*C13 and H19 = B19*H13.

	A	B	C	D	E	F	G	H
12			Masses					
13			11.00	9.00	10.00	10.25	10.50	10.75
14	X-axis	1.000	11.00	9.00	10.00	10.25	10.50	10.75
15		0.500	5.50	4.50	5.00	5.13	5.25	5.38
16		-0.500	-5.50	-4.50	-5.00	-5.13	-5.25	-5.38
17		-1.000	-11.00	-9.00	-10.00	-10.25	-10.50	-10.75
18		-0.500	-5.50	-4.50	-5.00	-5.13	-5.25	-5.38
19		0.500	5.50	4.50	5.00	5.12	5.25	5.37

Figure 4: Array with all combinations $x_i m_i$

In the same manner, a similar table is constructed (Figure 5), in which the y_i coordinates (B26:B31) and the masses m_i (C25:H25) are multiplied. This is done in the array C26:H31, where the cell expressions are: C26 = B26*C25, H31 = B31*H25.

	A	B	C	D	E	F	G	H
24			Masses					
25			11.00	9.00	10.00	10.25	10.50	10.75
26	Y-axis	0.000	0.00	0.00	0.00	0.00	0.00	0.00
27		0.866	9.53	7.79	8.66	8.88	9.09	9.31
28		0.866	9.53	7.79	8.66	8.88	9.09	9.31
29		0.000	0.00	0.00	0.00	0.00	0.00	0.00
30		-0.866	-9.53	-7.79	-8.66	-8.88	-9.09	-9.31
31		-0.866	-9.53	-7.79	-8.66	-8.88	-9.09	-9.31

Figure 5: Array with all combinations $y_i m_i$

Then, one assignment table is built, where the optimal placement of the masses is calculated and displayed (Figure 6). For better visualization, the positions are placed in cells B38:B43 (B38 = C2), respectively - the masses in cells C37:H37 (C37 = G2). At first, the user may enter "0" in the array of cells C38:H43. In cells I38:I43, the values of this array are summed horizontally: I38 = SUM(C38:H38), and in cells C44:H44 – they are summed vertically: C44 = SUM(C38:C43). Crucial calculation is on bottom right: I44 = SUMPRODUCT(C14:H19,C38:H43)^2+SUMPRODUCT(C26:H31,C38:H43)^2. In this cell, the square value of the multiplied arrays from Figure 4 and Figure 6 is added to the square value of the multiplied arrays from Figure 5 and Figure 6, i.e. the right-hand side of expression (3) is applied.

	A	B	C	D	E	F	G	H	I
36			Masses						
37			11.00	9.00	10.00	10.25	10.50	10.75	
38	Position №	1	0	0	0	0	0	0	0
39		2	0	0	0	0	0	0	0
40		3	0	0	0	0	0	0	0
41		4	0	0	0	0	0	0	0
42		5	0	0	0	0	0	0	0
43		6	0	0	0	0	0	0	0
44			0	0	0	0	0	0	0.000

Figure 6: Initial view of the assignment table

Next, the Solver parameters are set (Figure 7). As *Objective*, the cell $I$44 is selected, searching for its minimum value. It has to be achieved through change of *Changing Variable Cells* $C$38:$H$43. Several *Constraints* are also set: the values in the array $C$38:$H$43 have to be integers and positive, also $C$44:$H$44 = 1 and $I$38:$I$43 = 1. These constraints ensure that each of the masses have to be indicated by "1" in the array $C$38:$H$43. One *Solving Method* is selected: GRG Nonlinear.

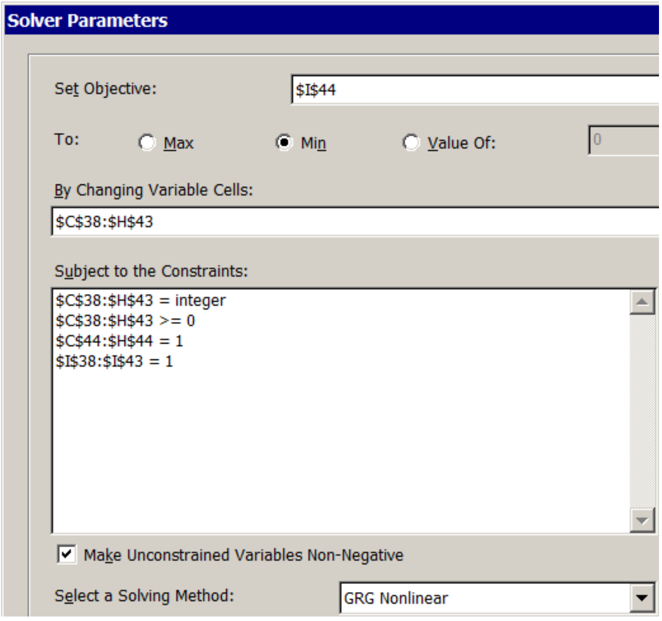

Figure 7: MS Excel Solver parameters menu

After starting the solving process, the solution to the problem appears (Figure 8). Here, it can be seen that the optimal arrangement derived is as follows (position/mass): 1/11.00, 2/10.25, 3/10.00, 4/10.50, 5/10.75, 6/9.00.

	A	B	C	D	E	F	G	H	I
36			Masses						
37			11.00	9.00	10.00	10.25	10.50	10.75	
38	Position №	1	1	0	0	0	0	0	1
39		2	0	0	0	1	0	0	1
40		3	0	0	1	0	0	0	1
41		4	0	0	0	0	1	0	1
42		5	0	0	0	0	0	1	1
43		6	0	1	0	0	0	0	1
44			1	1	1	1	1	1	0.250

Figure 8: Arrangement of the blades (masses)

Additionally, as it is illustrated in Figure 9, the eccentricity of the material system and the coordinates of the CoM could be calculated (although they are outside the scope of the task). For the first one, equation (5) is used, written in cell F49 = SQRT(I44/H2^2). To determine the x-axis of the CoM, the left-hand expression (2) is used, applied in cell F51 = SUMPRODUCT(C14:H19,C38:H43)/H2. Similarly, for the y-axis of the CoM, the right-hand expression (2) is applied in cell F53 = SUMPRODUCT (C26:H31,C38:H43)/H2.

	A–E	F–G
49	Eccentricity of the system:	0.00813
50		
51	X-axis of the center of mass:	-0.00407
52		
53	Y-axis of the center of mass:	0.00704

Figure 9: Eccentricity of the system and CoM coordinates

4. Results and Discussion

Typically, the runners have many more than 6 blades (for example, the runner in Figure 1 has 17). The calculating of the blades placement using MS Excel Solver is convenient. But, as the number of blades (masses) increases, the calculation logically starts to slow down, as the software analyzes more variants to find the optimal one. Since runners have an even distribution of the positions in a circle, one of the masses could be arbitrary fixed to any of the positions in advance (before the calculation). In this way, the work of MS Excel Solver could be accelerated. For example, let the mass 10.00 be fixed to position 1. This could be done by adding the following constraint in the MS Excel Solver parameters menu in Figure 7: $E$38 = 1. Thus, the following distribution (position/mass) is obtained more quickly: 1/10.00, 2/10.25, 3/11.00, 4/9.00, 5/10.75, 6/10.50. In this case, as it is expected, the eccentricity has the same minimum value: 0.00813.

The above means that there are several suitable mass distribution solutions which result in minimal eccentricity. Since they are equivalent as a practical application, the method under consideration here does not aim to describe all of them, but to find one of them which to be in use.

An analysis could be made about how useful the considered here preliminary arrangement is. If in the MS Excel Solver parameters menu (Figure 7) instead of “To: Min”, the choice is “To: Max”, the following distribution (position/mass) is obtained: 1/9.00, 2/10.25, 3/10.75, 4/11.00, 5/10.50, 6/10.00 and eccentricity of 0.04126. If the last value is compared to the minimum one found above (0.00813), the difference is more than fivefold. Hence, one random arrangement of the blades may lead to a large imbalance.

It should also be noted that here blades are considered as pointlike masses. With such an assumption, the CoM of each blade should be ideally coincide. But, since the mass of the blades differs, there is a probability that one fluctuation could exist in the position of the CoM for different blades which should slightly change their radius-vectors from the origin. In working conditions, a mensuration of the mass of blades is quick and easy, while an estimation of the position of their CoM is difficult. Therefore, in this preliminary balancing, only the mass of blades is taken into account (the same is done in practice, but the worker distributes the blades according to his subjective judgment).

5. Conclusions

The proposed method for preliminary static balancing allows one rapid optimal placement of blades achieving minimal eccentricity. In this case, the method was developed to support the fabrication of Francis water turbine runners, but it could be used elsewhere. In some cases, with a proper placement of blades (masses), a complete static balancing could be practically achieved.

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Author Profile

Chavdar Ognyanov Pashinski received the MS degree in Special Mechanical and Instrument Engineering (Small Arms) from Technical University of Sofia, Branch Plovdiv in 2002 and started working in the industry (PVD coatings, hydropower plants). He defended the PhD degree in Physical Sciences (Structure, Mechanical and Thermal Properties of the Condensed Matter) from the Central Laboratory of Applied Physics – Bulgarian Academy of Sciences in 2013 and began educational and scientific work in the two mentioned institutions. His interests comprised: PVD equipment and coatings, solid mechanics, hydraulic and pneumatic drives, small arms and robotics.