

An Exploratory Observation on Representing Even Numbers Using the Largest Prime Less than the Number

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Abstract: *This paper examines an observational approach related to the Strong Goldbach Conjecture, which states that every even integer greater than 2 can be expressed as the sum of two prime numbers or prime and 1. The study explores a heuristic method based on selecting the largest prime number less than a given even integer and analyzing the difference between the two values. Several numerical examples are presented to illustrate this pattern. The results show that, in many cases, subtracting the largest prime less than the given even number yields another prime, supporting known empirical behavior. However, the approach does not constitute a formal proof and remains a computational observation. The study highlights limitations related to identifying large primes and suggests directions for further analytical refinement.*

Keywords: Goldbach Strong Conjecture, Prime numbers, Even integers, number theory, heuristic methods

1. Introduction

There are so many conjectures about primes, which has not proven yet. Goldbach conjecture is a very simple and interesting property of primes, unfortunately it is also unsolved mystery for mathematicians yet. Mathematician Christian Goldbach discussed about it first time in a letter which is written by Christian Goldbach to Euler in 1742. The formal statement of Goldbach conjecture is “**every even integer greater than 2 can be expressed as the sum of two prime numbers**” (considering 1 as prime, which was prime that time). Today, primes are defined as integers greater than 1. Therefore, the modern version states even integers greater than 2 are the sum of two odd primes or sum of odd prime and 1. Leonard Euler replied that the strong version (sum of two primes for even numbers) was equivalent and likely true, though he couldn't prove it because prime numbers become more sparse as numbers get larger, but they are distributed irregularly. The conjecture connects addition (summing to n) with multiplication (the definition of primes), making it notoriously difficult. We will observe a way to write even numbers as two primes or prime and 1 in this paper.

For example:

- 1) 4 is an even number which can be written as the sum of 1 and 3 or 2 and 2.
- 2) 6 is an even number and written as sum of 1 and 5 or 3 and 3.
- 3) $8 = 7 + 1 = 5 + 3$
- 4) $10 = 7 + 3 = 5 + 5$
- 5) $12 = 11 + 1 = 7 + 5$ and so on...

2. Methodology Review

Statement: Mathematician Goldbach states that “every even number can be written as sum of two primes or sum of a prime and 1.

For necessarily: We know that the sum of two odd positive integers is always an even number then at the basis of this we consider odd prime numbers, nothing their sums which sum of two odd prime numbers or one prime and 1 are always an even positive integer.

$$\text{i.e. } P' + P'' = 2N$$

where N is any positive integer \geq or = to 2 and P' is greatest odd prime less than 2N, and P'' is any prime number or 1.

For sufficient: Let N is any positive integer \geq or = to 2 and P' is odd prime positive integer which is greatest prime $< 2N$.

Then we have to prove that P'' is also a prime positive integer or 1.

Now 2N is any even integer from $N = 1, 2, 3, 4, 5, 6, 7, 8, 9, \dots$

So, we can write that

$$2N = P' + P''. \text{ Means } 2N - P' = P'' \quad (1)$$

From here we should know that P' is greatest prime which is less than 2N.

Now, using a trial method;

For example: First step putting $N=2$ in equation (1)

Then $2N=4$ and $P' = 3$ because 3 is greatest prime less than 4. So, equation (1): $2N - P' = P''$; putting values in equation $4 - 3 = P''$, we get $P'' = 1$

Hence, we say that 4 is even number which can be represent in form of addition of prime number 3 and 1.

Second step putting $N = 3$, then $2N = 6$, and $P' = 5$ because 5 is greatest prime less than 6.

Putting in equation (1): $2N - P' = P''$; $6 - 5 = P''$ implies that $P'' = 1$.

Hence, we say that 6 is even number which can be written as addition of prime number 5 and 1.

Third step putting $N = 5$, then $2N = 10$, and $P' = 7$ because 7 is greatest prime number less than 10.

Putting in equation (1), we get $2N - P' = P''$ implies that $10 - 7 = P''$.

Hence, we say that 10 is even number which can be written as sum of primes 7 and 3.

Fourth step putting $N = 14$, then $2N = 28$ and $P' = 23$; because 23 is the greatest prime less than 28. Now putting in equation (1): $2N - P' = P''$ implies that $28 - 23 = P''$, thus we get $P'' = 5$.

Hence, we say that 28 is even number which can be written as sum of primes 23 and 5.

Fifth step putting $N = 48$, then $2N = 96$, we have $P' = 89$ because 89 is the greatest prime less than 96. Putting in equation (1): $2N - P' = P''$ implies that $96 - 89 = P''$, we get $P'' = 7$.

Hence, we say that 96 is even number which can be written as sum of primes 89 and 7.

More other representation: -

$P' + P'' = 2N$ where $2N$ is the sum of greatest prime (P') and prime or one is 1 (P'') of them

$7 + 1 = 8$ where 8 is the sum of greatest prime $7 < 8$ and 1

$11 + 1 = 12$ where 12 is the sum of greatest prime $11 < 12$ and 1

$13 + 1 = 14$ where 14 is the sum of greatest prime $13 < 14$ and 1

$13 + 3 = 16$ where 16 is the sum of greatest prime $13 < 16$ and 3

$17 + 1 = 18$ where 18 is the sum of greatest prime $17 < 18$ and 1

$19 + 1 = 20$ where 20 is the sum of greatest prime $19 < 20$ and 1

$19 + 3 = 22$ where 22 is the sum of greatest primes $19 < 22$ and 3

$23 + 1 = 24$ where 24 is the sum of greatest prime $23 < 24$ and 1

$23 + 3 = 26$ where 26 is the sum of primes 23 and 3

$29 + 1 = 30$ where 30 is the sum of prime 29 and 1

$31 + 1 = 32$ where 32 is the sum of prime 31 and 1

$31 + 3 = 34$ where 34 is the sum of primes 31 and 3

$31 + 5 = 36$ where 36 is the sum of primes 31 and 5

$37 + 1 = 38$ where 38 is the sum of prime 37 and 1

$37 + 3 = 40$ where 40 is the sum of primes 37 and 3

$41 + 1 = 42$ where 42 is the sum of prime 41 and 1

$41 + 3 = 44$ where 44 is the sum of primes 41 and 3

$89 + 3 = 92$ where 92 is the sum of primes 89 and 3

$97 + 3 = 100$ where 100 is the sum of primes 97 and 3

$103 + 1 = 104$ where 104 is the sum of prime 103 and 1

$149 + 1 = 150$ where 150 is the sum of prime 149 and 1

$199 + 1 = 200$ where 200 is the sum of prime 199 and 1

$499 + 1 = 500$ where 500 is the sum of prime 499 and 1

$997 + 3 = 1000$ where 1000 is the sum of primes 997 and 3, and so on.

Hence, we can say that any two primes addition makes a even number. i.e. every even number can be written as sum of a prime number or 1.

3. Literature Review

(i) Proof based on partitions of a new formulation of a set of even numbers: Sankai, Daniel, Loyford Njagi, and Josphine Mutembei introduced a new formulation of the form $E_{ij} = n_i + n_j + (n_j - n_i)^n$ or all^n belongs to N . By demonstrating that there exist at least a pair of prime numbers

in these odd partitions, the fact that the sum of any two prime numbers is even and there exists infinitely many prime numbers.

4. Analysis

As we see that we can write any even number in sum of primes or 1. Where one prime is greatest prime less than $2N$, when we consider larger even number or written as sum of primes or 1 as we tends to find a greatest prime number less than $2N$, here the problem creates that how to find that prime because we have no sequence or other method for finding a prime at any position in numbers.

5. Conclusion

This study presents an observational method for expressing even integers as sums involving the largest prime less than the number. The examples suggest that subtracting this prime often yields another prime, aligning with known computational evidence for the Strong Goldbach Conjecture. However, the method does not provide a formal proof and is limited by the lack of a general argument and the difficulty of identifying large primes efficiently. Future work should focus on developing a theoretical justification or probabilistic framework to support these observation

References

- [1] Christian Goldbach letter to L. Euler, 1742.
- [2] Sankai, Daniel, Loyford Njagi, and Josphine Mutembei 2024 "A detailed proof of the strong Goldbach conjecture based on partitions of a new formulation of a set of even numbers" (2024).