Analyses of Creep behavior in Composite Disc with Thickness Variation

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Abstract: ***This study focuses on analyzing the impact of thermal gradients on plastic stress patterns and creep deformation in a rotating isotropic disc with variable hyperbolic thickness subjected to thermal gradients. The findings reveal that thermal residual stresses significantly affect both stresses and strain-rates behavior in functionally graded material discs. Therefore, accounting for thermal gradients is essential for improving performance and ensuring optimal structural design.***

Keywords: Composites, Creep, functionally graded material, Thermal gradients

1. Introduction

Rotating discs are important mechanical components widely employed in various engineering applications, including automotive systems, aircraft engines, compressors, turbine rotors, and flywheels. In many industrial applications, these discs operate under high thermal gradients and elevated rotational speeds. Such demanding working conditions generate significant stresses within the disc material, making creep analysis an important area of research. Considering these factors, the present study investigates the creep behaviour of a functionally graded material (FGM) rotating disc under conditions with and without thermal gradients.

2. Analysis

The constitutive equations for creep are described under multiaxial stress, are expressed,

$$\dot{\varepsilon}_r = \frac{\dot{\bar{\varepsilon}} \times \left[x(r) - \frac{\left(H/F\right)}{G/F + H/F} \right] \left(\frac{G}{F} + \frac{H}{F} \right)}{\left[x(r)^2 - 2\frac{\left(H/F\right)}{G/F + H/F} x(r) + 1 \right]^{1/2}} = \frac{d\dot{u}_r}{dr} \quad (4.1)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\bar{\varepsilon}} \times \left[\left(1 + \frac{H}{F}\right) - \frac{H}{F} x(r) \right]}{\left[x^2 - 2\frac{\left(H/F\right)}{G/F + H/F} x(r) + \frac{\left(1 + H/F\right)}{G/F + H/F} \right]^{1/2}} = \frac{\dot{u}_r}{r} \quad (4.2)$$

$$\dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}} \left\{ -G\sigma_r - F\sigma_\theta \right\} \quad (4.3)$$

The effective strain rate of disc is assumed, as described by following Sherby's law [1977],

$$\dot{\bar{\varepsilon}} = \left[M\left(\bar{\sigma} - \sigma_0 \right) \right]^n \quad (4.4)$$

where $\dot{\bar{\varepsilon}}, M, \bar{\sigma}, \sigma_0, n$ are the effective strain rate, creep parameter, effective stress, threshold stress, the stress exponent.

Dividing Eq. (4.1) by Eq. (4.2) and integrating the resulting equation by taking limit a to r on both sides,

$$\frac{\dot{u}_r}{\dot{u}_{r_i}} = \exp \int_a^r \frac{\phi(r)}{r} dr \quad (4.5)$$

where, $x(r) = \frac{\sigma_r}{\sigma_\theta}$, is the ratio of radial and tangential stresses at any radius and $\dot{u}_r = du/dt$, is the radial deformation rate and $\dot{u}_{r_i}$, is the radial deformation rate at the inner radius.

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The equilibrium force in the radials direction is,

$$\frac{d}{dr}\left(r\, h\, \sigma_r \right) + \rho \omega^2 r^2 h = h\sigma_\theta \quad (4.6)$$

Boundary Conditions are $\sigma_r(a) = 0 = \sigma_r(b)$ (4.7)

Dividing Eq. (4.5) by r and equated to Eq. (4.2),

$$\frac{\bar{\sigma} - \sigma_0}{\psi(r)} = \frac{\left(\dot{u}_{r_i}\right)^{1/n}}{M} \quad (4.8)$$

Where,

$$\psi(r) = \left\{ \frac{\left[x(r)^2 - \frac{2\left(H/F\right)}{G/F + H/F} x(r) + \frac{1 + H/F}{G/F + H/F} \right]^{1/2}}{\frac{1 + H/F}{\sqrt{G/F + H/F}} - \frac{H/F}{\sqrt{G/F + H/F}} x(r)} \frac{\dot{u}_r}{r.\dot{u}_{r_i}} \right\}^{1n} \quad (4.9)$$

and

$$M(r) = e^{-35.38} P^{0.2077} T(r)^{4.98} V^{-0.622} \quad (4.10)$$

$$\sigma_0(r) = -0.03507 P + 0.01057\, T(r) + 1.00536 - 2.11916 \quad (4.11)$$

Simplify Eq. (4.8) into Eq. (4.1) to get the tangential stress (σ_θ),

$$\sigma_\theta = \frac{\bar{\sigma} - \sigma_0}{\psi(r)} \psi_1(r) + \psi_2(r) \quad (4.12)$$

where,

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$$\psi_1(r) = \frac{\psi(r)}{\left\{\left[x(r)^2 - 2\frac{H/F}{G/F + H/F}x(r) + \frac{1 + H/F}{G/F + H/F}\right]\right\}^{1/2}} \quad (4.13)$$

$$\psi_2(r) = \frac{\sigma_0}{\left\{\left[x(r)^2 - 2\frac{H/F}{G/F + H/F}x(r) + \frac{1 + H/F}{G/F + H/F}\right]\right\}^{1/2}} \quad (4.14)$$

$$\sigma_{\theta_{avg}} = \frac{1}{b-a}\int_a^b \sigma_\theta \, dr \quad (4.15)$$

where, $\sigma_{\theta_{avg}}$ is the average tangential stress.

Now radial stress can be expressed by integrating Eq. (4.6),

$$\sigma_r(r) = \frac{1}{r.h}\left[\int_a^r \sigma_\theta \, dr - \frac{\omega^2 \rho (r^3 - a^3)}{3}\right] \quad (4.16)$$

For a disc, the thermal conductivity can be calculated as,

$$K(r) = \frac{[100 - V(r)]K_m + V(r)K_d}{100} \quad (4.17)$$

Where $K_m = 247\, W/m\, K$ is matrix conductivity and $K_d = 100\, W/m\, K$ is diserpersoid conductivity.

$$T(r) = 619.69 + 0.6083\ r - 0.0208\ r^2 + 3.27\times10^{-4} r^3 - 1.96\times1(\quad (4.18)$$

where $T(r)$ is the temperature taken from Gupta et al.[4].

3. Results and Discussion

The study has been taken out for anisotropic FGM discs to analysis the consequence of thermal gradients on plastic stress distributions and strain rates and computer program has been made for this study. This study has been done for the anisotropic FGM discs containing silicon carbide whisker in a matrix of pure aluminum in presence of thermal gradients and obtained results are compared for both the discs in absence of thermal gradients to analyze the effect of thermal gradients. The distribution of tangential stresses in anisotropic whisker reinforced discs for presence and absence of thermal gradients has been shown in figure 1. An anisotropic disc with thermal gradients has little higher the tangential near the inner radius and slightly lowers near the outer radius in disc with absence of thermal gradients. The variation of radial in anisotropic whisker reinforced discs for presence and absence of thermal gradients has been shown in figure 2. The change in the magnitude of radial stress distribution is very small in the anisotropic disc with/without thermal gradients. The distribution of tangential strain rate in anisotropic rotating discs along the radius in the disc for presence and absence of thermal gradients has

Table 1

Density of disc material $\rho = 2812.4\, kg/m^3$

Angular velocity, $\omega = 15{,}000\ rpm$

Inner radius of disc, $a = 31.75mm$

Outer radius of disc, $b = 152.4mm$

Particle size, $P = 1.7\ \mu m$

Particle content, $V = 20\%$

Operating temperature, $T = 623\ K$

Creep parameters for whisker reinforced disc:

$m = 53.0\times10^{-4}\ s^{-1/8}/MPa$ and $\sigma_0 = 52.83$ MPa

The ratio of anisotropic constants,

$G/F = 1.34,\ H/F = 1.64$

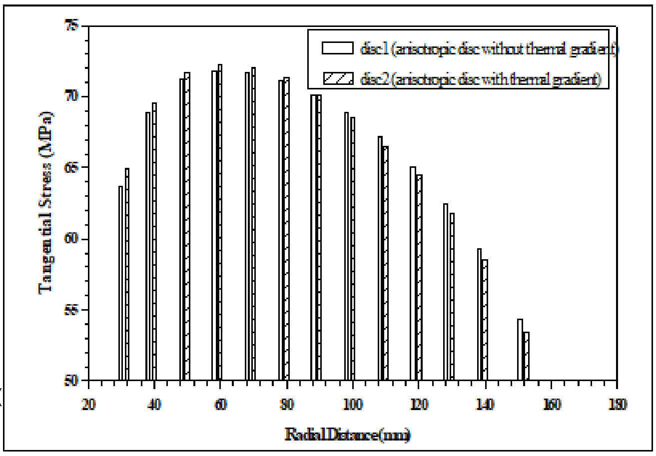

Figure 1: Variation of tangential stresses in anisotropic disc with/without thermal gradients

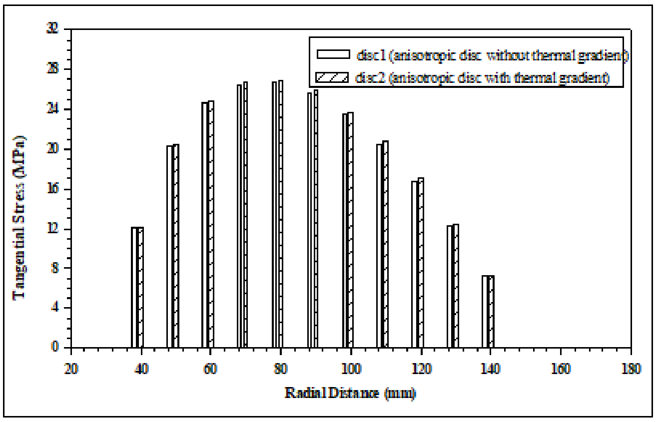

Figure 2: Variation of radial stresses in anisotropic disc with/without thermal gradients.

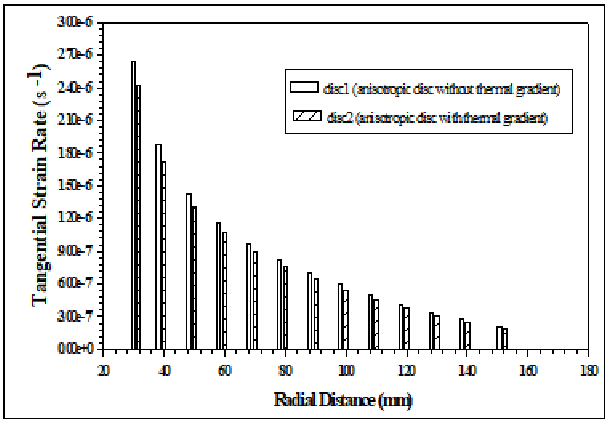

Figure 3: Variation of tangential strain rates in anisotropic disc with/without thermal gradients.

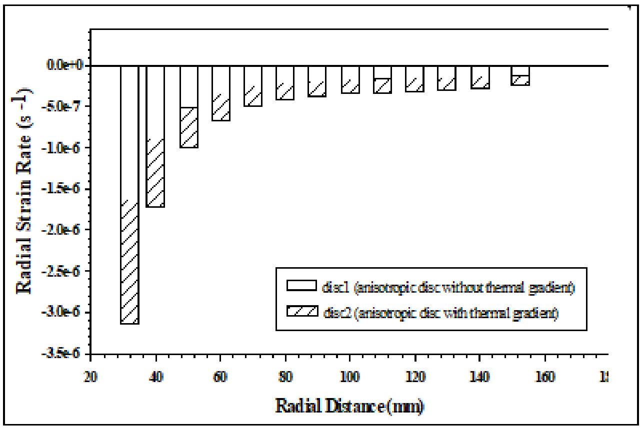

Figure 4: Variation of radial strain rates in anisotropic disc with/without thermal gradients

been shown in figure 3. The magnitude due to thermal gradients is smaller compared to disc without thermal gradients. But, the trend of variation of tensile strain rate in tangential direction remains the same in the anisotropic discs with/without thermal gradients. In figure 4, the radial strain rate has been obtained in the anisotropic whisker reinforced discs with thermal gradients and the results have been compared with those obtained for an anisotropic whisker reinforced discs without thermal gradients. By employing thermal gradients in anisotropic discs, the magnitude of radial strain rate can be reduced as compared to anisotropic discs without thermal gradients.

4. Conclusion

The effect of thermal gradients significantly affects the creep behavior in an anisotropic whisker reinforced rotating disc although the effect of thermal gradients on stresses is relatively small. So, this aspect which is requiring for safe designing of a rotating disc should be taken care of.

References

[1] Bhatnagar, N.S., Kulkarni, P.S. and Arya, V.K. [1986] “Steady state creep of orthotropic rotating discs of variable thickness” *Nuclear Engineering and Design* 91(2), 121-141.

[2] Callioglu, H., Demir, E. and Sayer, M. [2011] “Thermal stress analysis of functionally graded rotating discs”, *Scientific Research and Essays* 6(6), 3437 - 3446.

[3] Garg, M., Salaria, B. S. and Gupta, V. K. [2012] “Analysis of steady-state creep in a functionally graded rotating disc of variable thickness”, *Composites: Mechanics, Computations, Applications, an International Journal* 3(2), 171-188.

[4] Gupta, V.K., Singh, S.B., Chandrawat, H.N. and Ray, S. [2005] “Modeling of creep behaviour of a rotating disc in presence of both composition and thermal gradients”, *Journal of Engineering Materials and Technology* 127(1), 97-105.

[5] Ma, B.M. [1960] “Creep analysis for rotating solid discs of variable thickness”, *Journal Franklin Institute* 269(5), 408-419.

[6] Mishra, R.S. and Pandey, A.B. [1990] “Some observations on the high-temperature creep behaviour of 6061 Al-SiC composites”, *Metallurgical Transactions*, 21A(7), 2089-2091.

[7] Pandey, A.B., Mishra, R.S. and Mahajan, Y.R. [1992] “Steady state creep behavior of silicon carbide particulate reinforced aluminium composites” *Acta Metallurgica Materialia* 40(8), 2045-2052.

[8] Sherby, O.D., Klundt, R.H. and Miller, A. K. [1977] “Flow stress, subgrain size and subgrain stability at elevated temperature”, *Metallurgical Transactions* 8A, 843-850.

[9] Singh, S.B. and Ray, S. [2001] “Steady-state creep behavior in an isotropic functionally graded material rotating disc of Al-SiC composite”, *Metallurgical Transactions* 32A(7), 1679-1685.

[10] Von Mises, R. [1913] “Mechanics of solids in the plastically deformable state”, *NASA, Technical Memorandom 88488*, 1986.

[11] Wahl, A.M., Sankey, G.O., Manjoine, M.J. and Shoemaker, E. [1954] “Creep tests of rotating risks at elevated temperature and comparison with theory” *Journal of Applied Mechanics* 76, 225-235.