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Modeling the Relationship Between Maximum Subsidence Angle and Coal Seam Dip Angle for Predicting Ground Movement in Underground Mining

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Abstract: ***This study develops regression-based models to predict the maximum subsidence angle as a function of coal seam dip angle for underground coal mining in the Quang Ninh coal basin, Vietnam. Data from representative mines were analyzed by separating seam dip conditions below and above 45 degrees. Five regression models were evaluated, including linear, logarithmic, polynomial, power, and exponential functions. Comparative accuracy assessment using Root Mean Squared Error showed that second-order polynomial regression provided the best predictive performance in both dip-angle groups, with RMSE values of approximately 0.72 degrees for dip angles below 45 degrees and 0.64 degrees for dip angles above 45 degrees. The developed models provide a practical tool for improving subsidence prediction, safety pillar design, and hazard zoning in underground mining operations.***

Keywords: Underground mining; Ground subsidence; Subsidence prediction; Coal seam dip angle; Regression modeling; Rock mass deformation; Mining geomechanics.

1. Introduction

In underground coal mining, the movement and deformation of the rock strata surrounding the extraction area are inevitable processes. When the goaf (mined-out area) is not completely filled or thoroughly treated, these displacements propagate toward the surface, forming a subsidence basin and causing various ground deformation phenomena. Accurate forecasting of movement parameters is not only a technical requirement but also a pivotal task for risk control, the protection of civil structures and technical infrastructure, and ensuring the safety of the mining production process itself.

Among the predicted parameters, the maximum subsidence angle (θ) plays a particularly vital role. It is a directional quantity used to determine the location of the maximum subsidence point and the overall geometry of the subsidence basin. Empirical studies have demonstrated that the maximum subsidence angle is not a fixed constant but varies complexly, directly influenced by geological factors and mining technology, with the coal seam dip angle (δ) being the most significant factor.

The history of rock mechanics and mining engineering has recorded various approaches to solving the problem of displacement prediction. According to the synthesis by [1], these methods can be categorized into five main groups:

1) **Analytical Method:** Based on the laws of elasticity, plasticity, and rock mechanics to establish mathematical models that explain displacement mechanisms. The advantage of this method lies in its rigorous logic; however, it often faces practical application challenges due to the assumption that rock masses are homogeneous and continuous, a condition rarely found in nature.
2) **Approximation Function Method:** Utilizes mathematical functions (such as probability or exponential functions) to fit actual subsidence curves obtained from monitoring data. While highly practical, this method requires a sufficiently large volume of observational data and is often site-specific to a particular mine.
3) **Influence Function Method:** Built on the principle of superposition, this method establishes the relationship between small-scale mining units and overall surface deformation. It is highly effective for forecasting in areas with geological conditions similar to previously monitored sites.
4) **Physical Modeling Method (Equivalent Materials):** Simulates geological structures using materials with corresponding mechanical properties at a specific scale. This method allows for the visual observation of fracturing and subsidence processes but involves high costs and lengthy preparation times.
5) **Numerical Modeling Method:** Leveraged by computer processing, software (such as FLAC, Plaxis, Phase2) enables the simulation of complex relationships between movement parameters and geo-technical mining conditions [2]. However, the accuracy of numerical models depends entirely on the selection of the constitutive model and input parameters.

The Quang Ninh coal basin is the largest underground mining region in Vietnam, characterized by extremely complex hydrogeological and engineering geological conditions. Coal seams here exhibit significant variations in dip angles, ranging from gently inclined to vertical. Applying movement parameters from other countries or mining regions to Quang Ninh often results in substantial errors, complicating the design of safety pillars and environmental impact assessments.

Despite numerous studies, establishing a specific quantitative relationship between the maximum subsidence angle and the coal seam dip angle tailored to the specific characteristics of Quang Ninh mines remains limited. Addressing this reality, this paper focuses on establishing mathematical regression equations to predict the maximum subsidence angle. Research data is synthesized and standardized from actual mining operations at representative mines such as Mao Khe, Vang Danh, Ha Lam, and Mong Duong. The results of this study are expected to provide a rapid, highly reliable forecasting tool, contributing to the precision of rock movement management in the Quang Ninh coal basin.

2. Methods

The seam dip angle (δ) is a critical parameter for predicting ground subsidence, ranging from 0° to 90°. However, the inclination of the seam is classified into different levels depending on the specific research objectives. Seams with a dip angle from 0° to 35° are categorized as gently inclined seams, those from 36° to 54° are inclined seams, and those from 55° to 90° are classified as steeply dipping seams.

To predict surface subsidence using approximation or influence functions, it is essential to establish the relationship between the maximum subsidence angle (θ) and the seam dip angle (δ) across these varying inclinations [3]. This relationship is expressed through different equations depending on the specific location and geological conditions of the coal seam.

Based on the synthesized results and analysis of the relationship between the maximum subsidence angle and the seam dip angle at several Chinese coal mines, [2] categorized the findings according to two types of seam inclinations as follows:

For seams with a dip angle less than or equal to 45°, the relationship between the maximum subsidence angle and the coal seam dip angle as expressed in Equation (1).

$$\theta = 90^\circ - 0.68\delta \qquad (1)$$

For seams with a dip angle greater than 45°, the relationship between the maximum subsidence angle and the coal seam dip angle as expressed in Equation (2).

$$\theta = 28.8^\circ + 0.68\delta \qquad (2)$$

Equations (1) and (2) are widely utilized for predicting ground surface subsidence in Chinese coal mines.

Similar research findings regarding the relationship between the maximum subsidence angle and the seam dip angle have also been studied in several other countries. In Poland, mining scientists have proposed a formula representing the relationship between the maximum subsidence angle and the seam dip angle, applicable under the condition where δ≤ 60° as expressed in Equation (3).

$$\theta = 90^\circ - 0.67\delta \qquad (3)$$

In the United Kingdom, mining scientists have proposed a quadratic formula representing the relationship between the maximum subsidence angle and the seam dip angle [4], applicable under the condition where δ≤ 30° as expressed in Equation (4).

$$\theta = 90^\circ - 1.333\delta + 0.0148\,\delta^2 \qquad (4)$$

For the Donets Basin in Ukraine, the maximum subsidence angle is calculated [5] using the empirical formula (5):

$$\theta = 90^\circ - \delta \qquad (5)$$

In Vietnam, the determination of the maximum subsidence angle is calculated [6], [7] according to formula (6):

$$\theta = 90^\circ - K.\,\delta \qquad (6)$$

Where K is a coefficient determined by the ratio K = h/H, in which h represents the overburden thickness and H is the mining depth.

Research results from various countries worldwide indicate a close correlation between the maximum subsidence angle and the seam dip angle.

Based on the formulas representing the relationship between the maximum subsidence angle and the seam dip angle in several countries, it is observed that under certain conditions, when the seam dip angle is 0° or 90°, the maximum subsidence angle equals 90°. Furthermore, when the seam dip angle is 45°, the subsidence angle reaches its minimum value.

By synthesizing both cases where the seam dip angle varies from 0° to 90°, the relationship between the maximum subsidence angle and the seam dip angle has been established in the form of a sine function, as expressed in Equation (7):

$$\theta = 90^\circ - A.(\sin 2\delta)^2 \qquad (7)$$

Where A is a coefficient that depends on the geological conditions of the study area. Based on data collected from Chinese coal mines, the coefficient was determined to be A = 28.5°; consequently, expression (7) can be rewritten as formula (8):

$$\theta = 90^\circ - 28.5^\circ(\sin 2\delta)^2 \qquad (8)$$

The aforementioned analyses demonstrate a clear correlation between the maximum subsidence angle and the coal seam dip angle. As this relationship is expressed through various types of equations, establishing a precise mathematical link is essential for accurately predicting the maximum subsidence angle based on the known inclination of the coal seam.

3. Results

Based on research findings regarding the relationship between the maximum subsidence angle and the coal seam dip angle in various countries, it is evident that the equations representing this link vary significantly depending on the specific geological conditions of each study area. To establish a representative relationship for the geological conditions of the Quang Ninh coal basin, data on maximum subsidence angles and seam dip angles categorized into cases less than or equal to 45° and greater than 45° have been collected, as shown in Table 1.

Table 1: Data on dip angles and maximum subsidence angles at selected coal seams in the Quang Ninh region [8]

Case of seam dip angles less than 45°			Case of seam dip angles greater than 45°		
No	Seam dip angle (degree)	Maximum subsidence angle (degree)	No	Seam dip angle (degree)	Maximum subsidence angle (degree)
1	12	78	13	45	39
2	15	74	14	49	45
3	16	72	15	54	55
4	22	60	16	57	60
5	23	59	17	63	69
6	24	57	18	65	72
7	25	56	19	69	76
8	26	55	20	72	78
9	28	53	21	75	81
10	30	51			
11	37	46			
12	39	44			

Based on these data, Excel software was used to construct a chart representing the relationship between the maximum subsidence angle and the seam dip angle, from which regression equations were established for five different function types: linear, logarithmic, polynomial, power, and exponential. Figure 1 illustrates the regression equations for cases where the seam dip angle is less than 45°. To evaluate the accuracy of these equations, 10 data pairs from Table 1 were used for the regression analysis, while the 11th and 12th pairs were excluded from the regression to serve as validation. Subsequently, the established regression equations were utilized to predict the maximum subsidence angle based on known seam dip angles.

The regression equations representing the relationship between the maximum subsidence angle and the coal seam dip angle for cases where the dip angle is less than 45° are as follows:

1) Linear function regression equation:

$$\theta = 96.459 - 1.5843\delta; R^2 = 0.9727$$

2) Logarithmic function regression equation.

$$\theta = 158.31 - 31.658\ln(\delta); R^2 = 0.9904$$

3) Polynomial function regression equation.

$$\theta = 110.78 - 3.04828\delta + 0.0348\delta^2; R^2 = 0.9940$$

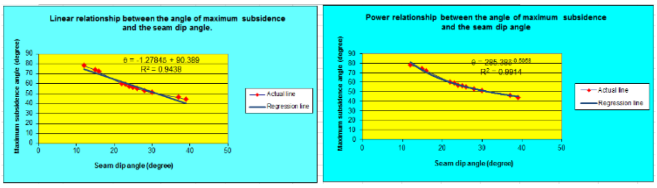
4) Power function regression equation.

$$\theta = 275.78\delta^{-0.4941}; R^2 = 0.9868$$

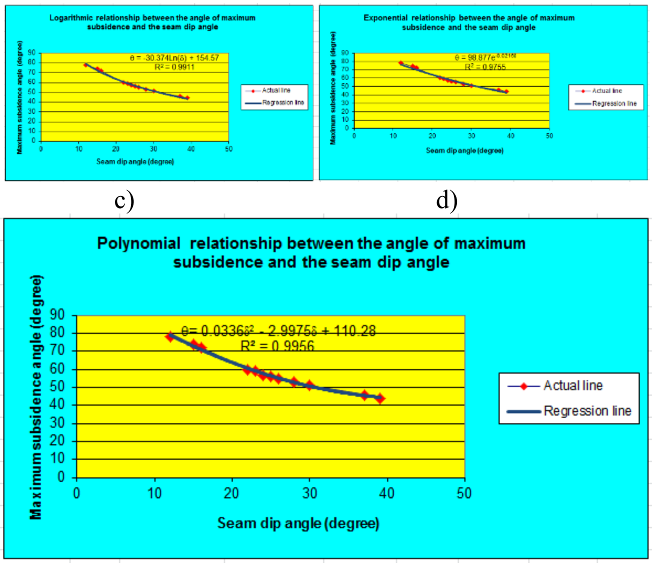
5) Exponential function regression equation.

$$\theta = 106.11e^{-0.02518\delta}; R^2 = 0.9905$$

The relationship curve (Figure 1) indicates that for seam dips below 45°, an increase in the dip angle results in a decrease in the angle of maximum subsidence.



a) b)



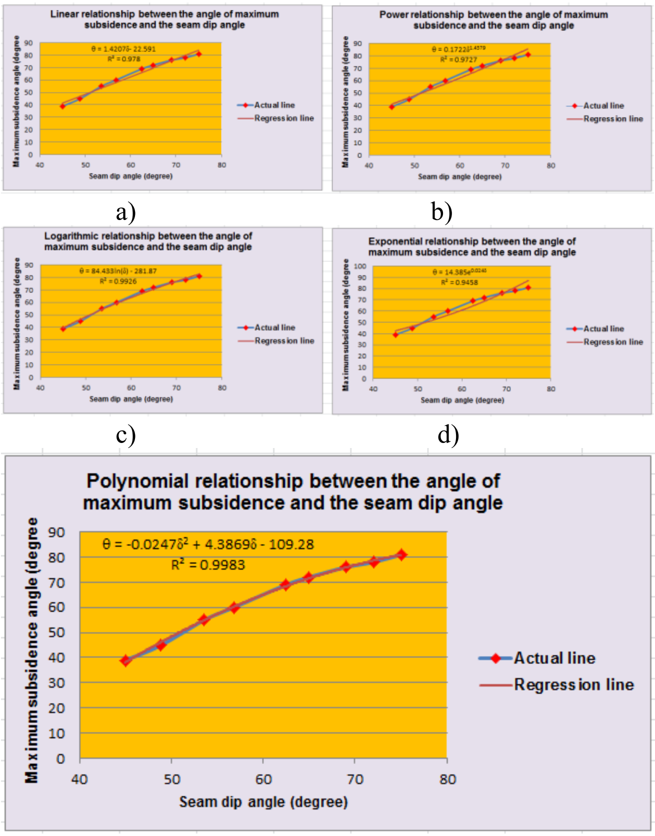
c) d)

e)

Figure 1: Relationship between the maximum subsidence angle and the seam dip angle (for dip angles less than 45°)

In cases where the coal seam dip angle greater than 45°, the relationship between the maximum subsidence angle and the seam dip angle as shown in Figure 2.

According to the relationship between the maximum subsidence angle and the coal seam dip angle for cases where the dip angle exceeds 45°, as the dip angle of the seam increases, the maximum subsidence angle also increases.



a) b)

c) d)

e)

Figure 2: Relationship between the maximum subsidence angle and the seam dip angle (for dip angle greater than 45°).

In figures 1, 2, a) Linear relationship; b) Power relationship
c) Logarithmic relationship; d) Exponential relationship;
e) Polynomial relationship.

According to the graph showing the relationship between the maximum subsidence angle and the coal seam dip angle for angles exceeding 45° (Figure 2), as the seam dip angle increases, the maximum subsidence angle also increases.

For coal seam dip angles greater than 45°, the regression equations between the maximum subsidence angle and the seam dip angle take the following forms:

1) Linear function regression equation:
$$\theta = 1.4207 - 22.591\delta; R^2 = 0.9780$$
2) Logarithmic function regression equation.
$$\theta = 84.433\ln(\delta) - 281.87; R^2 = 0.9926$$
3) Polynomial function regression equation.
$$\theta = -0.0247\delta^2 + 4.8369\delta - 109.28; R^2 = 0.9983$$
4) Power function regression equation.
$$\theta = 0.1722\delta^{1.4379}; R^2 = 0.927$$
5) Exponential function regression equation.
$$\theta = 14.385e^{0.024\delta}; R^2 = 0.9458$$

The correlation coefficient R in the above equations is used to determine the strength of the relationship in the regression model and as expressed in Equation (9).

$$R^2 = \frac{n.\sum_{i=1}^{n}(y_i^{'})^2 - (\sum_{i=1}^{n} y_i^{'})^2}{n.\sum_{i=1}^{n}(y_i)^2 - (\sum_{i=1}^{n} y_i)^2} \qquad (9)$$

Where:
n - number of sampled pairs (or sample size).
y_i' - the ith predicted value calculated from the regression equation.
y_i - the ith actual value obtained through empirical sampling.

To evaluate the forecasting accuracy of the regression equation, in addition to the correlation coefficient R, it is necessary to calculate the Root Mean Squared Error (RMSE) of the predicted values using the following formula (10):

$$RMSE = \pm\sqrt{\frac{[\Delta_i^2]}{n-1}} \qquad (10)$$

Where: Δ_i is the deviation between the actual value and the ith predicted value.

Based on the established regression equations representing the relationship between the maximum subsidence angle and the seam dip angle, the maximum subsidence angle can be predicted for both cases: seam dip angles less than 45° (Table 2) and greater than 45° (Table 3)

Table 2: Predicting the maximum subsidence angle based on coal seam dip for dip angles less than 45°

No	Seam dip angle (degree)	Actual subsidence angle (degree)	Predicted subsidence angle based on regression functions (degree)					Discrepancy between actual and regression-based values (degree)				
			Linear	Loga	Poly	Power	Exp	Linear	Loga	Poly	Power	Exp
1	12	78	75.0	79.1	79.1	81.2	76.3	3.0	-1.1	-1.1	-3.2	1.7
2	15	74	71.2	72.3	72.9	72.5	71.5	2.8	1.7	1.1	1.5	2.5
3	16	72	69.9	70.4	70.9	70.2	70.0	2.1	1.6	1.1	1.8	2.0
4	22	60	62.3	60.7	60.6	59.8	61.5	-2.3	-0.7	-0.6	0.2	-1.5
5	23	59	61.0	59.3	59.1	58.4	60.2	-2.0	-0.3	-0.1	0.6	-1.2
6	24	57	59.7	58.0	57.7	57.2	58.9	-2.7	-1.0	-0.7	-0.2	-1.9
7	25	56	58.4	56.8	56.3	56.0	57.6	-2.4	-0.8	-0.3	0.0	-1.6
8	26	55	57.2	55.6	55.1	54.9	56.4	-2.2	-0.6	-0.1	0.1	-1.4
9	28	53	54.6	53.4	52.7	52.9	54.0	-1.6	-0.4	0.3	0.1	-1.0
10	30	51	52.0	51.3	50.6	51.1	51.7	-1.0	-0.3	0.4	-0.1	-0.7
11	37	46	37.8	44.0	45.6	46.3	41.9	8.2	2.0	0.4	-0.3	4.1
12	39	44	34.7	42.3	44.8	45.1	39.9	9.3	1.7	-0.8	-1.1	4.1
							RMSE:	**±4.32**	**±1.22**	**±0.72**	**±1.26**	**±2.34**

Table 3: Predicting the maximum subsidence angle based on coal seam dip for dip angles greater than 45°

No	Seam dip angle (degree)	Actual subsidence angle (degree)	Predicted subsidence angle based on regression functions (degree)					Discrepancy between actual and regression-based values (degree)				
			Linear	Loga	Poly	Power	Exp	Linear	Loga	Poly	Power	Exp
1	45	39	41.3	39.5	38.1	41.0	42.4	-2.3	-0.5	0.9	-2.0	-3.4
2	49	45	46.7	46.4	46.0	46.1	46.4	-1.9	-1.6	-1.2	-1.3	-1.6
3	54	55	53.6	54.3	54.9	52.8	52.1	1.4	0.7	0.1	2.2	2.9
4	57	60	58.1	59.2	60.2	57.4	56.2	1.9	0.8	-0.2	2.6	3.8
5	63	69	66.2	67.3	68.4	65.8	64.5	2.8	1.7	0.6	3.2	4.5
6	65	72	69.8	70.6	71.5	69.6	68.5	2.2	1.4	0.5	2.4	3.5
7	69	76	75.4	75.6	75.8	75.9	75.4	0.6	0.4	0.2	0.1	0.6
8	72	78	79.7	79.2	78.5	80.7	81.0	-1.7	-1.2	-0.5	-2.7	-3.0
9	75	81	84.0	82.7	80.8	85.5	87.0	-3.0	-1.7	0.2	-4.5	-6.0
RMSE:								**±2.22**	**±1.29**	**±0.64**	**±2.76**	**±3.79**

4. Discussion

4.1 Analysis of the correlation between the maximum subsidence angle and the seam dip angle

The research results indicate a close relationship between the maximum subsidence angle (θ) and the dip angle of the coal seam (δ). Unlike horizontal coal seams, where the point of maximum subsidence typically coincides with the center of the mined-out area in inclined seams, the maximum subsidence point tends to shift towards the dip side.

As the seam dip angle δ increases, the gravitational component acting along the dip direction intensifies the sliding of the overlying strata. This phenomenon alters the morphology of the subsidence basin, causing the maximum subsidence angle θ to decrease gradually as δ increases. Observational data suggest that the correlation curve follows a linear or non-linear trend, depending on the geomechanical properties of the surrounding rock mass.

4.2. Establishment of mathematical models

The relationship between θ and δ is not static but is significantly governed by the geological structure. Through empirical analysis, this study has established high-reliability quadratic polynomial models to predict this relationship:

1) For seam dip angles less than 45°:

$$\theta = 110.78 - 3.0482\delta + 0.0348\delta^2; R^2 = 0.9940$$

2) For seam dip angles greater than 45°:

$$\theta = -0.0247\delta^2 + 4.8369\delta - 109.28; R^2 = 0.9983$$

4.3. Practical Significance in Mining Operations

Accurately determining the maximum subsidence angle θ based on the dip angle δ plays a vital role in mine planning and management:

- Identification of hazardous zones: It allows for the precise prediction of the location subjected to maximum surface subsidence. This enables the effective demarcation of safety zones to protect overlying infrastructure and residential areas.
- Optimization of safety pillars: It helps minimize coal resource loss by designing optimal safety pillars based on the precise calculation of the asymmetric subsidence basin boundaries.

4.4. Influencing Factors and Limitations

Although the relationship between θ and δ has been established, several marginal factors must be considered:

- Mining depth (H): As mining progresses to deeper levels, the influence of the dip angle on the surface may diminish due to self-compaction and the bulking characteristics of the caved rock mass.
- Pore water pressure: Fluctuations in pore water pressure within inclined seams can alter the frictional properties of the rock layers, potentially leading to unpredictable variations in the subsidence angle θ.

5. Conclusion

This study established empirical relationships between coal seam dip angle and maximum subsidence angle for underground coal mining conditions in the Quang Ninh coal basin. Among five tested regression models, the quadratic polynomial function provided the highest predictive accuracy for both seam dip categories, with strong goodness-of-fit and low RMSE values. The developed models offer a practical engineering tool for subsidence forecasting, mine planning, and surface hazard management. Future studies should validate the models using larger datasets and incorporate additional geological variables such as mining depth, stratigraphy, and hydrogeological conditions.

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