International Journal of Scientific Engineering and Research (IJSER)
www.ijser.in
ISSN (Online): 2347-3878
Volume 1 Issue 4, December 2013

Error Minimization for Faithful Reconstruction of Analog Signal

Adepoju Gafari Adepoju[1], Bisiriyu Akeem Olawale[2]

[1]Department of Electronic and Electrical Engineering, Ladoke Akintola University of Technology, P. M. B. 4000, Ogbomoso, Oyo State, Nigeria

[2]Department of Electrical and Electronic Engineering, Osun State Polytechnic, P.M.B. 301, Iree, Osun State, Nigeria

Abstract: ***Sampling is one of the important stages in transmitting analog signal using digital technique. It involves the assignment of the sampling frequency which determines how faithfully the sampled signal can represent the analog signal. A careful choice of the sampling frequency is very crucial especially to reduce loss of information .In this study, MATLAB program was used to sample a 5-V,50H-z sinusoidal signal at different sampling frequencies of 225Hz, 275Hz, 100Hz, 125Hz, 1025Hz and 1075Hz. Also, from the sampled signals, the analog signal was reconstructed and the error of reconstruction was checked in order to determine the accuracy at each sampling frequency. It is discovered that the error of reconstruction decreases as the sampling frequency increases. At the sampling frequency of 1025Hz, the error was 4.0856 x10^{-14}, which is approximately zero. It implies that sampling above the Nyquist rate is very crucial to faithful representation and hence, reconstruction of the analog signal.***

Keywords: sampling frequency, reconstruction, error, analog.

1. Introduction

It is the interest of this paper to study the sampling of analog signal at some frequencies; to study if at each frequency, there is a good representation of the analog signal and hence,to study if there is going to be a faithful reconstruction of the analog signal from the samples taken at each sampling frequency.

Advancement in digital technology has made discrete-time systems inexpensive, light- weight, programmable and easily reproducible. Hence, processing of discrete-time signal is preferred even though we have a large number of continuous-time signals in the modern world [1]. Analog signals are otherwise referred to as continuous-time signals. To process analog signal by digital technique, there is the need to convert it into digital signal through three steps of sampling, quantization and binary coding as shown in fig 1.

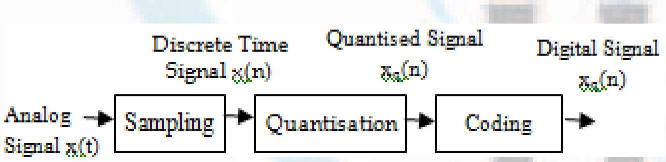


Figure 1: Stages in producing digital signal from analog signal

Quantisation is the conversion of a discrete-time continuous valued signal into a discrete-time, discrete valued digital signal. Digital signal values are a finite set of possible values).Coding implies that each discrete value is represented by a b-bit binary sequence [2].

1.1 Sampling

Sampling is the process of obtaining a discrete-time signal from a continuous-time signal at regular time-intervals. The time interval is the sampling period. That is, sampling is a bridge between continuous-time and discrete-time signal [3]. Fig.2 (b) shows a sampled version of a continuous-time signal in fig.2 (a). The samples are equally spaced at the sampling period, Ts [4].

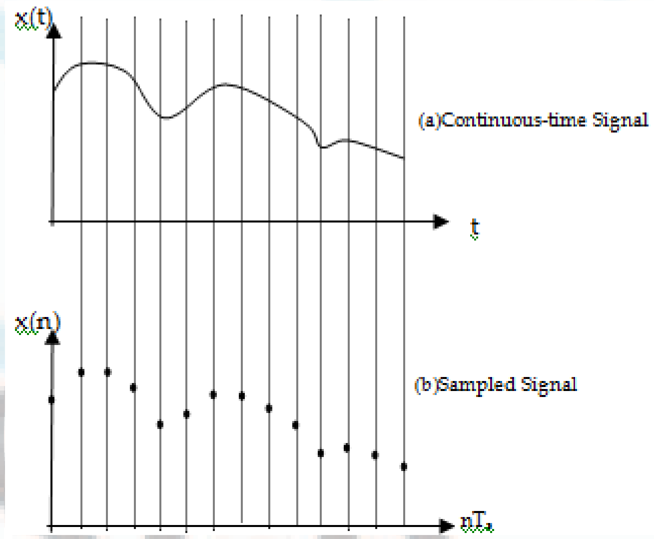


Figure 2: Continuous-time signal and Sampled Signal

Continuous-time sinusoidal signal can be expressed as [2],

$$x(t) = A\sin(2\pi f t) \quad -\infty < t < \infty \qquad 1$$

Where, A is the amplitude of the signal and f is the frequency in Hz ,

On the other hand, discrete-time sinusoidal signal may be expressed as

$$x(n) = A\sin(2\pi F n) \quad -\infty < n < \infty \qquad 2$$

Where, n : integer variable
A : the amplitude of the signal
F is the frequency in cycles per sample.
Hence, a continuous-time signal is defined for all values of

time within an infinite interval of time while a discrete-time signal is a sequence of signal values defined in discrete points of time [5]. The relationship between F and f is such that,

$$F = \frac{f}{f_s} \qquad 3$$

Where, f_s is the sampling frequency in Hz. The conversion of x(t) to x(n) is achieved by a continuous-time to discrete-time converter [6].

B. Relationship between x(t) and x(n)

Equation (4) shows the relationship between the variable t of analog signal and the variable n of discrete-time signal is,

$$t = \frac{n}{f_s} \quad or \qquad 4$$

$$t = nT$$

The fundamental difference between analog signal and discrete-time signal is the frequency range. The highest frequency in continuous-time signal is infinite while that in the discrete-time signal is $\frac{f_s}{2}$ [2].

2. Material and Methods

Interpolation refers to the process of estimating the unknown data values for specific locations using the known data values for other points. Some of the methods of estimation are:

(i) Zero-order-hold interpolation
(ii) Nearest-neighbour interpolation
(iii) Linear interpolation
(iv) Cubic-order-hold (COH) interpolation
(v) Sinc function interpolation

In the zero-order hold interpolation, a given sample value is held for the sample interval until the next sample is received. The resulting signal is a staircase waveform which requires an appropriately designed analog post-filter for accurate waveform reconstruction [4].

The nearest-neighbor interpolation also results in a stair-case like reconstruction as in zero-order hold but with discontinuities at the mid-points between sample points instead of at the sample points [4]. In linear interpolation, the reconstruction is a continuous function that just connects the sample values with straight lines.

The cubic-order-hold interpolation uses spline interpolation for a smoother estimate of the analog signal between samples. Hence this interpolation does not require an analog post-filter

A. Sinc Interpolation

Sinc interpolation was adopted in this study. It corresponds to the impulse response of a linear time-invariant ideal low-pass filter .It interpolates a continuous function that passes through the uniformly-spaced data samples filter [7]. MATLAB has provision for implementing sinc function. For a sinc function, the implementation expression is as in equation (6).

$$x_r(t) = \sum_{n_1=-\infty}^{n_2=\infty} x(n) \sin c(f_s(t - nT_s)) \qquad 6$$

However, the interpolation was done in the finite range n_1=0 and n_2=102 as in equation (7).

$$x_r(m\Delta t) = \sum_{n=n_1}^{n=n_2} x(n) \sin c(f_s(m\Delta t - nT_s)) \qquad 7$$

The time grid was done in the range of $0 \le m\Delta t \le 0.1$ which was equivalent to five times the period of the analog signal. Using MATLAB software, a continuous-time signal $x(i) = 5\ Sin\ (2\pi 50t)$ was sampled at a variable sampling frequency f_s of 225,275,100,125,1025 and 1075Hz to produce $x(n)$ .It is determined if there will be a faithful representation of x(t) by its samples at each of the varying sampling frequency. Hence, the reconstruction was carried out with the error determined in each case.

Fig.4 and 5 show the flow chart for the plotting the sampled and the reconstructed signals.

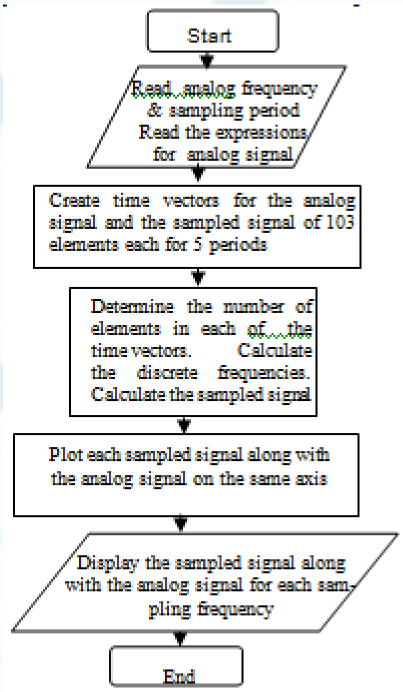


Figure 4: Flow chart to plot the sampled signals

ISSN (Online): 2347-3878
Volume 1 Issue 4, December 2013

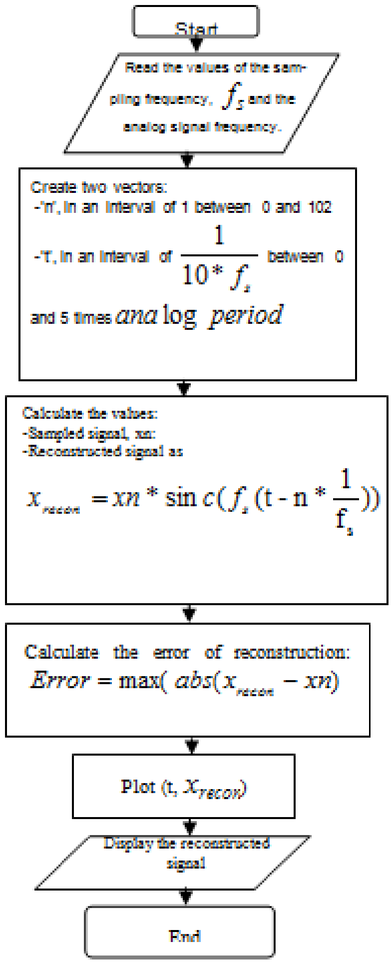


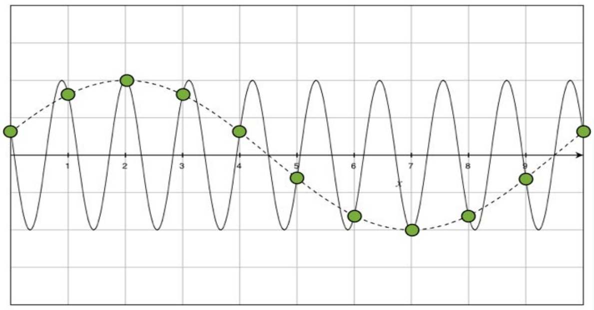
Figure 5: Flow chart to plot the reconstructed signals

3. Theory/Calculation

Uniform sampling corresponds to taking samples of continuous-time signals at equally spaced time Intervals. On the other hand, non-uniform sampling often arises in time-interleaved where a signal is passed through multiple parallel channels, each uniformly sampling the signal at the same rate. The output samples of the channels are then multiplexed to obtain a full discrete-time representation of the signal [7]

A. Inadequate Sampling of analog signal

Aliasing is a phenomenon that can occur due to improper sampling, where the original signal is indistinguishable when reconstructing it from its samples. This usually occurs because the sampling rate is inadequate [8].In fig.3, the solid line shows the analog signal of 0.9 Hz frequency that is being sampled at a sampling rate of 1 Hz. The dots show the samples of the 0.9 Hz signal. In reconstructing the signal from its samples, the dashed line signal of frequency 0.1 Hz is produced which is not an accurate representation of the input signal [9].



Source: Neil, 2012

Figure 3: Illustrating distortion caused by aliasing

In essence, aliasing happens whenever an analog signal is not sampled at a high enough frequency.

B. Reconstructing Continuous-time signal from the Sampled Signal

In reconstructing a continuous-time signal from discrete-time samples, the knowledge of the sampling rate is very important. Spectra analysis by Fourier transform has shown that the spectrum of a signal sampled yields the original spectrum replicated in the frequency domain at the sampling frequency. In essence, the sampling operation has left the original input spectrum intact, merely replicating it periodically in the frequency domain with a spacing of the sampling frequency. This implies that if $X_{sampled}(f)$ denotes the Fourier transform of the sampled signal) while X(f) denotes the transform of the original signal, then $X_{sampled}(f)$ is the sum of the two spectra components as shown in equation (5).

$X_{sampled}(f) = X(f) + X_{High}(f)$ eq. 5

Where $X_{High}(f)$ is the replicated versions of X (f) that constitute harmonic versions of the original signal.

C. Reconstruction Conditions

There are two conditions to be fulfilled as follows:

1. Band-limiting of the analog signal before it is sampled. This involves removing high frequency spectra components. It requires the use of an ideal low-pass filter.
2. The sampling frequency f_s must be greater than twice the maximum frequency f_{max}, present in the signal. This minimum sampling frequency, known as the Nyquist rate, is the minimum distance between the spectra copies, each with bandwidth f_{max}. Nyquist sampling theorem provides a description for the nominal sampling rate to avoid distortion due to aliasing [8].

It says that, in order to ensure a faithful reconstruction, the original signal must be sampled at an appropriate rate as described in the sampling theorem [10].A continuous-time signal can be reconstructed exactly from its samples if the samples are taken at a sampling rate that is greater than twice the maximum frequency [3].n practice, sampling is usually higher to provide some margin and make the filtering requirements less critical [11].In addition, interpolation needs to be done to estimate the value of the signal between samples [4].

4. Results and Observations

Fig. 6, fig.7 and fig.8 show the result obtained when an analog signal $x(t) = 5\ Sin\ (2\pi 50t)$ was sampled at sampling frequencies 225Hz, 275Hz, 100Hz, 125Hz, 1025Hz,and 1075Hz respectively. It is to be noted that, for clarity of explanation, the analog signal and the sampled signal are plotted on the same axes.

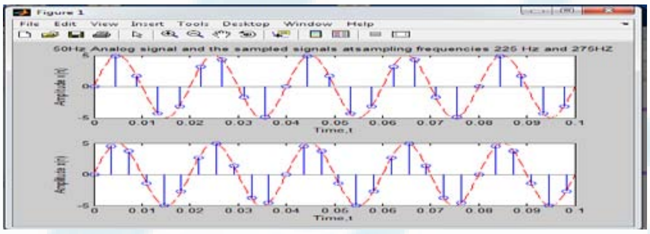


Figure 6: Sampled signal sampling frequencies of 225 Hz and 275Hz respectively

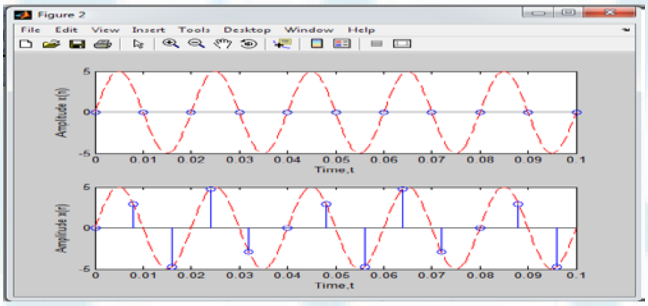


Figure 7: Sampled signal sampling frequencies of 100 Hz and 125Hz respectively

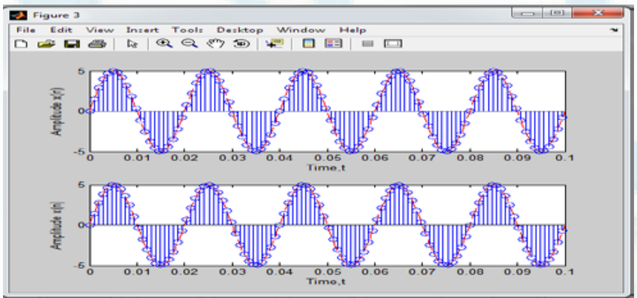


Figure 8: Sampled signal sampling frequencies of 1025 Hz and 1075Hz respectively

Fig.9 and fig.10 show the reconstructed analog signals from the samples at the sampling frequencies of 1025Hz and 275Hz respectively.

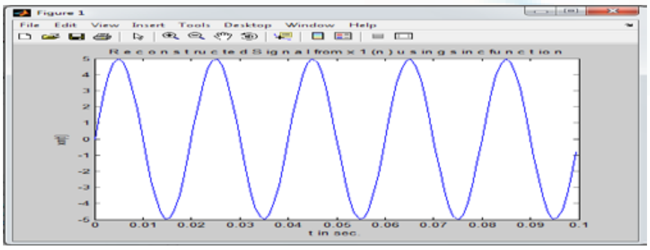


Figure 9: Reconstructed signal from the sampled signal at sampling frequency of 1025 Hz:Error of reconstruction= 4.0856 x10^{-14}

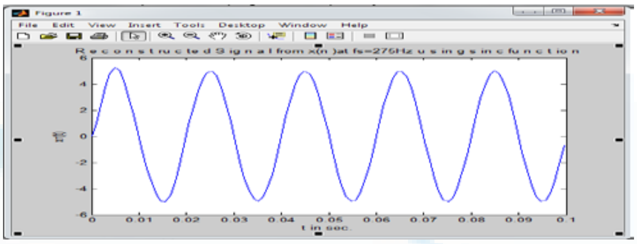


Figure 10: Reconstructed signal from the sampled signal at sampling frequency of 275 Hz: Error of reconstruction= 0.5145

Fig.11 shows the reconstructed signal from the sampled signal at the sampling frequency of 225Hz.Fig.12 and fig.13 show the reconstructed signal from the sampled signal at the sampling frequencies of 100Hz and 125Hz respectively.

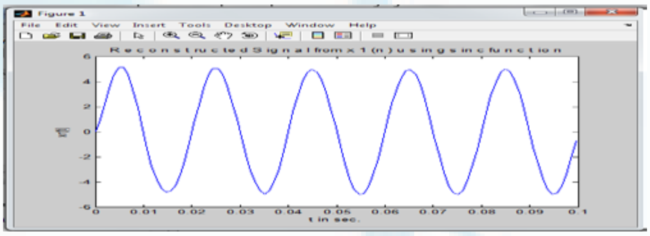


Figure 11: Reconstructed signal from the sampled signal at sampling frequency of 225 Hz: Error of reconstruction= 0.6893

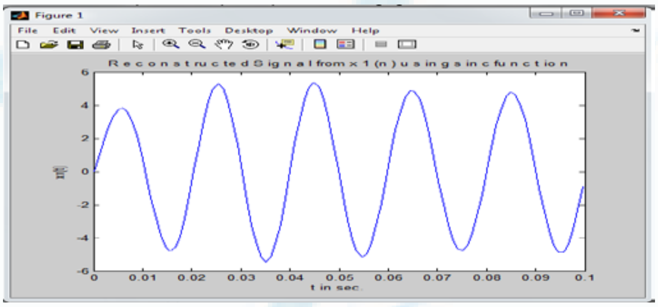


Figure 12: Reconstructed signal from the sam pled signal at sampling frequency of 125 Hz : Error of reconstruction=1.48

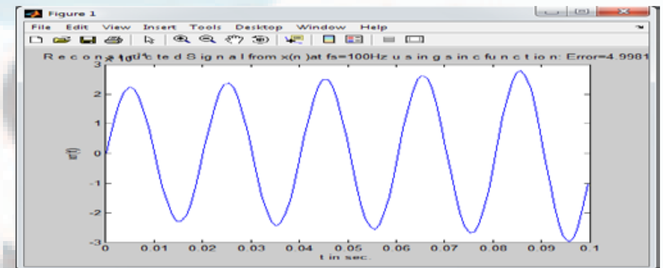


Figure 13: Reconstructed signal from the sampled signal at sampling frequency of 100 Hz: Error of reconstruction=4.9981

5. Observation

It will be observed that maximum error occurs at the sampling frequency of 100Hz which is twice the frequency of the analog signal that was sampled initially (i.e 50Hz). This is observed to be the Nyquist rate. Also, the error reduces as the sampling frequency increases.

6. Conclusion and Recommendation

Nyquist sampling theorem requires that, adequate representation and reconstruction is achieved when the sampling is done at more than twice the maximum frequency component in the analog signal. It is important to mention that the input signal contains one frequency only. The error of reconstruction decreases as the sampling frequency increased above the Nyquist sampling rate. It is to be emphasized that, for approximately zero error of reconstruction of analog signal, the sampling should be done at a frequency that is greater than the Nyquist rate. This avoids the distortion that may arise from reconstructing the analog from its samples.

References

[1] Sanjay,S.Communication Systems (Analog and Digital),S.K. Kataria & Sons Publisher of Engineering and Computer Books,4760-61/23, Ansari Road,Darya Ganj,New Delhi,2008.
[2] Aliyazicioglu, Z.Continuous-Time and Discrete-Time Signal Sampling of Analog Signals.Lecture Notes on ECE 308. Electrical and Computer Engineering Department.Cal Poly Pomona,2004
[3] Peter,C.Sampling & Discrete signals.Lecture Notes on E2.5 Signals &Linear Systems.Department of Electrical & Electronic Engineering Imperial College London,2011
[4] Sanjeev, R.K.Lecture Notes for ELE201 Introduction to Electrical Signals and Systems.Retrieved on 5th May, 2011 from http://www.princeton.edu/~cuff/ele201/kulkarni_text/dig it izn.pdf.2002.
[5] Finn, H.Discrete-time signals and systems.Tech Teach. Retrieved on 10th October, 2012 from http://techteach.no/publications/ discretetime_signals_ systems/discrete.pdf.2005.
[6] So, H.C.Sampling and Reconstruction of Analog Signals Retrieved from http://www.ee.cityu.edu.hk/~hcso/ ee3202_4.pdf, 2011.
[7] ShayM.Alan,V.O.Sinc Interpolation of Nonuniform Samples.Retrieved on 16th June,2011 from http://www.rle.mit.edu/dspg/documents/Maymon_Sinc_ Interpolation2011.pdf.2007
[8] Bruno, A.O. Aliasing.PSC 129-Sensory Processes. Retrieved on 8th March, 2011 from http://redwood.berkeley.edu/bruno /npb261/aliasing.pdf. 2000.
[9] Neil, M.Signal Processing: Aliasing. Retrieved on 9th June, 2013 from http://www.xyobalancer.com/_blog/XYO_Balancer_Blog/p o st/ Signal_Processing_Aliasing.2012.
[10] Amit, D.Digital Coding of Analog Signal. Electronics Engineering Department,Sardar Vallabhbhai National Institute of Technology,Surat-395007.2009 [11] Maxim Integrated,Filter Basics: Anti-Aliasing.Retrieved on 9th June 2012 from
[11] http://www.maximintegrated.com/appnotes/index.mvp/i d/928.2012.

Author Profile

Gafari Abiola ADEPOJU was born in Nigeria and received the B.Tech. Hons Degree in Electronic and Electrical Engineering from the Ladoke Akintola University of Technology (LAUTECH), Ogbomoso, M.Sc. degree in Electrical and Electronics Engineering from University of Lagos and Ph.D from LAUTECH, Ogbomoso, Nigeria. He is a Senior Lecturer in the Department of Electronic and Electrical Engineering, LAUTE LAUTECH, Ogbomoso, Nigeria. His research areas of interest are power system Analysis, Steady state and dynamic stability analyses of Power systems and Application of FACTS controllers in Power Transmission systems

Akeem Olawale BISIRIYU was born in Nigeria and received the B.Tech. Degree in Electronic and Electrical Engineering from the Ladoke Akintola University of Technology (LAUTECH), Ogbomoso, M.Eng. degree in Electrical and Electronics Engineering from University of Ilorin. Presently, he is a Ph.D student of LAUTECH, Ogbomoso, Nigeria. He is a Senior Lecturer in the Department of Electrical and Electronic Engineering of Osun State Polytechinic, Iree.Osun State of Nigeria. His research areas of interest are pattern recognition of moving object detection, tracking and classification.