

# Portfolio Rebalancing Model Using Fuzzy Optimization

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**Abstract:** *Uncertainty in stock market is a major issue for all the investor. Researchers have proposed portfolio selection as one of the measures to overcome uncertainty of stock market. The financial market is highly volatile and investor needs to change his/her investment strategy from time to time. Investor can construct portfolio using his/her own aspiration level with respect to return, dividend, risk, liquidity and transaction cost. In this paper portfolio is constructed and rebalanced using fuzzy optimization technique with user different aspiration level.*

**Keywords:** Portfolio selection, semi absolute deviation, fuzzy number, transaction cost, S-shape membership functions.

## 1. Introduction

Uncertainty in stock market is a prolonged issue and there is no exact solution to solve the uncertainty problem. The stock market is an uncertain, complex, dynamic and noisy system. Researchers have proposed portfolio selection as one of the measures to overcome complication and uncertainty of stock market. Portfolio selection is a challenging problem as stocks do not follow a predefined or steady pattern. Investors consider return and risk as the two fundamental factors. The financial market is highly volatile and investor needs to change his/her investment strategy from time to time. In this paper portfolio is rebalanced in the presence of dividend, transaction cost and fuzzy turnover rate. The portfolio is rebalanced after every year because in Indian Stock Market the profit earned after a year is exempted from tax liability. The equity market with 25 Stocks is considered for creating and rebalancing the portfolio.

## 2. Related Work

Harry Markowitz [1] in 1952 gave most significant theory for portfolio selection. The main principle of mean variance model was to use expected return of portfolio as investment return & variance as investment risk. Markowitz's mean variance model has lead to development of number of models. Konno and Yamazaki [2] proposed mean absolute model as an alternative to mean variance model keeping all positive features of mean variance model. Markowitz [1] developed Mean Variance optimization model which became a very common quantitative model in finance today by for constructing an optimal portfolio. The MV model allocates each asset in the portfolio a proportion of the investment amount by taking into consideration each asset's returns, risk and the correlations between the assets. Konno et.al [2] developed a linear programming model using Mean Absolute Deviation as risk function, thus replacing variance in Markowitz's MV model. The LP model was however equivalent to Markowitz's model when it possess a multivariate normal distribution of the asset returns. Markowitz [3] transformed the general mean-semi variance

portfolio optimization problem into a general mean-variance optimization problem. Konno et.al [4] used mean-variance objective function and extended it to include skewness in portfolio optimization problem. Konno et.al [5] utilized a variant of the MV model by imposing fixed transaction cost and cardinality constraints on problems with up to 54 assets. Speranza[6] proposed semi-absolute deviation to evaluate risk in portfolio selection model. S.C.Liu et.al [7] proposed mean-variance-skewness model for portfolio selection with transaction costs assuming that the transaction cost is a V-shaped function of the difference between the existing portfolio and a new one. Chen et. al. [8] proposed Portfolio optimization of equity mutual funds with fuzzy return rates and risks. Liu [9] has solved portfolio optimization problem using fuzzy technique. He represented asset returns by fuzzy data. Chen et.al [10] proposed mean-variance-skewness model for optimal portfolio selection in intuitionistic fuzzy environment. Firstly, membership and non-membership functions of object and constrain functions were defined. Secondly, intuitionistic fuzzy programming model was presented based on intuitionistic fuzzy "min-max" operator. Pankaj Gupta et. al[11]., morphed mean-variance optimization portfolio model into semi-absolute deviation model, They applied multi criteria decision making via fuzzy mathematical programming to develop comprehensive models of asset portfolio optimization for the investors' pursuing either of the aggressive or conservative strategies. Yong Fang et. al. [12], proposed a linear programming model for portfolio rebalancing with transaction cost, they illustrated the behavior of the proposed model using data from the Shanghai Stock Exchange.

## 3. Problem Formulation

It is assumed that an investor allocates his/her wealth among  $n$  assets. The investor starts with an existing portfolio and rebalances the portfolio while reallocating assets but does not invest additional capital during the rebalancing. It is assumed that  $x_i$  is the proportion of total funds invested in the  $i^{th}$  asset,  $\delta_i \in \{0,1\}$  indicates the absence or presence

of  $i^{th}$  assets in portfolio,  $r_i$  is expected rate of return of  $i^{th}$  asset without transaction costs is given by  $r_i = \frac{1}{T} \sum_{t=1}^T r_{it}$ ,  $d_i$  is expected dividend earned for  $i^{th}$  asset

is given by  $d_i = \frac{1}{T} \sum_{t=1}^T d_{it}$ , where  $r_{it}$  and  $d_{it}$  is determined

by historical data.  $x_i^+$  is the proportion of an asset  $i, i=1,2,\dots,n$  which is bought by the investor and  $x_i^-$  is the proportion of an asset  $i, i=1,2,\dots,n$  which is sold by the investor.

The transaction cost of an  $i^{th}$  asset can be expressed as  $p(x_i^+ + x_i^-)$ , where  $p$  is the rate of transaction costs of an asset. Therefore the total transaction costs can be expressed as  $\sum_{i=1}^n p(x_i^+ + x_i^-)$ . The investor does not invest the additional capital during the portfolio rebalancing process.

Thus

$$\sum_{i=1}^n (x_i^0 + x_i^+ - x_i^-) + \sum_{i=1}^n p(x_i^+ + x_i^-) = 1$$

where  $x_i^0$  is the proportion of an  $i^{th}$  asset owned by the investor at the time of portfolio creation. The expected net return on the portfolio after paying transaction costs is

$$\sum_{i=1}^n r_i (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-)$$

The expected net dividend earned on the portfolio can be expressed as

$$\sum_{i=1}^n d_i (x_i^0 + x_i^+ - x_i^-)$$

The semi-absolute deviation of return on the portfolio  $x = (x_1, x_2, \dots, x_n)$  over the past period  $t, t = 1, 2, \dots, T$  can be represented as

$$w_t(x) = \left| \min \left\{ 0, \sum_{i=1}^n (r_{it} - r_i) x_i \right\} \right| = \frac{\left| \sum_{i=1}^n (r_{it} - r_i) x_i \right| + \sum_{i=1}^n (r_i - r_{it}) x_i}{2} \max \left\{ \sum_{i=1}^n r_i (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-), \sum_{i=1}^n d_i (x_i^0 + x_i^+ - x_i^-) \right\}$$

where  $x_i = x_i^0 + x_i^+ - x_i^-$ . The expected semi-absolute deviation of the return of the portfolio  $x = (x_1, x_2, \dots, x_n)$  below the expected return can be represented as

$$w(x) = \frac{1}{T} \sum_{t=1}^T w_t(x) = \sum_{t=1}^T \frac{\left| \sum_{i=1}^n (r_{it} - r_i) x_i \right| + \sum_{i=1}^n (r_i - r_{it}) x_i}{2T}$$

where  $x_i = x_i^0 + x_i^+ - x_i^-$ ,  $w(x)$  is used to measure the portfolio risk.

In literature it has been observed that turnover rates of an asset is used to measure liquidity [10]. The turnover rate of an asset is the proportion of turnover volume to tradable volume of an asset, and is a factor that tells the liquidity of an asset. Therefore turnover rate has been used to measure liquidity. Turnover rates of an asset have been treated as fuzzy numbers as it cannot be predicted accurately. Possibility theory has been proposed by Zadeh [13] and advanced by Dubois and Prade [14] and fuzzy variables are associated with possibility distributions. Afterwards, the liquidity of an asset by the possibility distribution was approached by [11], [12]

A fuzzy number A is called trapezoidal with tolerance interval  $[a, b]$ , left width  $\alpha$  and right width  $\beta$  if its membership function takes the form:

$$A(t) = \begin{cases} 1 - \frac{a-t}{\alpha} & \text{if } a - \alpha \leq t \leq a \\ \frac{\alpha}{\alpha} & \text{if } a \leq t \leq b \\ 1 - \frac{t-b}{\beta} & \text{if } b \leq t \leq b + \beta \\ 0 & \text{Otherwise} \end{cases}$$

Let the trapezoidal fuzzy number  $\tilde{L}_i = (La, Lb, \alpha, \beta)$  denote turnover rate of the  $i^{th}$  asset. Then, the turnover rate of the

portfolio  $x = (x_1, x_2, \dots, x_n)$  is  $\sum_{i=1}^n \tilde{L}_i x_i$ . Using the fuzzy

extension principle [13], the crisp possibilistic mean value of the turnover rate of the portfolio  $x = (x_1, x_2, \dots, x_n)$  is given

as

$$E(\tilde{L}(x)) = E \left( \sum_{i=1}^n \tilde{L}_i x_i \right) = \sum_{j=1}^n \left( \frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6} \right) x_j$$

$$\max \left\{ \sum_{i=1}^n r_i (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-), \sum_{i=1}^n d_i (x_i^0 + x_i^+ - x_i^-) \right\}$$

$$\min \sum_{t=1}^T \frac{\left| \sum_{i=1}^n (r_{it} - r_i) x_i \right| + \sum_{i=1}^n (r_i - r_{it}) x_i}{2T}$$

Subject to

$$\sum_{j=1}^n \left( \frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6} \right) x_j \geq L$$

$$\sum_{i=1}^n (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-) = 1$$

$$x_i = x_i^0 + x_i^+ - x_i^-, i = 1, 2, \dots, n$$

$$l_i \delta_i \leq x_i^+ \leq u_i \delta_i, i = 1, 2, \dots, n$$

$$0 \leq x_i^- \leq x_i^0, i = 1, 2, \dots, n$$

where  $L$  is a constant decided by investor (decision maker) on his/her own degree of satisfaction

$l_i$  = lower bound of proportion that can be invested in an asset

$u_i$  = upper bound of proportion that can be invested in an asset

The expression becomes a nonlinear and non smooth function of  $x = (x_1, x_2, \dots, x_n)$  in the third objective function

due to the absolute value. Therefore eliminating the absolute function of the third objective function, the P1 problem can be transformed into the following problem:

$$(P2) \max \sum_{i=1}^n r_i (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-) \quad f(x) = \frac{1}{1 + \exp(-\alpha x)}$$

$$\max \sum_{i=1}^n d_i (x_i^0 + x_i^+ - x_i^-)$$

$$\min \frac{1}{T} \sum_{t=1}^T y_t$$

subject to

$$\sum_{j=1}^n \left( \frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6} \right) x_j \geq L$$

$$\sum_{i=1}^n (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-) = 1$$

$$y_t + \sum_{i=1}^n (r_{it} - r_i) x_i \geq 0, t = 1, 2, \dots, T$$

$$x_i = x_i^0 + x_i^+ - x_i^-, i = 1, 2, \dots, n$$

$$l_i \delta_i \leq x_i^+ \leq u_i \delta_i, i = 1, 2, \dots, n$$

$$0 \leq x_i^- \leq x_i^0, i = 1, 2, \dots, n$$

$$y_t \geq 0, i = 1, 2, \dots, n$$

where  $L$  is a constant decided by investor (decision maker) on his/her own degree of satisfaction

$l_i$  = lower bound of proportion that can be invested in an asset

$u_i$  = upper bound of proportion that can be invested in an asset

The above problem P2 is a multi objective linear programming problem and there are several powerful multiple objective linear programming algorithms to solve it efficiently.

#### 4. Portfolio Rebalancing Model based on Fuzzy Decision Theory

Social, economical conditions & behavior of investor have a huge effect on future returns, so optimization approach is not always appropriate, a satisfactory approach is much better. An investor has levels for expected return and perceived risk. In literature various types of membership functions have been proposed such as a tangent type of a membership function [15], an exponential membership function [16], linear membership function [17,18] and many more, to express vague aspiration levels of an investor. Watada [19] used a non-linear S-shaped membership function to express investor aspiration level of return rate and risk. The proposed strategy has been used by many researchers [9] [10] as investor is in a position to tell only his expectations or aspiration. The approach is also termed as satisfactory approach as investor aspiration level is primarily governed by his or her need. This membership function is

$$f(x) = \frac{1}{1 + \exp(-\alpha x)}$$

where  $\alpha, 0 < \alpha < \infty$  is a fuzzy parameter which measures the degree of vagueness. Here  $\alpha$  determines the shapes of membership function and also different values of  $\alpha$  may reflect different investors aspiration levels. This function has a similar shape to the tangent hyperbolic function but it is much simpler to handle than that.

Here following non-linear S-shape membership functions is used to express the vague aspiration levels of the investor's expected returns, dividend, risk and liquidity of the portfolio. In portfolio rebalancing model i.e. (P2 model) of asset portfolio selection, the three objectives (return, dividend and risk) and the constraint on the liquidity of the portfolio are considered to be vague and uncertain.

Membership function of the goal for expected portfolio return is given by

$$\mu_r(x) = \frac{1}{1 + \exp[-\alpha_r \left( \sum_{i=1}^n r_i x_i - r_M \right)]}$$

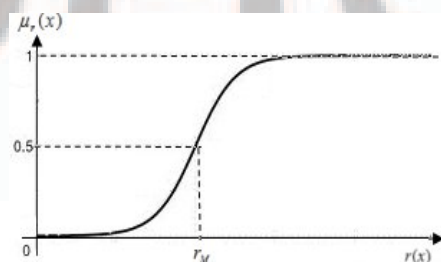


Figure 1.0 : Membership function of the goal for expected return.

where  $r_M$  is the mid-point (i.e. middle aspiration level for the portfolio return) where the membership function value is 0.5 and  $\alpha_r$  can be given by the investor based on his/her own degree of satisfaction for the for the portfolio return.

Membership function of the goal for expected portfolio dividend is given by

$$\mu_d(x) = \frac{1}{1 + \exp[-\alpha_d \left( \sum_{i=1}^n d_i x_i - d_M \right)]}$$

where  $d_M$  is the mid-point (i.e. middle aspiration level for the portfolio dividend) where the membership function value is 0.5 and  $\alpha_d$  can be given by the investor based on his/her own degree of satisfaction for the portfolio dividend .

Membership function of the goal for portfolio risk is given by

$$\mu_w(x) = \frac{1}{1 + \exp[\alpha_w (w(x) - w_M)]}$$

where  $w_M$  is the mid-point (i.e. middle aspiration level for the portfolio risk) where the membership function value is 0.5 and  $\alpha_w$  can be given by the investor based on his/her own degree of satisfaction for the portfolio risk .

Membership function of the goal for portfolio liquidity is given by

$$\mu_l(x) = \frac{1}{1 + \exp[-\alpha_l (E(\tilde{L}(x)) - l_M)]}$$

where  $l_M$  is the mid-point (i.e. middle aspiration level for the portfolio liquidity ) where the membership function value is 0.5 and  $\alpha_l$  can be given by the investor based on his/her own degree of satisfaction for the portfolio liquidity.

Here  $\alpha_r, \alpha_d, \alpha_w$  and  $\alpha_l$  determine the shapes of membership function  $\mu_r(x), \mu_d(x), \mu_w(x)$  and  $\mu_l(x)$  respectively, where  $\alpha_r > 0, \alpha_d > 0, \alpha_w > 0$  and  $\alpha_l > 0$ . As we increase the value of parameters  $\alpha_r, \alpha_d, \alpha_w$  and  $\alpha_l$ , the value of vagueness decreases.

In the above defined fuzzy membership function using Bellman-Zadeh's maximization principle [20]; the fuzzy multi objective asset portfolio selection problem is formulated as follows:

(FP1) max  $\eta$

Subject to

$$\eta \leq \mu_r(x)$$

$$\eta \leq \mu_d(x)$$

$$\eta \leq \mu_w(x)$$

$$\eta \leq \mu_l(x)$$

$$\sum_{i=1}^n (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-) = 1$$

$$y_t + \sum_{i=1}^n (r_{it} - r_i) x_i \geq 0, \quad t = 1, 2, \dots, T$$

$$x_i = x_i^0 + x_i^+ - x_i^-, \quad i = 1, 2, \dots, n$$

$$l_i \delta_i \leq x_i^+ \leq u_i \delta_i, \quad i = 1, 2, \dots, n$$

$$0 \leq x_i^- \leq x_i^0, \quad i = 1, 2, \dots, n$$

$$y_t \geq 0, \quad t = 1, 2, \dots, n$$

$$0 \leq \eta \leq 1$$

where  $l_i$  = lower bound of proportion that can be invested in an asset

$u_i$  = upper bound of proportion that can be invested in an asset

FP1 is a non linear programming problem. It can be written as constraints involving exponential function as follows:

$$\alpha_r \left[ \left\{ \sum_{i=1}^n r_i x_i - \sum_{i=1}^n p(x_i^+ + x_i^-) \right\} - r_M \right] \geq \log \frac{\eta}{1-\eta}$$

$$\alpha_d \left( \sum_{i=1}^n d_i x_i - d_M \right) \geq \log \frac{\eta}{1-\eta}$$

$$-\alpha_w \left( \frac{1}{T} \sum_{i=1}^T y_i - w_M \right) \geq \log \frac{\eta}{1-\eta}$$

$$\alpha_l \left[ \sum_{j=1}^n \left( \frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6} \right) x_j - l_M \right] \geq \log \frac{\eta}{1-\eta}$$

Putting  $\xi = \log \frac{\eta}{1-\eta}$  then  $\eta = \frac{1}{1 + \exp(-\xi)}$ . Since

the logistic function is monotonically increasing, so maximizing  $\eta$  makes  $\xi$  maximize. Therefore, the problem (FP1) can be transformed into an equivalent linear programming problem as follows:

(FP2)

$$\max \xi$$

Subject to

$$\xi \leq \alpha_r \left[ \left\{ \sum_{i=1}^n r_i x_i - \sum_{i=1}^n p(x_i^+ + x_i^-) \right\} - r_m \right]$$

$$\xi \leq \alpha_d \left[ \sum_{i=1}^n d_i x_i - r_d \right]$$

$$\xi + \frac{\alpha_w}{T} \sum_{i=1}^T y_i \leq \alpha_d w_m$$

$$\xi - \alpha_l \sum_{j=1}^n \left( \frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6} \right) x_j \leq -\alpha_l l_m$$

$$\sum_{i=1}^n (x_i^0 + x_i^+ - x_i^-) - \sum_{i=1}^n p(x_i^+ + x_i^-) = 1$$

$$y_t + \sum_{i=1}^n (r_{it} - r_i) x_i \geq 0, \quad t = 1, 2, \dots, T$$

$$x_i = x_i^0 + x_i^+ - x_i^-, \quad i = 1, 2, \dots, n$$

$$l_i \delta_i \leq x_i^+ \leq u_i \delta_i, \quad i = 1, 2, \dots, n$$

$$0 \leq x_i^- \leq x_i^0, \quad i = 1, 2, \dots, n$$

$$y_t \geq 0, \quad i = 1, 2, \dots, n$$

$$\xi \geq 0$$

Where,  $\alpha_r$ ,  $\alpha_d$ ,  $\alpha_w$  and  $\alpha_l$  are parameters decided by investor (decision maker) on his/her own degree of satisfaction. The shape parameters are experientially decided by the investors'. Also different parameter values may reflect different investor's aspiration levels. Therefore, it is convenient for different investors to formulate investment strategies using the proposed portfolio rebalancing model. The parameters  $r_M$ ,  $d_M$ ,  $w_M$ ,  $l_M$  (middle aspiration levels) can be obtained by approximating and taking average of necessary and sufficient levels.

### 5. Experimental Study

In this paper fuzzy portfolio rebalancing model is used to reallocate the investor's assets. The financial market is highly volatile and investor needs to change his/her investment strategy from time to time. In this study portfolio is rebalanced after one year because in Indian Stock Market the profit earned after a year is exempted from tax liability. It is assumed that the investor owns an existing portfolio and he/she will not invest additional capital during the portfolio rebalancing process. The model has been analyzed on data set of Bombay Stock Exchange (BSE). 25 Stocks are picked keeping in mind that the constructed portfolio covers a diversified area. The stocks have been selected from Auto Industry, Banking sector, Software Industry, Power Sector etc. The FP2 formulation is equivalent to linear programming problem and it is solved on LINGO Software. The rate of transaction cost depends upon broker but it is normally not more than 2.5% of purchased/sold amount.

Here fixed transaction cost 0.5% was taken. For each portfolio, groups of six stocks were prepared based on the data set, using historical data from April 1, 2009 through March 31, 2012, Table 1A shows the exchange of all the assets. Table 2A, 2B, 2C contains the return and dividend from each of the assets. Table 3A, 3B, 3C contains the expected risk of the assets. The liquidity profile is based on the daily turnover rate for each of the assets, the future turnover rates of the assets are assumed to be trapezoidal fuzzy numbers. Since the future turnover rates of the assets are assumed to be trapezoidal fuzzy numbers, we need to estimate the tolerance interval, the left spread and the right spread of the fuzzy numbers. Moreover, the value of these parameters can be obtained by using Delphi Method [20].

For illustration purpose, a method to find the estimate of the fuzzy turnover rates for Stock 1 (Table 4A) is discussed in detail. First, historical data for daily turnover rates from April 1, 2009 through March 31, 2012 to calculate the frequency of historical turnover rates is used. Most of the historical turnover rates fall into the intervals [0.0008, 0.0012], [0.0012, 0.0016], [0.0016, 0.0020] and [0.0020, 0.0024]. The mid-points of the intervals [0.0008, 0.0012] and [0.0020, 0.0024] as the left and the right end points of the tolerance interval, are taken. Thus, the tolerance interval of the fuzzy turnover rate become [0.0010, 0.0022]. Similarly, fuzzy turnover rates of all 25 Stocks were obtained (Table 4A, 4B, 4C): Portfolio creation and rebalancing by using FP2 model is shown in Table 5A to 10D.

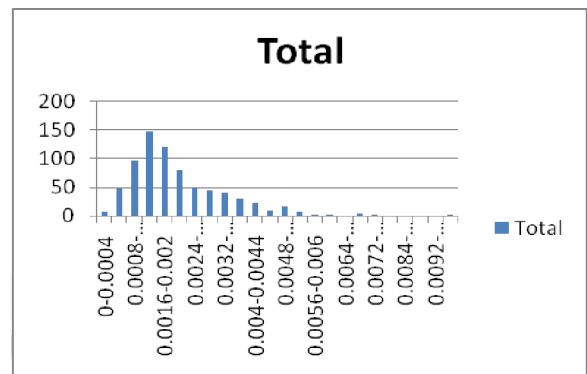


Figure 2.0: Frequency Turnover Rate of Stock 1

The 25 financial assets as above comprise of the population from which an attempt to construct a portfolio comprising of 6 assets with the corresponding upper and lower bounds of capital budget allocation was done. The objective is to maximize the degree of satisfaction in regard to maximization of portfolio returns, portfolio dividend, portfolio liquidity and minimization of portfolio risk.

To avoid large number of small investments and to ensure a sufficient diversification of the investment the lower and upper bounds on the investment in individual assets are set in such a way that there exists a feasible solution.

The large value of  $r_M, d_M, w_M, l_M$  reflects aggressive and optimistic view [12] and results are shown in table 5A to 7D and which are given by the investor are small. They are as follows:

- (i) Portfolio creation when  $r_M = 0.102, d_M = 0.0025, w_M = 0.042, l_M = 0.0055$
- (ii) Portfolio rebalancing after one year when  $r_M = 0.037, d_M = 0.0048, w_M = 0.03, l_M = 0.003$
- (iii) Portfolio rebalancing again after one year when  $r_M = 0.011, d_M = 0.0098, w_M = 0.04, l_M = 0.003$

The small value of  $r_M, d_M, w_M, l_M$  reflects conservative and pessimistic view [12] and results are shown in table 8A to 10D and which are given by the investor are small. They are as follows

- (i) Portfolio creation when  $r_M = 0.096, d_M = 0.002, w_M = 0.033, l_M = 0.0035$
- (ii) Portfolio rebalancing after one year when  $r_M = 0.0034, d_M = 0.004, w_M = 0.026, l_M = 0.0023$

- (iii) Portfolio rebalancing again after one year when  $r_M = 0.0034, d_M = 0.004, w_M = 0.026, l_M = 0.0023$

**Table 1A:** The Stock ID of 25 Assets

Stock ID	1	2	3	4	5	6	7	8	9
Stock	ACC	AXIS	Bajaj	Bharti	BHEL	BOB	BPCL	DLF	Grasim
Stock ID	10	11	12	13	14	15	16	17	18
Stock	HCL	HPCL	ICICI	IOC	ITC	Mahindra	Maruti	NTPC	ONGC
Stock ID	19	20	21	22	23	24	25		
Stock	PNB	SAIL	SBI	Tata	Tata	TCS	Wipro		

**Table 2A:** Expected Return & Dividend of the Stock for First Year

Stock ID	1	2	3	4	5	6	7	8	9
Return	0.0442	0.0867	0.1073	-0.0077	0.0401	0.0788	0.0331	0.0367	0.0554
Dividend	0.0033	0.0020	0.0029	0.0003	0.0011	0.0032	0.0016	0.0009	0.0016
Stock ID	10	11	12	13	14	15	16	17	18
Return	0.1083	0.0225	0.0852	0.0345	0.03	0.089	0.0572	0.0073	0.0226
Dividend	0.0033	0.0017	0.0026	0.0016	0.0017	0.0021	0.0004	0.0018	0.0033
Stock ID	19	20	21	22	23	24	25		
Return	0.0689	0.0778	0.0522	0.1242	0.0931	0.0363	0.09		
Dividend	0.0062	0.0025	0.0030	0.0028	0.0064	0.0009	0.0013		

**Table 2B:** Expected Return & Dividend of the Stock for Second Year

Stock ID	1	2	3	4	5	6	7	8	9
Return	0.0096	0.0998	0.0347	0.0101	-0.019	0.0317	0.0149	-0.0192	-0.0092
Dividend	0.0087	0.0044	0.0081	0.0004	0.0014	0.0086	0.0032	0.0009	0.0016
Stock ID	10	11	12	13	14	15	16	17	18
Return	0.0235	0.0136	0.0098	0.0106	0.0253	0.0245	-0.006	-0.0123	0.0066
Dividend	0.0045	0.0038	0.0029	0.0028	0.0045	0.002	0.0006	0.0018	0.0049
Stock ID	19	20	21	22	23	24	25		
Return	0.0097	-0.031	0.0218	0.0372	-0.004	0.0321	-0.003		
Dividend	0.0025	0.0025	0.0015	0.0069	0.0032	0.0031	0.0026		

**Table 2C:** Expected Return & Dividend of the Stock for Third Year

Stock ID	1	2	3	4	5	6	7	8	9
Return	0.0187	-0.0113	0.0179	-0.0077	-0.0379	-0.0079	0.0124	-0.0076	0.0086
Dividend	0.0080	0.0080	0.0105	0.0003	0.0012	0.0113	0.0032	0.0009	0.0011
Stock ID									
Return	0.001	-0.0158	-0.0153	-0.0177	0.0128	-0.0059	0.0062	-0.0061	-0.006
Dividend	0.0050	0.0044	0.0033	0.0020	0.0020	0.0024	0.0008	0.0020	0.0009
Stock ID									
Return	-0.0186	-0.0451	-0.0178	0.0171	-0.0252	0.0007	-0.0044		
Dividend	0.0045	0.0021	0.0023	0.0023	0.0048	0.0026	0.0020		

**Table 3A:** Expected Risk of the Stock for First Year

Stock ID	1	2	3	4	5	6	7	8	9
Risk	0.0348	0.0357	0.0371	0.0350	0.0263	0.0318	0.0336	0.0542	0.0373
Stock ID	10	11	12	13	14	15	16	17	18
Risk	0.0470	0.0355	0.0517	0.0308	0.0186	0.0356	0.0379	0.0164	0.0261
Stock ID	19	20	21	22	23	24	25		
Risk	0.0281	0.0420	0.0407	0.0599	0.0422	0.0429	0.0316		

**Table 3B:** Expected Risk of the Stock for Second Year

Stock ID	1	2	3	4	5	6	7	8	9
Risk	0.0207	0.0219	0.0296	0.0279	0.0157	0.0231	0.0340	0.0440	0.0382
Stock ID	10	11	12	13	14	15	16	17	18
Risk	0.0253	0.0478	0.0292	0.0344	0.0207	0.0235	0.0317	0.0163	0.0249
Stock ID	19	20	21	22	23	24	25		
Risk	0.0241	0.0247	0.0222	0.0312	0.0340	0.0191	0.0254		

**Table 3C: Expected Risk of the Stock for Third Year**

Stock ID	1	2	3	4	5	6	7	8	9
Risk	0.0232	0.0354	0.0259	0.0185	0.0220	0.0285	0.0322	0.0454	0.0207
Stock ID	10	11	12	13	14	15	16	17	18
Risk	0.0253	0.0280	0.0322	0.0118	0.0103	0.0273	0.0306	0.0210	0.0155
Stock ID	19	20	21	22	23	24	25		
Risk	0.0284	0.0401	0.0374	0.0495	0.0318	0.0218	0.0252		

**Table – 4A: Fuzzy Turnover Rates of the Stock for First Year**

Stock ID	1				2				3			
$\tilde{L}$	0.0016	0.0033	0.0012	0.0036	0.0025	0.0070	0.0017	0.0077	0.0004	0.0022	0.0001	0.0025
Stock ID	4				5				6			
$\tilde{L}$	0.0007	0.0019	0.0005	0.0021	0.0006	0.0018	0.0004	0.0020	0.0007	0.0019	0.0005	0.0021
Stock ID	7				8				9			
$\tilde{L}$	0.0010	0.0022	0.0008	0.0024	0.0021	0.0045	0.0017	0.0049	0.0006	0.0018	0.0004	0.0020
Stock ID	10				11				12			
$\tilde{L}$	0.0006	0.0019	0.0008	0.0021	0.0021	0.0045	0.0017	0.0049	0.0029	0.0053	0.0025	0.0057
Stock ID	13				14				15			
$\tilde{L}$	0.0002	0.0005	0.0001	0.0005	0.0008	0.0017	0.0006	0.0018	0.0020	0.0044	0.0016	0.0048
Stock ID	16				17				18			
$\tilde{L}$	0.0010	0.0028	0.0007	0.0031	0.0003	0.0006	0.0002	0.0006	0.0003	0.0009	0.0002	0.0010
Stock ID	19				20				21			
$\tilde{L}$	0.0004	0.0022	0.0001	0.0025	0.0010	0.0022	0.0008	0.0024	0.0016	0.0034	0.0013	0.0037
Stock ID	22				23				24			
$\tilde{L}$	0.0050	0.0110	0.0040	0.0120	0.0071	0.0131	0.0061	0.0141	0.0007	0.0013	0.0006	0.0014
Stock ID	25											
$\tilde{L}$	0.0006	0.0009	0.0005	0.0009								

**Table 4B: Fuzzy Turnover Rates of the Stock for Second Year**

Stock ID	1				2				3			
$\tilde{L}$	0.0008	0.002	0.0006	0.0022	0.0015	0.0030	0.0012	0.0032	0.0006	0.003	0.0002	0.0034
Stock ID	4				5				6			
$\tilde{L}$	0.0008	0.0014	0.0007	0.0015	0.0006	0.0012	0.0005	0.0013	0.0004	0.001	0.0003	0.0011
Stock ID	7				8				9			
$\tilde{L}$	0.0009	0.0018	0.0007	0.0019	0.0021	0.0036	0.0018	0.0038	0.0003	0.0015	0.0001	0.0017
Stock ID	10				11				12			
$\tilde{L}$	0.0006	0.0015	0.0004	0.0016	0.0012	0.0024	0.001	0.0026	0.0024	0.0048	0.002	0.0052
Stock ID	13				14				15			
$\tilde{L}$	0.0002	0.0004	0.0001	0.0005	0.0004	0.001	0.0003	0.0011	0.0021	0.0033	0.0019	0.0035
Stock ID	16				17				18			
$\tilde{L}$	0.0005	0.0029	0.0001	0.0033	0.0002	0.0004	0.0001	0.0005	0.0004	0.0006	0.0003	0.0007
Stock ID	19				20				21			
$\tilde{L}$	0.0006	0.0016	0.0005	0.0017	0.0004	0.001	0.0003	0.0011	0.0016	0.0034	0.0013	0.0037
Stock ID	22				23				24			
$\tilde{L}$	0.0037	0.0073	0.0031	0.0079	0.0047	0.0089	0.004	0.0096	0.0006	0.0012	0.0005	0.0013
Stock ID	25											
$\tilde{L}$	0.0004	0.0006	0.0003	0.0007								

**Table – 4C: Fuzzy Turnover Rates of the Stock for Third Year**

Stock ID	1				2				3			
$\tilde{L}$	0.0007	0.0019	0.0005	0.0021	0.0031	0.0055	0.0027	0.0059	0.0008	0.0023	0.0005	0.0025
Stock ID	4				5				6			
$\tilde{L}$	0.0006	0.0018	0.0004	0.002	0.0007	0.0019	0.0005	0.0021	0.0005	0.0011	0.0004	0.0012
Stock ID	7				8				9			
$\tilde{L}$	0.0006	0.0012	0.0005	0.0013	0.0024	0.0048	0.002	0.0052	0.0003	0.0009	0.0002	0.001
Stock ID	10				11				12			
$\tilde{L}$	0.0006	0.0018	0.0004	0.002	0.001	0.0028	0.0007	0.0031	0.0021	0.0045	0.0017	0.0049
Stock ID	13				14				15			
$\tilde{L}$	0.0002	0.0005	0.0001	0.0005	0.0004	0.0014	0.0003	0.0015	0.0016	0.0042	0.0011	0.0047
Stock ID	16				17				18			
$\tilde{L}$	0.001	0.0028	0.0007	0.0031	0.0002	0.0004	0.0001	0.0005	0.0003	0.0006	0.0002	0.0006
Stock ID	19				20				21			
$\tilde{L}$	0.0006	0.0015	0.0004	0.0016	0.0003	0.0009	0.0002	0.001	0.0024	0.0052	0.002	0.0056
Stock ID	22				23				24			
$\tilde{L}$	0.0032	0.0068	0.0026	0.0074	0.0020	0.0041	0.0017	0.0045	0.0003	0.0021	0.0001	0.0024
Stock ID	25											
$\tilde{L}$	0.0003	0.0009	0.0002	0.0010								

**6. Portfolio Creation**

**Table 5A**

Membership grade  $\eta$  , obtain return, obtain dividend, obtain risk, obtain liquidity when  $r_M = 0.102, d_M = 0.0025,$

$$w_M = 0.042, l_M = 0.0055$$

$\eta$	$\xi$	$\alpha_r$	$\alpha_d$	$\alpha_w$	$\alpha_l$	obtain return	obtain dividend	obtain risk	obtain liquidity
0.513247	0.053350	350	4000	800	1600	0.1026	0.00252	0.04195	0.00555
0.514921	0.0597400	400	4500	900	1800				
0.516569	0.0663450	450	5000	1000	2000	0.1026	0.00255	0.04196	0.00556

**Table 5B**

Portfolio creation when  $r_M = 0.102, d_M = 0.0025, w_M = 0.042, l_M = 0.0055, \alpha_r = 350, \alpha_d = 4000, \alpha_w = 800, \alpha_l = 1600,$

Stock	1	2	3	4	5	6	7	8	9
Proportion	0.0000	0.1154	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock	10	11	12	13	14	15	16	17	18
Proportion	0.1190	0.0000	0.2156	0.0000	0.0000	0.1000	0.0000	0.0000	0.0000
Stock	19	20	21	22	23	24	25		
Proportion	0.0000	0.0000	0.0000	0.3500	0.0000	0.0000	0.1000		

**Table-5C**

Portfolio creation when  $r_M = 0.102, d_M = 0.0025, w_M = 0.042, l_M = 0.0055, \alpha_r = 400, \alpha_d = 4500, \alpha_w = 900, \alpha_l = 1800,$

Stock	1	2	3	4	5	6	7	8	9
Proportion	0.0000	0.1154	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock	10	11	12	13	14	15	16	17	18
Proportion	0.1190	0.0000	0.2156	0.0000	0.0000	0.1	0.0000	0.0000	0.0000
Stock	19	20	21	22	23	24	25		
Proportion	0.0000	0.0000	0.0000	0.3500	0.0000	0.0000	0.1000		



Table-5D

Portfolio creation when  $r_M = 0.102$ ,  $d_M = 0.0025$ ,  $w_M = 0.042$ ,  $l_M = 0.0055$ ,  $\alpha_r = 450$ ,  $\alpha_d = 5000$ ,  $\alpha_w = 1000$ ,  $\alpha_l = 2000$ ,

Stock	1	2	3	4	5	6	7	8	9
Proportion	0.0000	0.1154	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock	10	11	12	13	14	15	16	17	18
Proportion	0.1190	0.0000	0.2156	0.0000	0.0000	0.1000	0.0000	0.0000	0.0000
Stock	19	20	21	22	23	24	25		
Proportion	0.0000	0.0000	0.0000	0.35	0.0000	0.0000	0.1000		

7. Portfolio Rebalancing After One Year

Table- 6A

Membership grade  $\eta$ , obtain return, obtain dividend, obtain risk, obtain liquidity when  $r_M = 0.037$ ,  $d_M = 0.0048$ ,  $w_M = 0.03$ ,  $l_M = 0.003$

$\eta$	$\xi$	$\alpha_r$	$\alpha_d$	$\alpha_w$	$\alpha_l$	obtain return	obtain dividend	obtain risk	obtain liquidity
0.5152	0.0608	900	1600	500	2400	0.0412	0.0066	0.0382	0.0034
0.6641	0.6817	1000	1800	600	2600	0.0412	0.0066	0.0382	0.0034
0.6804	0.7556	1100	2000	700	2800	0.0412	0.0066	0.0382	0.0034

Table 6B

Portfolio rebalancing when  $r_M = 0.037$ ,  $d_M = 0.0048$ ,  $w_M = 0.03$ ,  $l_M = 0.003$ ,  $\alpha_r = 900$ ,  $\alpha_d = 1600$ ,  $\alpha_w = 500$ ,  $\alpha_l = 2400$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0.2808	0	0	0.0807	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0	0	0.2156	0	0	0.1	0	0	0
Purchased ratio	0	0	0	0	0	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0.1		

Table-6C

Portfolio rebalancing when  $r_M = 0.037$ ,  $d_M = 0.0048$ ,  $w_M = 0.03$ ,  $l_M = 0.003$ ,  $\alpha_r = 1000$ ,  $\alpha_d = 1800$ ,  $\alpha_w = 600$ ,  $\alpha_l = 2600$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0.281	0	0	0.081	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0	0	0.216	0	0	0.1	0	0	0
Purchased ratio	0	0	0	0	0	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0	0.1	
Purchased ratio	0	0	0	0	0	0.05	0		

Table 6D

Portfolio rebalancing when  $r_M = 0.037$ ,  $d_M = 0.0048$ ,  $w_M = 0.03$ ,  $l_M = 0.003$ ,  $\alpha_r = 1100$ ,  $\alpha_d = 2000$ ,  $\alpha_w = 700$ ,  $\alpha_l = 2800$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0.3	0	0	0.1	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0	0	0.2	0	0	0.1	0	0	0
Purchased ratio	0	0	0	0	0	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0	0.1	
Purchased ratio	0	0	0	0	0	0	0	0.1	0

8. Portfolio Rebalancing Again After One Year

Table- 7A

Membership grade  $\eta$ , obtain return, obtain dividend, obtain risk, obtain liquidity when  $r_M = 0.011$ ,  $d_M = 0.0098$ ,  $w_M = 0.04$ ,  $l_M = 0.003$

$\eta$	$\xi$	$\alpha_r$	$\alpha_d$	$\alpha_w$	$\alpha_l$	obtain return	obtain dividend	obtain risk	obtain liquidity
0.5257	0.103	400	300	600	3000	0.0128	0.00822	0.082	0.00312
0.5293	0.1175	450	350	700	3200	0.0128	0.00823	0.082	0.00312
0.5329	0.1317	500	400	800	3400	0.0128	0.00823	0.082	0.00315

**Table-7B**

Portfolio rebalancing when  $r_M = 0.011, d_M = 0.0098, w_M = 0.04, l_M = 0.003, \alpha_r = 400, \alpha_d = 300, \alpha_w = 600, \alpha_l = 3000$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0.0793	0	0	0	0	0	0	0
Purchased ratio	0.05	0	0	0	0	0	0.1	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0.119	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0	0	0.11	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0		
Purchased ratio	0	0	0	0	0	0	0		

**Table-7C**

Portfolio rebalancing ratio when  $r_M = 0.011, d_M = 0.0098, w_M = 0.04, l_M = 0.003, \alpha_r = 450, \alpha_d = 350, \alpha_w = 700, \alpha_l = 3200$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0.0786	0	0	0	0	0	0	0
Purchased ratio	0.05	0	0	0	0	0	0.1	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0.119	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0	0	0.109	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0		
Purchased ratio	0	0	0	0	0	0	0		

**Table-7D**

Portfolio rebalancing ratio after one year when  $r_M = 0.011, d_M = 0.0098, w_M = 0.04, l_M = 0.003, \alpha_r = 500, \alpha_d = 400, \alpha_w = 800, \alpha_l = 3400$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0.078	0	0	0	0	0	0	0
Purchased ratio	0.05	0	0	0	0	0	0.1	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0.11	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0	0	0.109	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0		
Purchased ratio	0	0	0	0	0	0	0		

**9. Portfolio Creation**

**Table 8A**

Membership grade  $\eta$ , obtain return, obtain dividend, obtain risk, obtain liquidity when  $r_M = 0.096, d_M = 0.002, w_M = 0.033, l_M = 0.0035$

$\eta$	$\xi$	$\alpha_r$	$\alpha_d$	$\alpha_w$	$\alpha_l$	obtain return	obtain dividend	obtain risk	obtain liquidity
0.5644	0.2589	350	4000	800	1600	0.0947	0.0022	0.0327	0.0044
0.5644	0.2589	400	4500	900	1800	0.0947	0.0022	0.0327	0.0044
0.5816	0.3295	450	5000	1000	2000	0.0947	0.0022	0.0327	0.0044

**Table 8B**

Portfolio creation when  $r_M = 0.096, d_M = 0.002, w_M = 0.033, l_M = 0.0035, \alpha_r = 350, \alpha_d = 4000, \alpha_w = 800, \alpha_l = 1600$

Stock	1	2	3	4	5	6	7	8	9
Proportion	0	0.4	0	0	0	0	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Proportion	0.1	0	0.1	0	0	0.1	0	0	0
Stock	19	20	21	22	23	24	25		
Proportion	0	0	0	0.1358	0	0	0.2142		

**Table-8C**

Portfolio creation when  $r_M = 0.096, d_M = 0.002, w_M = 0.033, l_M = 0.0035, \alpha_r = 400, \alpha_d = 4500, \alpha_w = 900, \alpha_l = 1800$

Stock	1	2	3	4	5	6	7	8	9
Proportion	0	0.4	0	0	0	0	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Proportion	0.1	0	0.1	0	0	0.1	0	0	0
Stock	19	20	21	22	23	24	25		
Proportion	0	0	0	0.1	0	0	0.2143		

**Table 8D**

Portfolio creation when  $r_M = 0.096, d_M = 0.002, w_M = 0.033, l_M = 0.0035, \alpha_r = 450, \alpha_d = 5000, \alpha_w = 1000, \alpha_l = 2000$

Stock	1	2	3	4	5	6	7	8	9
Proportion	0.00	0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Stock	10	11	12	13	14	15	16	17	18
Proportion	0.10	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.00
Stock	19	20	21	22	23	24	25		
Proportion	0.00	0.00	0.00	0.1356	0.00	0.00	0.2144		

**10. Portfolio Rebalancing After One Year**

**Table- 9A**

Membership grade  $\eta$ , obtain return, obtain dividend, obtain risk, obtain liquidity when  $r_M = 0.0034, d_M = 0.004, w_M = 0.026, l_M = 0.0023$

$\eta$	$\xi$	$\alpha_r$	$\alpha_d$	$\alpha_w$	$\alpha_l$	obtain return	obtain dividend	obtain risk	obtain liquidity
0.5016	0.0065	900	1600	500	2400	0.05473	0.00576	0.02609	0.00305
0.5019	0.0077	1000	1800	600	2600	0.05475	0.00577	0.02609	0.00306
0.5022	0.0089	1100	2000	700	2800	0.05479	0.00579	0.02613	0.00309

Table 9B

Portfolio rebalancing when  $r_M = 0.0034, d_M = 0.004, w_M = 0.026, l_M = 0.0023, \alpha_r = 1000, \alpha_d = 1800, \alpha_w = 600, \alpha_l = 2600$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0.05	0	0	0.144	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0	0	0.05	0	0	0.1	0	0	0
Purchased ratio	0	0	0	0	0	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0.1		
Purchased ratio	0	0	0	0	0	0.054	0		

Table 9C

Portfolio rebalancing when  $r_M = 0.0034, d_M = 0.004, w_M = 0.026, l_M = 0.0023, \alpha_r = 1100, \alpha_d = 2000, \alpha_w = 700, \alpha_l = 2800$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0	0	0	0.1441	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0	0	0	0	0	0.1	0	0	0
Purchased ratio	0	0	0	0	0	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0.1		
Purchased ratio	0	0	0	0	0	0.0535	0		

Table-9D

Portfolio rebalancing when  $r_M = 0.0034, d_M = 0.004, w_M = 0.026, l_M = 0.0023, \alpha_r = 900, \alpha_d = 1600, \alpha_w = 500, \alpha_l = 2400$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0.1	0	0	0.1	0	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0	0	0.1	0	0	0.1	0	0	0
Purchased ratio	0	0	0	0	0	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0	0	0	0.1		
Purchased ratio	0	0	0	0	0	0	0.1		

11. Portfolio Rebalancing Again After One Year

Table- 10A

Membership grade  $\eta$ , obtain return, obtain dividend, obtain risk, obtain liquidity when  $r_M = 0.0103, d_M = 0.0094, w_M = 0.034, l_M = 0.002$

$\eta$	$\xi$	$\alpha_r$	$\alpha_d$	$\alpha_w$	$\alpha_l$	obtain return	obtain dividend	obtain risk	obtain liquidity
0.5162	0.0649	400	300	600	3000	0.0009	0.0065	0.0339	0.0027
0.5189	0.0758	450	350	700	3200	0.0009	0.0065	0.0339	0.0027
0.5216	0.0866	500	400	800	3400	0.0009	0.0065	0.0339	0.0027

Table 10B

Portfolio rebalancing when  $r_M = 0.0103, d_M = 0.0094, w_M = 0.034, l_M = 0.002, \alpha_r = 400, \alpha_d = 300, \alpha_w = 600, \alpha_l = 3000$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0.0509	0	0	0	0	0	0	0
Purchased ratio	0.05	0	0	0	0	0	0.05	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0.1	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0	0	0.0634	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0.01	0	0	0		
Purchased ratio	0	0	0	0	0	0	0		

Table-10C

Portfolio rebalancing ratio when  $r_M = 0.0103, d_M = 0.0094, w_M = 0.034, l_M = 0.002, \alpha_r = 450, \alpha_d = 350, \alpha_w = 700, \alpha_l = 3200$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0.0509	0	0	0	0	0	0	0
Purchased ratio	0.05	0	0	0	0	0	0.05	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0.1	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0	0	0.0634	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0.01	0	0	0		
Purchased ratio	0	0	0	0	0	0	0		

Table-10D

Portfolio rebalancing ratio after one year when  $r_M = 0.0103, d_M = 0.0094, w_M = 0.034, l_M = 0.002, \alpha_r = 500, \alpha_d = 400, \alpha_w = 800, \alpha_l = 3400$

Stock	1	2	3	4	5	6	7	8	9
Sold ratio	0	0.0509	0	0	0	0	0	0	0
Purchased ratio	0.05	0	0	0	0	0	0.05	0	0
Stock	10	11	12	13	14	15	16	17	18
Sold ratio	0.1	0	0	0	0	0	0	0	0
Purchased ratio	0	0	0	0	0.0634	0	0	0	0
Stock	19	20	21	22	23	24	25		
Sold ratio	0	0	0	0.01	0	0	0		
Purchased ratio	0	0	0	0	0	0	0		

12. Conclusion

In this paper, fuzzy optimization technique is used to construct portfolio and to rebalance it, using S shaped aspiration function. Investor can construct portfolio using his/her own aspiration level. In this model five parameters return, dividend, risk, liquidity and transaction cost are used. The rebalancing is done after every year and pessimistic and optimistic view of investor is considered for rebalancing the portfolio to give him maximum profit.

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