

# Two Dimensional Consolidations for Clay Soil of Non-Homogeneous and Anisotropic Permeability

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**Abstract:** In this research a computer program was modified to solve two-dimensional consolidation problems. The modified computer program results showed a very good agreement compared with the analytical solution of two-dimensional consolidation. The non-homogeneity in one and two directions with the an-isotropic effect of permeability was included in the program. It was found that the increasing of anisotropic ratio of permeability increases the average degree of consolidation while the increase in non-homogeneous permeability ratio decreases the average degree of consolidation according to the drainage face taken there. This effect should be taken in to account when dealing with the analysis of consolidation of compressible soil and average degree of consolidation.

**Keywords:** Finite Element, Consolidation, permeability

## 1. Introduction

The conventional Terzaghi theory [1] is derived based on assumptions such as the soil fluid flow is in one direction only and also that the soil is homogenous and isotropic. These assumptions are most often-violated in real life situations. They are functioned to simulate a special case of subjecting soil layer to uniform load such as fill load. The homogeneous and isotropic soil means that every soil parameters are constant with any direction and so the permeability of soil is assumed constant along the depth of soil layer.

Two and three-dimensional consolidation behavior that governed by two and three diffusion equations were presented by many investigators to overcome the defect of assuming fluid flow in one direction [2, 3, 4]. Rendulic-Terzaghi equation is considered as extending to Terzaghi's equation. For simple geometry and one dimensional consolidation, analytical solution can be used, otherwise numerical analysis such finite difference and finite element method are utilized. Using two dimensional consolidation analyses is more realistic than one dimensional consolidation.

A considerable amount of research is achieved to include the effect of non homogeneous and isotropic soil parameters into account when analyzing consolidation behavior. Analytical solution for consolidation in two layer soil column was provided by Zhu and Yin (2005)[5]. A finite difference approach was used to solve the consolidation settlement for a layer with variable compressibility, permeability and coefficient of consolidation [6]. Xie et al. presented a fully explicit analytical solution for one dimensional consolidation of two layered soils with partially drained boundaries. Amiri and Esmaelly [8] investigated the effect of variable permeability and compressibility on the consolidation behavior.

In the present work, the permeability is considered variable in one or two different direction and is included in the finite

element code of two dimensional consolidation settlement problem. The study contribute in investigating the effect of an isotropic permeability and non homogeneous permeability on the average degree of consolidation

## 2. Governing Equations

The consolidation governing equation for excess pore water pressure  $u$  is

$$c_{vx} \frac{\partial^2 u}{\partial x^2} + c_{vy} \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t} \quad (1)$$

Where  $c_{vx}$ ,  $c_{vy}$  are the coefficient of consolidation in  $x$ , and  $y$  direction and  $C_v = \frac{k}{\gamma_w m_v}$

It is practically accepted to assume  $C_v$  as a constant through the consolidation progressing since  $k$  and  $m_v$  both decrease through the process. Where  $K$  is the permeability and  $m_v$  is the coefficient of volume change.

According to Griffiths and Smith [9], the discretization of equation 1 using Galerkin method leads to the following equation

$$[k]\{u\} + [m_m] \frac{du}{dt} = \{0\} \quad (2)$$

Where  $[K]$  and  $[m_m]$  are the fluid conductivity and mass matrices respectively.

The above first order time dependent equation can be solved by finite difference as follow using element assembly method

$$[K]\{U\}_0 + [M_m] \frac{dU}{dt}_0 = \{0\}_0 \quad (3)$$

$$[K]\{U\}_1 + [M_m] \frac{dU}{dt}_1 = \{0\}_1 \quad (4)$$

A third equation advances the solution from 0 to 1 using a weighted average of the gradient at the beginning and end of time interval, thus,

$$\{U\}_1 = \{U\}_0 + \Delta t \left( (1-\theta) \frac{dU}{dt}_0 + \theta \frac{dU}{dt}_1 \right) \quad (5)$$

$$([M_m] + \theta \Delta t [K])\{U\}_1 = ([M_m] + (1-\theta)\Delta t [K])\{U\}_0 \quad (6)$$

The above equation gives the distribution of excess pore water pressure [10]

### 3. Verification of the Computer Program

To check the accuracy of the results of finite element program, a problem which represents the dissipation of excess pore water pressure from a square of soil [9]. The square of soil is previous from four directions, and because of symmetry, it can be used as a part of square of soil (Figure 1). The load is uniform and the coefficient of consolidation is constant with time and equal in magnitude. The pressure at the center of the mesh is calculated at time interval 0.1, and then the results are plotted in Figure 2, where the results are compared with series solution values obtained by [11]. The crude finite element idealization gives excellent results. The solution of the finite element program is compared with analytical solution giving good agreement.

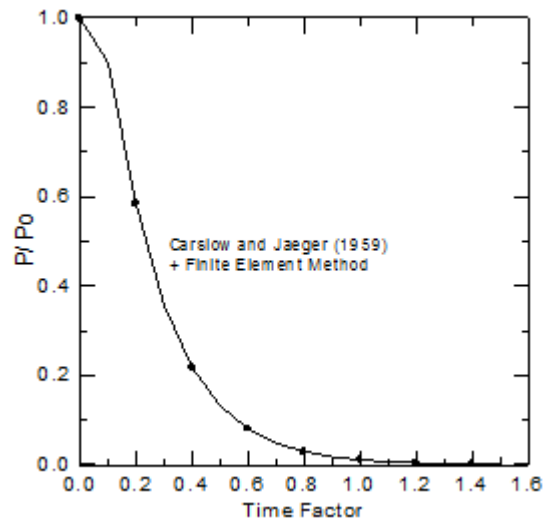


Figure 2: Comparisons of finite element results with series results, After Smith and Griffiths 1998

### 4. Variation of Permeability

The finite element program was used to study the effect of the variable permeability on the average degree of consolidation for the two-dimensional consolidation problem. The variation of permeability can be discussed as follows:

1. The permeability in x direction is different from that in y direction (anisotropic permeability).
2. The permeability is variable with depth in one direction (non-homogeneous permeability in one direction)
3. The permeability is variable with depth in each direction (non-homogeneous permeability in two direction)

The problem will depend on the type of variation of permeability (if it is stratified, linear or nonlinear variation) and if the type of variation is the same in two directions or different. The program can be modified to include all these types of variation of permeability. In this study the variation of permeability is used as linear variation with depth. The sketch for the permeability variation with depth has been introduced to simplify obtaining the average degree of consolidation from these variables (Figure 3).

The variables considered in this study are as follows:

1.  $k_x$  is the permeability in x direction
2.  $k_y$  is the permeability in y direction
3.  $k_{x1}/k_{y1}$  is the anisotropic permeability ratio (1-10)
4.  $k_{x1}/k_{x2}$  is the non-homogeneous permeability ratio in the x direction (1-8)
5.  $k_{y1}/k_{y2}$  is the non-homogeneous permeability ratio in the y direction (1-8)
6.  $H_h/H_v$  is the ratio of drainage path in x direction to drainage path in y direction.
7. X is the distance from the centerline of the mesh to the right side in x direction.

To study the effect of the factor ( $X/H_h$ ) and the anisotropic permeability ratio ( $k_x/k_y$ ) on the average degree of consolidation, two cases are studied. For the first case the

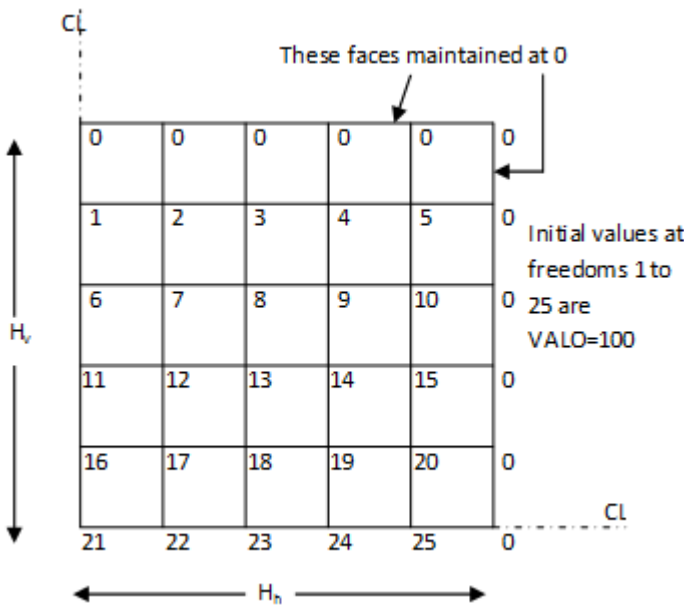
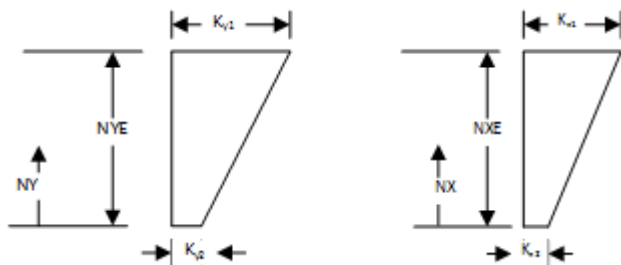


Figure 1: Grid of the verification problem

permeability of soil is considered isotropic and homogeneous and the excess pore pressures are recorded at different location from the centerline of the mesh,  $X/H_h$ . The second case considered the permeability of soil as anisotropic parameter and the excess pore pressures are recorded at two locations from the centerline of the mesh  $X/H_h = 0$  and  $X/H_h = 0.4$ .

Figure (4) shows the effects of factor ( $X/H_h$ ) on the average degree of consolidation for values (0, 0.2, 0.4, 0.6, 0.8). It shows that the average degree of consolidation increases with the increasing of ( $X/H_h$ ). The increase of ( $X/H_h$ ) means the region that the average degree of consolidation is calculated close to the drainage face. Figure (5) shows the effect of anisotropic ratio of permeability on the average degree of consolidation for different ratio of  $X/H_h$  (0, and 0.4), and different ratio of anisotropic ratio of permeability (1, 4, 6, 10). The average degree of consolidation increases at anisotropic ratio of permeability greater than one. Also increase of  $X/H_h$  will increase the average degree of consolidation.



$$K_{NY} = k_{y1} - (NY(k_{y1} - k_{y2}) / NYE) \quad K_{NX} = k_{x1} - (NX(k_{x1} - k_{x2}) / NXE)$$

Figure 3: Linear variation of permeability

### 5. Effect of Non Homogeneous Permeability Ratio

The results of the finite element program for all cases studied are presented as average degree of consolidation,  $U_{av}$ . It can be computed at any location in the mesh by the following formula ( $U_{av} = 1 - u_e / u_o$ ) where  $u_e$  is the excess pore water pressure and  $u_o$  is the initial pore water pressure.

For the case of non-homogeneous permeability ratio in y direction  $k_{y1}/k_{y2} = 8$  and anisotropic permeability ratio  $k_{x1}/k_{y1} > 1$ , when the anisotropic permeability ratio increases from 4 to 10, keeping constant value of non-homogeneous permeability ratio, the average degree of consolidation increases from 0.7 to 0.92 (Table 1). The increase in  $U_{av}$  is due to the increase in anisotropic permeability ratio.

Effect of the factor ( $X/H_h$ ) at which the average degree of consolidation was calculated are also presented in Table 1. Two cases are discussed the first is for  $U_{av}$  computed at  $X/H_h = 0$  and the second is for  $U_{av}$  computed at  $X/H_h = 0.4$ . The  $U_{av}$  calculated at  $X/H_h = 0.4$  is greater than  $U_{av}$  calculated at  $X/H_h = 0$  because this region is close to the right face of the mesh which is idealized as a drainage face in the presented problem.

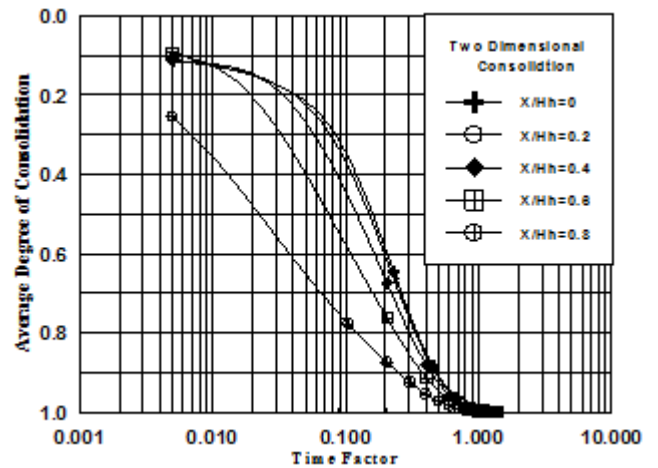


Figure 4: average degree of consolidation at different location  $X/H_h$  for homogeneous and isotropic soil.

Table 1: The effect of the anisotropic ratio of permeability and distance ratio  $X/H_h$  on the average degree of consolidation

$H_v/H_h$	$K_x/K_y$	$X/H_h$	$T_v$	$K_{y1}/K_{y2}$	$U_{av}$
1	10	0	0.03	8	0.5
1	10	0.4	0.03	8	0.6
1	4	0	0.1	8	0.7
1	10	0	0.1	8	0.92

For the case of non-homogeneous permeability in two directions together with the anisotropic permeability the  $U_{av}$  increases due to anisotropic of permeability of soil. With high permeability ratio the  $U_{av}$  decreases because the high permeability is concentrated close to the impervious face.

The high ratio of non-homogeneous permeability decreases the average degree of consolidation (Table 2) because the high permeability in x direction and y direction are close to the impervious side of the mesh.

Table 2: show the effect of the non-homogeneity in two direction and the distance ratio  $X/H_h$  on the average degree of consolidation

$H_v/H_h$	$K_x/K_y$	$X/H_h$	$T_v$	$K_{y1}/K_{y2}, K_{x1}/K_{x2}$	$U_{av}$
1	10	0	0.1	1.1	0.94
1	10	0	0.1	8.8	0.58
1	10	0.4	0.1	8.8	0.63

Table 2 also shows the result of average degree of consolidation for the case of two dimensional consolidation for soil with anisotropic permeability and non homogeneous permeability in x and y direction. The average degree of consolidation is calculated at two location represented by  $X/H_h$ . It is shown that the average degree of consolidation increases as the location closes to the face of drainage.

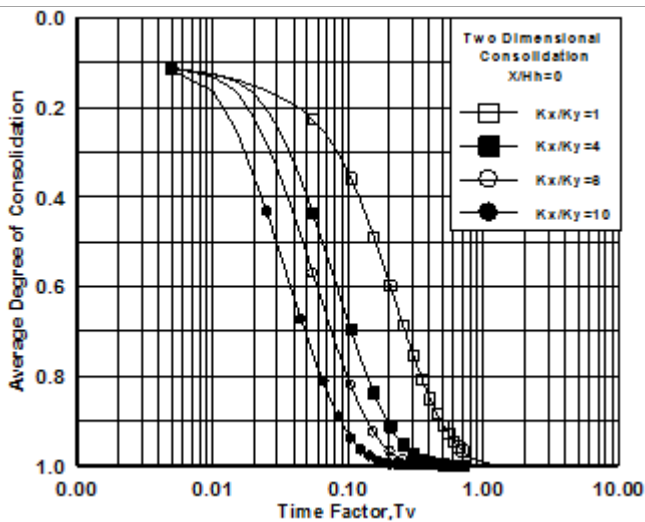


Figure 5: average degree of consolidation at  $X/H_h=0$  for different anisotropic permeability ratio

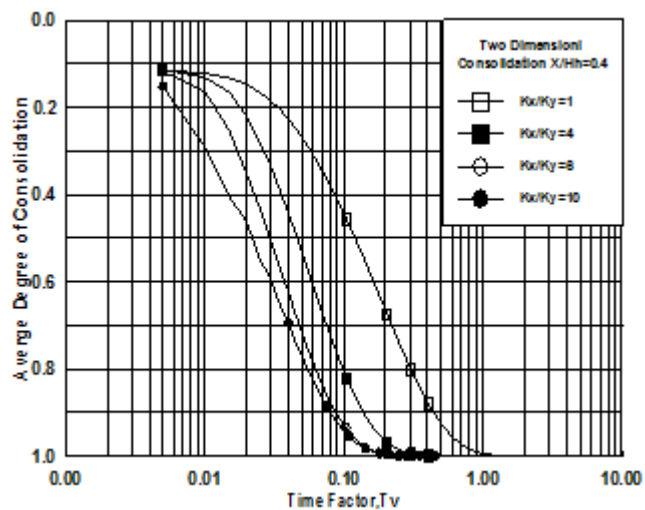


Figure 6: average degree of consolidation at  $X/H_h=0.4$  for different anisotropic permeability ratio

### 6. Effect of Anisotropic and Homogeneous Permeability

The study also considered rectangular mesh of dimensions  $H_v$  and  $H_h = 2H_v$  (Figure 7). Three cases are studied, the first case is for soil with isotropic permeability and non-homogeneous permeability in y or/and in x direction where the  $U_{av}$  is calculated at  $X/H_h = 0.4$ . The effect of anisotropic permeability on the  $U_{av}$  are studied in the other two cases by using anisotropic permeability ratio of  $k_{x1}/k_{y1} = 4$  and 10. The non-homogeneous and anisotropic permeability reflects the nature of assumed soils which approximately represent real soil.

Figure 8a shows the result of  $U_{av}$  for case of soil with isotropic and non-homogeneous permeability in y direction and homogeneous permeability in x direction.

Figure 8b shows the results of  $U_{av}$  for the case of soil with isotropic and non-homogeneous permeability in x and y direction. From the two Figures, it is found the effect of non-homogeneous permeability ratio in x-direction has slightly effect on  $U_{av}$ . It is attributed to that the increase of ratio of

non-homogeneous permeability in x-direction means increase in permeability in the region that is far from the drainage face of the soil while the region close to the drainage face still has the same value of permeability.

Figure 9a and Figure 10a show the  $U_{av}$  versus time factor,  $T_v$ , for the same previous case of soil with non-homogeneous permeability in y direction and homogenous permeability in x direction but for anisotropic permeability  $k_{x1}/k_{y1} = 4$  and  $k_{x1}/k_{y1} = 10$ . The results of  $U_{av}$  are approximately the same. The anisotropic permeability has no effect on the  $U_{av}$ .

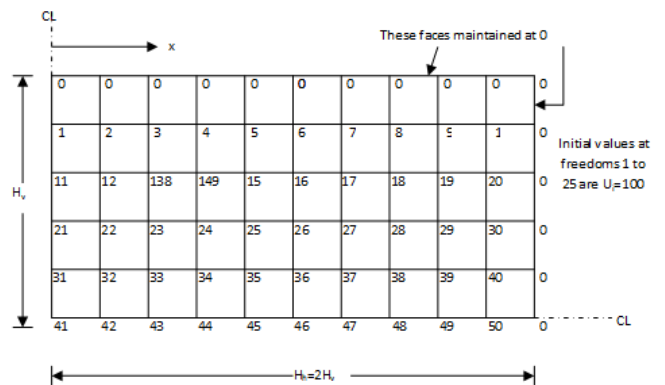


Figure 7: Grid of the studied problem

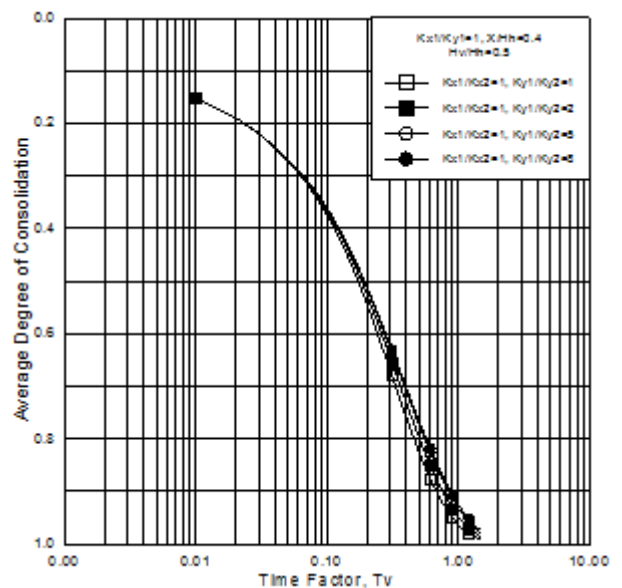
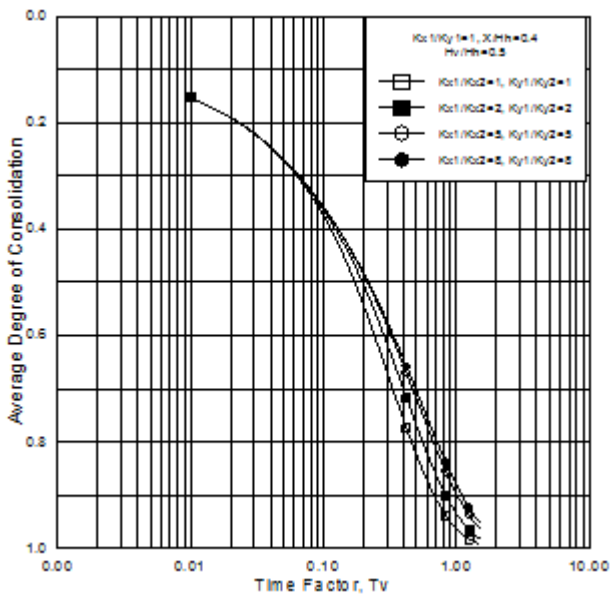
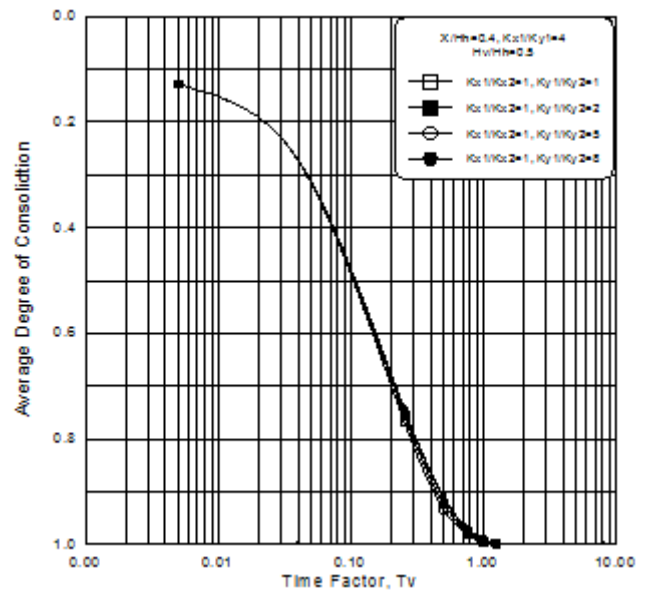


Figure 8a: Average degree of consolidation,  $U_{av}$ , versus time factor,  $T_v$ , for soil with isotropic permeability,  $k_{x1}/k_{y1} = 1$ , homogeneous permeability in x direction and non-homogeneous permeability in x direction

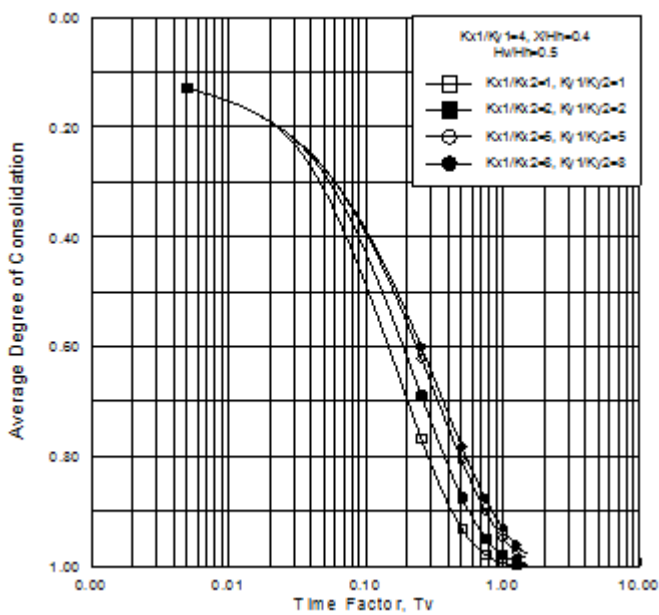
This may be attributed to that the high permeability is close to the impervious face. Figure 9b and Figure 10b show the result of  $U_{av}$  against  $T_v$  for soil with non-homogeneous permeability in both x and y direction and with same ratio of anisotropic permeability. It is obvious that the  $U_{av}$  increases as non-homogeneous permeability increases from  $k_{x1}/k_{x2} = 1$  to  $k_{x1}/k_{x2} = 8$ .



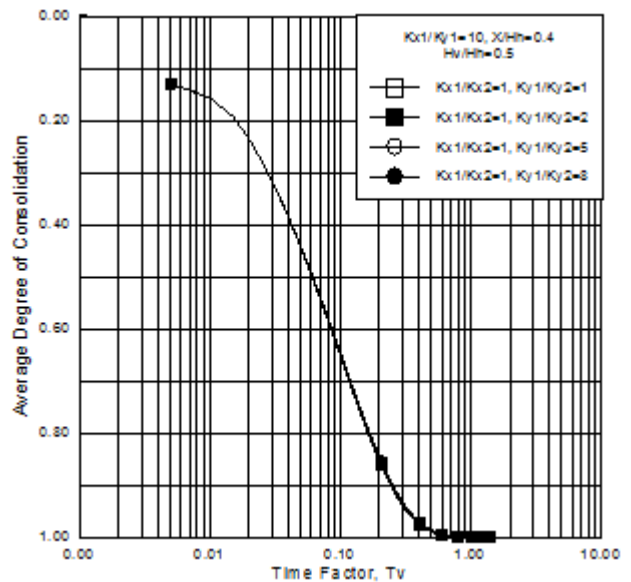
**Figure 8b:** Average degree of consolidation,  $U_{av}$ , versus time factor,  $T_v$ , for soil with isotropic permeability,  $k_{x1}/k_{y1} = 0$ , non-homogeneous permeability in y direction and non-homogeneous permeability in x direction



**Figure 9b:** Average degree of consolidation,  $U_{av}$ , versus time factor,  $T_v$ , for soil with anisotropic permeability,  $k_{x1}/k_{y1} = 4$ , non-homogeneous permeability in y direction and non-homogeneous permeability in x direction

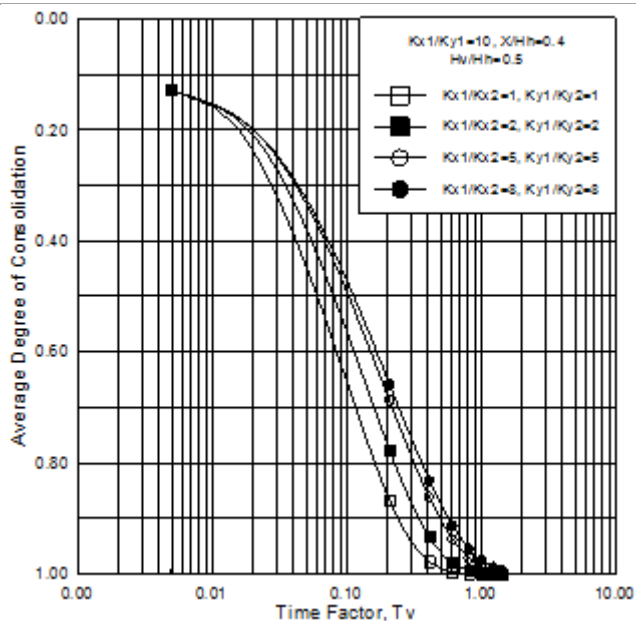


**Figure 9a:** Average degree of consolidation,  $U_{av}$ , versus time factor,  $T_v$ , for soil with anisotropic permeability,  $k_{x1}/k_{y1} = 4$ , non-homogeneous permeability in y direction and homogeneous permeability in x direction



**Figure 10a:** Average degree of consolidation,  $U_{av}$ , versus time factor,  $T_v$ , for soil with anisotropic permeability,  $k_{x1}/k_{y1} = 10$ , non-homogeneous permeability in y direction and homogeneous permeability in x direction





**Figure 10b:** Average degree of consolidation,  $U_{av}$ , versus time factor,  $T_v$ , for soil with anisotropic permeability  $k_{x1}/k_{y1}=10$ , non-homogeneous permeability in y direction and non-homogeneous permeability in x direction

## 7. Conclusion

In this study a computer program which depends on the uncoupled solution using finite element method has been used to solve problem of two-dimensional consolidation settlement. This computer program is a good tool to study the non-homogeneous and anisotropic permeability effects on the average degree of consolidation. According to the study, the following conclusions are drawn:

1. It is concluded that the average degree of consolidation decreases with the decrease of permeability with depth "non-homogeneous permeability in y direction.
2. The soil layer of low permeability which located close to the drainage face reduces the average degree of consolidation.
3. The average degree of consolidation increases when the anisotropic permeability ratio increases more than one
4. Including real variation of permeability in the soil to compute the average degree of consolidation has to be depended on in design.

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