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Characteristic of Threshold Level in a Stochastic Model

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Abstract: ***A departure of people is common in much organization. Once a large number of departures from the organization reach a certain threshold level, it could be consider as a threshold break. The time to achieve threshold is an important feature of the organization. In this paper the stochastic model is proposed to get the expectation and variance of the time to achieve the threshold level.***

Keywords: Exponential distribution, Geometric distribution, Threshold time

1. Introduction

Any departure of people can induce down to the organization, and a large number of vacancies may even cause a 'threshold break' of the organization. If the total amount of departure of people crosses a particular level, the organization reaches down in economic status which will be called a threshold break point. The organization suffers a heavy loss of manpower and it cannot be run economically without recruitment when the number of exits of personnel crosses the threshold level. Thus, the time to achieve the threshold level is an important characteristic for the management of the organization.

2. Stochastic Model

Stochastic model is to set up a projection model which looks at a single policy, an entire portfolio or an entire company. But rather than setting investment returns according to their most likely estimate, for example, the model uses random variations to look at what investment conditions might be like. Based on a set of random outcomes, the experience of the company is projected, and the outcome is noted. Then this is done again with a new set of random variables. In fact, this process is repeated thousands of times. At the end, a distribution of outcomes is available which shows not only the most likely estimate but what ranges are reasonable too. Stochastic models help to assess the interactions between variables, and are useful tools to numerically evaluate quantities. Stochastic modeling is for the purpose of estimating the probability of outcomes within a forecast to predict what conditions might be like under different situations. The random variables are usually constrained by historical data. In this paper a mathematical model is developed to obtain the expected time of breakdown point or the expected time to reach the threshold level, in the context of manpower planning with the assumptions that the times between decision epochs are independent and identically distributed (i.i.d.) random variable, the number of departure at each decision epoch are i.i.d. random variables and that the threshold level is a random variable following a Geometric distribution.

3. Stochastic Model on Expected Time

T - Continuous random variable denoting the time of threshold break of the organization
t - The time of occurrence of the decision.
G(t) - Cumulative distribution function of T.
$M_T(s)$ - the moment generating function of T.
$M_X(s)$ - moment generating function of X.

The number of decisions made in (0, t] from a renewal process $V_i(t) = F_i(t) - F_{i+1}(t)$

where $F_0(t) = 1$

Let E(T) and V(T) be the mean and variance of the time for achieve threshold level.

Probability of Continuous random variable to cross the threshold level T is greater than t is Exactly i decisions in (0, t] and the threshold level is not reached.

That is $P(T > t) = \sum_{i=0}^{\infty}[F_i(t) - F_{i+1}(t)]\beta^i$

Thus Cumulative distribution function of T is G(t)

$$G(t) = (1 - \beta)\sum_{i=1}^{\infty}F_i(t)\beta^{i-1}$$

The probability density function of t is given by

$$G'(t) = (1 - \beta)\sum_{i=1}^{\infty}f_i(t)\beta^{i-1}$$

The moment function of T is

$$M_T(s) = \frac{(1-\beta)M_X(s)}{1-\beta M_X(s)}$$

For s with $M_X(s)$ less than $\frac{1}{\beta}$ which gives

$$E[T] = \frac{E[X]}{1-\beta}$$

$$V(T) = \frac{(1-\beta)Var[X] + \beta(E[X])^2}{(1-\beta)^2}$$

Where $\beta = E[(1-\beta)U]$

Let X_i be the independent and identically distributed (i.i.d) exponential random variable with probability density function f(x) = $\frac{1}{a}e^{-\frac{x}{a}}$

Where x and a are positive values.

Let U_i be i.i.d poisson random variable with parameter λ then $\beta = e^{-\lambda\theta}$

Hence we obtain

$E[T] = \frac{\beta}{1-e^{-\lambda\theta}}$ and $V(T) = \frac{a^2}{(1-e^{-\lambda\theta})^2}$

the threshold level follows a Geometric distribution with parameter θ.

4. Conclusion

For fixed a, β and various values of λ, The mean threshold number decreases, the threshold break time also decreases, and as the rate of departure of people increases, the breakdown time of the organization decreases. This model suggests the time at which the organization should think of recruitment to replace the departure of personnel.

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