Trajectory Tracking of a 2-Link Robotic Manipulator Using Adaptive Terminal Sliding Mode Controller

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Abstract: This paper proposes an adaptive terminal sliding mode controller which combines adaptive control and sliding mode control to control a 2-link nonlinear robotic manipulator with uncertain parameters. We use an adaptive algorithm based on the concept of sliding mode control to alleviate the chattering phenomenon of control input. Adaptive laws are developed to obtain the gain of switching input and the boundary layer parameters. The stability and convergence of the robotic manipulator control system are guaranteed by applying the Lyapunov stability theorem. Simulation results demonstrate that the chattering of control input can be alleviated effectively. The proposed controller scheme can assure robustness against a large class of uncertainties and achieve good trajectory tracking performance.

Keywords: Trajectory tracking, robotic manipulators, sliding mode control, adaptive control, Lyapunov stability theorem

1. Introduction

A chattering free adaptive terminal sliding mode controller for uncertain systems is proposed in this paper. The actual control law is obtained by integrating the discontinuous derivative control signal and hence it is continuous. The sliding mode control is based on the design of a high-speed switching control law that drives the system’s trajectory onto a user-chosen hyperplane in the state space, also known as sliding surface. The main feature of sliding mode control are the following: (1) fast response and good transient performance; (2) robustness against a large class of perturbations or model uncertainties; and (3) the possibility of stabilizing some complex nonlinear systems which are difficult to stabilize by state feedback control laws. Chattering phenomenon can cause some problems such as saturation and heat for mechanical parts of robot manipulators or drivers. To reduce or eliminate the chattering, various papers have been reported by many researchers and classified into two most important methods, namely, boundary layer saturation method and estimated uncertainties method. Nonlinear control methodologies are more general because they can be used in both linear and nonlinear systems. These controllers can solve different problems such as, invariance to system uncertainties and resistance to the external disturbance. The most common non linear methodologies that have been proposed to solve the control problem consist of the following methodologies:

- Feedback Linearization Control Methodology
- Passivity-Based Control Methodology
- Sliding Mode Control Methodology
- Robust Lyapunov-Based Control Methodology
- Adaptive Control Methodology
- Artificial Intelligence-Based Methodology

The requirement of prior knowledge about the uncertainty bands for designing terminal sliding mode controllers is not a necessary requirement in the proposed controller. Trajectory tracking of a 2-link robotic manipulator which is a non-linear system with mismatched uncertainties also considered in our simulation study. Simulation results demonstrate that the proposed control strategy is successful in eliminating the undesired chattering in the control input while ensuring satisfactory stabilization as well as tracking performances. Hence the proposed controller is suitable for practical applications.

2. Description

2.1 Tracking Control of a Robotic Manipulator

Dynamics of an n-link robotic manipulator can be expressed as,

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + \tau_d \]

where \( \dot{q}, \ddot{q} \in \mathbb{R}^n \) represent the position, velocity and acceleration of the joints respectively.

\[ M(q) = M_0(q) + \Delta M(q) \in \mathbb{R}^{n \times n} \]

stands for the inertia matrix, \( C(q, \dot{q}) = C_0(q, \dot{q}) + \Delta C(q, \dot{q}) \in \mathbb{R}^{n \times n} \)

is the centripetal Coriolis matrix, \( G(q) = G_0(q) + \Delta G(q) \in \mathbb{R}^{n} \)

is the gravitational vector, \( \tau \in \mathbb{R}^n \) is the joint torque vector and \( \tau_d \in \mathbb{R}^n \) is the disturbance torque vector. Here \( M_0(q), C_0(q, \dot{q}), G_0(q) \) are the nominal terms and \( \Delta M(q), \Delta C(q, \dot{q}), \Delta G(q) \) represent the perturbations in the system matrices.

Then the dynamic model of the robotic manipulator can be written as,

\[ M_0(q)\ddot{\hat{q}} + C_0(q, \dot{\hat{q}})\dot{\hat{q}} + G_0(q) = \tau + \tau_d + F(q, \dot{q}, \ddot{q}) \]

where \( F(q, \dot{q}, \ddot{q}) = -\Delta M(q) - \Delta C(q, \dot{q}) - \Delta G(q) \in \mathbb{R}^n \)

is the lumped system uncertainty which is bounded by the following function

\[ ||F(q, \dot{q}, \ddot{q})|| \leq \rho_1 + \rho_2 ||q|| + \rho_3 ||\dot{q}|| \]

where \( \rho_1, \rho_2, \rho_3 \) are positive constants.

Suppose the control objective is to make the robotic manipulator track a reference trajectory. Let \( q^* \) and \( q \) be the
desired and actual position vectors. The tracking error and its derivatives are defined as

\[ e = q - q_d, \quad \dot{e} = \dot{q} - \dot{q}_d, \quad \ddot{e} = \ddot{q} - \ddot{q}_d \]

Using (1.1),

\[ \ddot{e} = M_0^{-1}(q)\tau + F(q, \dot{q}, \ddot{q}) - C_0(q, \dot{q})\dot{q} - G_0(q) \]  

(1.3)

The time derivative of (1.3) yields

\[ \frac{d}{dt} \ddot{e} = M_0^{-1}(q)\ddot{q} + \frac{d}{dt}(C_0(q, \dot{q})\dot{q} + G_0(q)) + M_0^{-1}(q)\dddot{q} + M_0^{-1}(q)\tau + M_0^{-1}(q)F(q, \dot{q}, \ddot{q}) - C_0(q, \dot{q})\dot{q} - G_0(q) \]

(1.4)

where \( F(q, \dot{q}, \ddot{q}) = M_0^{-1}(q)\dddot{q} + M_0^{-1}(q)\tau + M_0^{-1}(q)F(q, \dot{q}, \ddot{q}) \)

such that,

\( F(q, \dot{q}, \ddot{q}) \leq \rho_0 + \rho_1||q|| + \rho_2||q||^2 \)  

(1.5)

Here \( \rho_0, \rho_1, \rho_2 \) are positive constants.

**Remark 5.5:** The assumptions in the above inequalities are valid as the input disturbance \( \tau_d \) is assumed to be bounded, i.e. \( ||\tau_d|| < \chi \) where \( \chi \) is a positive constant. Furthermore, the modeling uncertainty \( F(q, \dot{q}, \ddot{q}) \) is also bounded by the assumption \( ||F(q, \dot{q}, \ddot{q})|| \leq \rho_0 + \rho_1||q|| + \rho_2||q||^2 \).

Let us consider the linear sliding surface as,

\[ s = \dot{e} + ce \]  

(1.6)

where \( c = \text{diag}(c_1, ..., c_n) \) is a design matrix. The first and second derivative of (1.6) can be obtained as,

\[ \dot{s} = \dot{e} + c \dot{e} \]

\[ \ddot{s} = \dddot{e} + c \dddot{e} \]

The nonsingular terminal sliding manifold (NTSM) for an n-link robotic manipulator is chosen as

\[ \sigma = s + \beta \]  

(1.7)

where \( \beta = \text{diag}(\beta_1, \beta_2, ..., \beta_n) \) is a design matrix.

Taking the derivative of (1.7) yields

\[ \dot{\sigma} = \dot{s} + p(s) + (p(s))^{-1} \dot{s} = \beta(p(q))^{-1}(s + (q/p)) \beta - \beta^{-1}(s + (q/p)) \beta - \beta^{-1} \]  

(1.8)

For an n-link robotic manipulator (1.0), if the NTSM manifold is chosen as (1.7), then the tracking error \( e \) will converge to zero if the time derivative of the control input is selected as,

\[ \dot{\tau} = u_1 + u_2 \]  

(1.9)

### 2.2 Adaptive Terminal Sliding Mode Controller

where,

\[ u_1 = -K^+M_0(q)\sigma - M_0(q)(\bar{B}_2^0 + \bar{B}_2^1||q||^2)\text{sign}(\sigma) \]

(2.0)

Here \( K^+ = \text{diag}(K_1^+, ..., K_n^+) \) is a positive matrix.

Defining the adaptation error as \( \hat{\theta} = \dot{\theta} \) and \( \hat{\theta} = \dot{\theta} \), the parameters \( \hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2 \) are to be estimated by using the adaptation law.

\[ \dot{\hat{\theta}}_0 = \frac{1}{v_0} \left( s^{(E)} \right)^{-1} \beta ||s^{(E)}||^2 ||\sigma|| \]  

(2.2)

\[ \dot{\hat{\theta}}_1 = \frac{1}{v_1} \left( s^{(E)} \right)^{-1} ||s^{(E)}||^2 ||\sigma|| \]  

(2.3)

\[ \dot{\hat{\theta}}_2 = \frac{1}{v_2} \left( s^{(E)} \right)^{-1} ||s^{(E)}||^2 ||\sigma|| \]  

(2.4)

Where \( v_0, v_1, v_2 \) are the positive tuning parameters. The dead zone technique is used to modify the adaptive tuning law as

\[ \dot{\hat{\theta}}_n = \frac{1}{v_n} \left( s^{(E)} \right)^{-1} \beta ||s^{(E)}||^2 ||\sigma|| \]  

(2.5)

where \( \epsilon \) is a small positive constant.

Now, the adaptive terminal sliding mode control law for the robotic manipulator is obtained as,

\[ u_1 = -K^+M_0(q)\sigma - M_0(q)(\bar{B}_2^0 + \bar{B}_2^1||q||^2)\text{sign}(\sigma) \]

(2.6)

A Lyapunov function is defined as \( V = \frac{1}{2} \sigma^2 \) and using the control law it is easy to find that,

\[ \dot{V} = \sigma \dot{\sigma} \]

\[ = \sigma [\beta^T F - k_s \sigma - k_c \text{sign}(\sigma)] \]

\[ = \sigma (Q - k_s \sigma - k_c \text{sign}(\sigma)) \]

\[ \leq Q\sigma^2 - k_s \sigma^2 \leq -n||\sigma|| \]
Clearly, the above equation implies that if, 

\[ k_1 \geq 0 \text{ and } k_2 > Q \]

the control law forces the sliding manifold \( \sigma \) to zero in finite time.

Using Lyapunov stability criterion,

The NTSM manifold \( \sigma \) can be shown to possess finite time reachability to zero which ensures that the tracking error of the robotic manipulator \( e = q - q_d \) converges to zero in finite time.

3. Simulation Results

The proposed chattering free adaptive terminal sliding mode (TSM) controller is applied for trajectory tracking of a two-link rigid robotic manipulator shown in Fig. 5.4. For the above two-link manipulator, the dynamic equation (5.27) has the following parameters,

\[
\begin{align*}
M(q) &= \begin{bmatrix}
    m_1 + m_2 & m_2 & 2m_1m_2 \cos(q_1) + 1 & m_2 + m_2 \cos(q_2) \\
    m_2 & m_1 + m_2 & 2m_1m_2 \sin(q_1) & m_1 + m_2 \\
    2m_1m_2 \cos(q_1) & 2m_1m_2 \sin(q_1) & m_1 & m_1 \\
    2m_1m_2 \sin(q_1) & 2m_1m_2 \cos(q_1) & m_1 & m_1 \\
\end{bmatrix} \\
C(q, \dot{q}) &= \begin{bmatrix}
    -m_2m_1 \sin(q_2)q_2^2 - 2m_2L_2 \sin(q_2)q_2 \dot{q}_2 \\
    m_2m_1 \sin(q_2)q_2^2 + 2m_2L_2 \sin(q_2)q_2 \dot{q}_2 \\
\end{bmatrix} \\
G(q) &= \begin{bmatrix}
    (m_1 + m_2)L_1 \cos(q_1) + m_2L_2 \cos(q_1 + q_2) \\
    m_2L_2 \cos(q_1 + q_2) \\
\end{bmatrix}
\end{align*}
\]

**Figure 1.1:** Configuration of a two-link robotic manipulator

Here \( q(t) = [q_{1}(t), q_{2}(t)]^{T} \) is the angular position vector where \( q_{1}(t) \) & \( q_{2}(t) \) are the angular positions of joints 1 & 2. \( M(q) \) is the inertia matrix, \( C(q, \dot{q}) \) is the centripetal Coriolis matrix. \( G(q) \) is the gravity vector and \( \tau = [\tau_1, \tau_2]^{T} \) is the applied torque. The two-link robotic manipulator has four inner states \( x_{1}(t) = \dot{q}_{1}(t), x_{2}(t) = \dot{q}_{2}(t), x_{3}(t) = \ddot{q}_{1}(t), \) two output states \( y_{1}(t) = q_{1}(t) \) and \( y_{2}(t) = \dot{q}_{2}(t) \) and two inputs \( u_{1}(t) = \tau_1 \) and \( u_{2}(t) = \tau_2 \). Friction terms are ignored. Table 1.0 lists the physical parameters of the two-link robotic manipulator considered in the simulation study.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>( l_1 )</td>
<td>Length of the first link</td>
<td>1m</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>Length of the second link</td>
<td>0.85m</td>
</tr>
<tr>
<td>( J_1 )</td>
<td>Moment of inertia of the D.C. motor 1</td>
<td>5kg - m</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>Moment of inertia of the D.C. motor 2</td>
<td>5kg - m</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>Mass of the link 1</td>
<td>0.5kg</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>Mass of link 2</td>
<td>1.5kg</td>
</tr>
<tr>
<td>( n_{1} )</td>
<td>Nominal Mass of link 1</td>
<td>0.4kg</td>
</tr>
<tr>
<td>( n_{2} )</td>
<td>Nominal Mass of link 2</td>
<td>1.2kg</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravitational constant</td>
<td>9.81m/s²</td>
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**Table 1.0:** Physical parameters of two-link robotic manipulator

The reference signals are \( q_{d1} = 1.25 - (7/5)e^{-t} + (7/20)e^{-4t} \) and \( q_{d2} = 1.25 - e^{-t} - (1/4)e^{-4t} \). The inertial states are selected as \( q_{1}(0) = 0, q_{2}(0) = 2.5, \dot{q}_{1}(0) = 0, \dot{q}_{2}(0) = 0 \). The external disturbances considered are \( \tau_{d1} = 2 \sin(t) + 0.5 \sin(200 \pi \cos(t)) \) and \( \tau_{d2} = 2 \cos(2t) + 0.5 \sin(200 \pi \cos(t)) \). The parameters of the proposed controller are selected as \( K = \text{diag}(0.023, 0.023), \beta = \text{diag}(0.05, 0.45) \) and \( R = \text{diag}(60, 60) \).

The simulations are carried out in MATLAB – Simulink platform by using ODE 4 solver with a fixed step size of 0.005sec.

The tracking response and control inputs obtained by using the NTSM controller.

**Figure 1.2:** Output tracking response of Joint 1 & Joint 2 with the controller
Simulation results obtained by applying the proposed adaptive TSM control laws (2.0) and (2.6) are shown in Fig. 1.4 - Fig. 1.7. It is observed from Fig. 1.4 that both the joints 1 and 2 track the reference trajectory faithfully. The control inputs applied to both the joints show no chattering as is evident in Fig. 1.5. The convergence plots of the estimated parameters $\hat{b}_0, \hat{b}_1, \hat{b}_2$ are shown in Fig. 1.6. The sliding surfaces and the sliding manifolds are plotted in Fig. 1.7 which confirms that these converge to zero quickly.
The output and input performances of the proposed adaptive TSM controller as well as the controllers designed by Feng et al. for the two-link robotic manipulator are tabulated in Table 1.2. It is noted that the proposed adaptive TSM controller offers comparable tracking performance by applying a smoother control input having minimal total variation as compared to the controllers designed by Feng et al. Moreover, the overall control energy spent in the case of the proposed adaptive TSM controller is not more than those in the other methods.

**Table 1.1: Comparison of controller performance**

<table>
<thead>
<tr>
<th>Types Of Controller</th>
<th>IAE</th>
<th>Total Variation</th>
<th>2-norm of input</th>
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<tr>
<td>Feng et al.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint 1</td>
<td>14.8</td>
<td>205.01</td>
<td>72.22</td>
</tr>
<tr>
<td>Joint 2</td>
<td>7.22</td>
<td>127.92</td>
<td>207.79</td>
</tr>
<tr>
<td>Proposed adaptive TSM controller</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint 1</td>
<td>7.01</td>
<td>114.29</td>
<td>114.61</td>
</tr>
<tr>
<td>Joint 2</td>
<td>4.05</td>
<td>74.02</td>
<td>140.03</td>
</tr>
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</table>

**5. Future Scope**

Discrete Sliding Mode Controllers will be easier to implement as compared to microcontrollers and digital signal processors (DSPs) which can also be used to control a huge number of continuous systems. The proposed design method may be extended by using intelligent controllers based on Neural Networks and Fuzzy Logic to incorporate flexibility and intelligence.

**References**


Author Profile

Saurav Chanda is pursuing 3rd Year B.Tech in Electronics & Communication Engineering from Dibrugarh University Institute of Engineering & Technology, Dibrugarh University, Assam. He is quite interested in the field of Robotics & Non Linear Control from the very outset.

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