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A Region Based Variational Method for Image Segmentation and Bias Correction with Application to MRI

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Abstract: ***This paper implements a region-based method for image segmentation, which is able to deal with intensity inhomogeneities in the segmentation. First, based on the MRI image model with bias error in it we derive local intensity clustering property of the image intensities, and define a local clustering criterion function for the image intensities in a neighborhood of each point. In a level set formulation, this criterion defines an energy in terms of the level set functions that represent a partition of the image domain and a bias field that accounts for the intensity inhomogeneity of the image. Therefore, by minimizing this energy, our method is able to simultaneously segment the image and estimate the bias field, and the estimated bias field can be used for bias correction. Post Enhancement is applied to improve the contrast and brightness which increases the SNR. In this paper, the performance is evaluated using the plots for SNR, MSE (Mean Square Error).***

Keywords: Bias correction, image segmentation, intensity in homogeneity, level set, MRI

1. Introduction

Intensity inhomogenity occurs due to various factors, such as spatial variations in illumination and imperfections of imaging devices, Intensity inhomogeneity occurs in the images which complicate many problems in image processing and computer vision. The level set method, originally used as numerical technique for tracking interfaces and shapes [14], has been increasingly applied to image segmentation in the past decade [2], [4], [5], In the level set method, contours or surfaces are represented as the zero level set of a higher dimensional function, usually called a level set function. With the level set representation, the image segmentation problem can be formulated and solved in a principled way based on well-established mathematical theories, including calculus of variations and partial differential equations (PDE).An advantage of the level set method is that numerical computations involving curves and surfaces can be performed on a fixed Cartesian grid without having to parameterize these objects. Moreover, the level set method is able to represent contours/surfaces with complex topology and change their topology in a natural way.

Existing level set methods for image segmentation can be categorized into two major classes: region-based models, and edge-based models [3], [7].Region-based models aim to identify each region of interest by using a certain region descriptor to guide the motion of the active contour. Level set methods are proposed based on a general piecewise smooth (PS) formulation originally proposed by Mumford and Shah. These methods do not assume homogeneity of image intensities, and therefore are able to segment images with intensity inhomogeneties. However, these methods are computationally too expensive and are quite sensitive to the initialization of the contour [10], which greatly limits their utilities. Edge-based models use edge information for image segmentation. These models do not assume homogeneity of image intensities, and thus can be applied to images with intensity inhomogeneities. From a generally accepted model of images with intensity inhomogeneities, we derive a local intensity clustering property, and therefore define a local clustering criterion function for the intensities in a neighborhood of each point. This local clustering criterion is integrated over the neighborhoodcenter to define energy functional, which is converted to a level set formulation. Minimization of this energy is achieved by an interleaved process of level set evolution and estimation of the bias field. As an application, our method can be used for segmentation and bias correction of magnetic resonance (MR) images.

2. Framework for Segmentation and Bias Correction

2.1 Image Model

In order to deal with intensity inhomogeneities in image segmentation, we formulate our method based on an image model that describes the composition of real-world images, in which intensity inhomogeneity is attributed to a component of an image. In this paper, we consider the following multiplicative model of intensity inhomogeneity. From the physics of imaging in a variety of modalities (e.g. camera and MRI), an observed image can be modelled as

$$I = bJ + n$$

Where J is the true image, b is the component that accounts for the intensity inhomogeneity, and n is additive noise. The component b is referred to as a bias field (or shading image).The true image J measures an intrinsic physical property of the objects being imaged, which is therefore assumed to be piecewise (approximately) constant. The bias field b is assumed to be slowly varying. The additive noise n can be assumed to be zero-mean Gaussian noise.

In this paper, we consider the image I as a function I : $\Omega \to \Re$ defined on a continuous domain Ω.The assumptions about the true image J and the bias field b can be stated more specifically as follows:

(A1)The bias field b is slowly varying, which implies that can be well approximated by a constant in a neighborhood of each point in the image domain.

(A2)The true image J approximately takes N distinct constant values $C1,\ldots\ldots,CN$ in disjoint regions $\Omega 1,\ldots..\Omega N$, respectively, where $\{\Omega_i\}_{i=1}^{N}$ forms a partition of the image domain,

$$\text{i.e. } \Omega = \bigcup_{i=1}^{N} \Omega_i \text{ and } \Omega_i \cap \Omega_j = \varphi \text{ for } i \neq j$$

2.2 Local Intensity Clustering Property

Region- based image segmentation methods typically rely on a specific region descriptor (e.g. intensity mean or a Gaussian distribution) of the intensities in each region to be segmented. However, it is difficult to give such a region descriptor for images with intensity inhomogeneities. Moreover, intensity inhomogeneities of ten lead to overlap between the distributions of the intensities in the regions $\Omega 1,\ldots..\Omega N$. Therefore, it is impossible to segment these regions directly based on the pixel intensities. Nevertheless, the property of local intensities is simple, which can be effectively exploited in the formulation of our method for image segmentation with simultaneous estimation of the bias field.

Based on the image model shown in section 2.1 and the assumptions A1 and A2, we are able to derive a useful property of local intensities, which is referred to as a local intensity clustering property as described and justified below. To be specific, we consider a circular neighborhood with a radius ρ centered at each point $y \in \Omega$, defined by $O_y \to \{x : |x-y| \leq \rho\}$. The partition $\{\Omega_i\}_{i=1}^{N}$ of the entire domain Ω induces a partition of the neighbourhood O_y, i.e., $\{oy \cap \Omega i\}_{i=1}^{N}$ forms a partition of oy. For a slowly varying bias field b, the values $b(x)$ for all x in the circular neighbourhood oy are close to $b(y)$ i.e., $b(x) \approx b(y)$ for $x \in oy$

Thus the intensities $b(x)J(x)$ in each sub region $oy \cap \Omega i$ are close to the constant $b(y)c_i$,

i.e. $b(x)J(x) \approx b(y)c_i$ for $x \in oy \cap \Omega i$

Then in view of the image model, we have

$$I(x) \approx b(y)c_i + n(x) \text{ for } x \in oy \cap \Omega i$$

Where n(x) is additive zero mean Gaussian noise. Therefore the intensities in the set

$I_y^i = \{I(x) : x \in oy \cap \Omega i\}$ form a cluster with cluster center $m \approx b(y)c_i$

The above described local intensity clustering property indicates that the intensities in the neighborhood oy can be classified into N clusters, with canters, $m \approx b(y)c_i$. This allows us to apply the standard K-means clustering to classify these local intensities.

Specifically, for the intensities $I(x)$ in the neighborhood oy, the K-means algorithm is an iterative process to minimize the clustering criterion [19], which can be written in a continuous form as

$$F_y = \sum_{i=1}^{N} \int_{oy} |I(x) - m_i|^2 \, u_i(x) dx$$

Where m_i is the cluster center of the I th cluster, u_i is the membership function of the region Ωi to be determined.

2.3 Local Clustering Criterion Function

In view of the above clustering criterion and the approximation of the cluster center by $m \approx b(y)c_i$, we define a clustering criterion for classifying the intensities in oy as

$$\varepsilon_y = \sum_{i=1}^{N} \int_{\Omega i \in O_y} K(y-x) \, |I(x) - b(y)ci|^2 dx$$

Where $K(y-x)$ is introduced as a nonnegative window function, also called kernel function, such that $K(y-x)$ =0 for $x \notin oy$ .With the window function, the clustering criterion function ε_y can be rewritten as

$$\varepsilon_y = \sum_{i=1}^{N} \int_{\Omega i} K(y-x) \, |I(x) - b(y)ci|^2 dx$$

The local clustering criterion function ε_y evaluates in the neighborhood oy given by the partition of $\{oy \cap \Omega i\}_{i=1}^{N}$ of oy .The smaller the value of ε_y , the better the classification. Naturally, we define the optimal partition $\{\Omega i\}_{i=1}^{N}$ of the entire domain as the one such that the local clustering criterion function ε_y is minimized for all y in Ω .Therefore, we need to jointly minimize ε_y for allin Ω .This can be achieved by minimizing the integral of ε_y with respect to y over the image domain.

Therefore, we define energy

$$\varepsilon = \int\left(\sum_{i=1}^{N}\int_{\Omega i} K(y-x)\,|\,I(x)-b(y)ci\,|^{2}dx\right)dy$$

Image segmentation and bias field estimation can be performed by minimizing this energy with respect to the regions $\Omega 1,\ldots.\Omega_N$, constants $C1,\ldots\ldots.,CN$, and bias field b .The Kernel function K is chosen as a truncated Gaussian function defined by

$$K(u)=\begin{cases}\dfrac{1}{a}e^{-|u|^{2}/2\sigma^{2}} & \text{for } |\,u\,| \le \rho\end{cases}$$

Where a is a normalization constant such that $\int K(u)=1$, σ is the standard deviation (or the scale parameter)of the Gaussian function, and ρ is the radius of the neighbourhood oy

3. Level Set Formulation and Energy Minimization

3.1 Level Set Formulaltion

Energy ε is expressed in terms of the regions $\Omega 1,\ldots.\Omega_N$ .It is difficult to derive a solution to the energy minimization problem from this expression of ε . The energy is converted to a level set formulation by level set functions, with a regularization functions. In the level set formulation, the energy minimization can be solved by using well-established variational methods [6]. In level set methods, a level set function is a function that takes positive and negative signs, which can be used to represent a partition of the domain Ω into two disjoint regions $\Omega 1$ and $\Omega 2$and. Let $\Omega \to \Re$ be a level set function, then its signs define two disjoint regions

$$\Omega_1:\{x:\phi(x)>0\}$$

$$\Omega_2:\{x:\phi(x)<0\}.$$

Which form a partition of the domain Ω .For the case of N >2, two or more level set functions can be used to represent N regions $\Omega 1,\ldots.\Omega_N$ .The level set formulation of the energy ε for the cases of N =2 and N >2, called two-phase and Multiphase formulations, respectively, will be given in the next two subsections.

The image domain is Ω into two disjoint regions $\Omega 1$ and $\Omega 2$ .In this case, a level set function ϕ is used to represent the two regions represented with their membership functions defined by $M1(\phi)=H(\phi)$ and $M2(\phi)=1-H(\phi)$ can be represented with tons and given .The regions, respectively, where H is the Heaviside function. Thus for the case of $N=2$, the energy can be expressed as the level set formulation

$$\varepsilon = \int\sum_{i=1}^{N}\left(\int K(y-x)\,|\,I(x)-b(y)c_i\,|^{2}\,dy\right)M_i(\phi(x))dx$$

Thus the level set function ϕ , the vector C and the bias field b are the variables of the energy .We can rewrite the above equation in the following form.

$$\varepsilon(\phi,c,b) = \int\sum_{i=1}^{N} e_i(x)M_i(\phi(x))dx$$

Where e_i is the function defined by

$$e_i(x) = \int K(y-x)\,|\,I(x)-b(y)c_i\,|^{2}\,dy$$

The functions e_i can be computed using the following equivalent expression:

$$e_i(x) = I^{2}1_K - 2c_iI(b*K) + c_i^{2}(b^{2}*K)$$

Where * is the convolution operation, and 1_K is the function defined by

1_K (x)= $K(y-x)$, which is equal to 1 everywhere except near the boundary of the image domain

The above defined energy $\varepsilon(\phi,c,b)$ is used as the data term in the energy of the proposed variational level set formulation which is defined by

$$\mathrm{F}(\phi,c,b) = \varepsilon(\phi,c,b) + \nu L(\phi) + \mu R(\phi)$$

With $L\,(\phi)$ and $R_p(\phi)$ being the regularization terms as defined below. The energy term $L\,(\phi)$ is defined by

$$L(\phi) = \int|\,\nabla H(\phi)\,|\,dx$$

Which computes the arc length of the zero level contour of level set function and therefore serves to smooth the contour by penalizing its arc length. The energy term is given by

$$R_p(\phi) = \int p(|\nabla \phi|dx$$

3.2 Energy Minimization

By minimizing this energy, we obtain the result of image segmentation given by the level set function ϕ and the estimation of bias field b .We give the solution to the energy minimization with respect to ea h variable as follows

Energy minimization with respect to ϕ

For fixed C and b , the minimization of $\mathrm{F}(\phi, c, b)$ with respect to ϕ can be achieved by using standard gradient descent method, namely, solving the gradient flow equation

$$\frac{\partial \phi}{\partial t} = -\frac{\partial F}{\partial \phi}$$

Energy minimization with respect to C:

For fixed ϕ and, the optimal C that minimizes the energy $\varepsilon(\phi, c, b)$ is given by

$$\hat{c}_i = \frac{\int (b * K) I u_i dy}{\int (b^2 * K) u_i dy}, i = 1,2,\ldots, N$$

with

$$u_i(y) = M_i(\phi(y))$$

Energy Minimization with Respect to b :

For fixed ϕ and C, the optimal b that minimizes the energy $\varepsilon(\phi, c, b)$, denoted by $\hat{b}$, is given by

$$\hat{b} = \frac{(IJ^{(1)}) * K}{J^{(2)} * K}$$

where

$$J^{(1)} = \sum_{i=1}^{N} c_i u_i$$

$$J^{(2)} = \sum_{i=1}^{N} c_i^2 u_i .$$

Note that the convolutions with a kernel function K confirms the slowly varying property of the derived optimal estimator $\hat{b}$ of the bias field

4. Parameters for implementation of the segmentation

In numerical implementation heavi side function H is replaced by smooth function that approximates H, called the smooth heavi side function.

$$H_{\epsilon}(x) = \frac{1}{2}\left[1 + \frac{2}{\pi} arc\left(\frac{x}{\epsilon}\right)\right]$$

Accordingly the dirac delta function which is the derivative of the heavi side function is replaced by derivative of H_{ϵ} which is computed by

$$\delta_{\epsilon}(x) = H'_{\epsilon}(x) = \frac{1}{\pi}\frac{\epsilon}{\epsilon^2 + x^2}$$

The choice of the parameters in our model is easy. Some of them, such as the parameters μ and the time step Δt , can be fixed as μ =1.0 and Δt =0.1. Our model is not sensitive to the choice of the parameters. The parameter σ and the size of the neighborhood O_y

5. Experimental Results

We demonstrate the method in the two-phase case (i.e.).The parameter σ is set to 4 for the experiments in this section. All the other parameters are set to the default values.

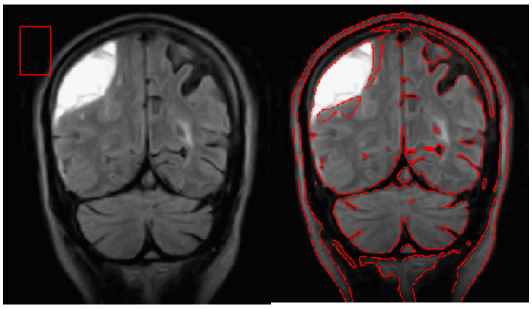

Figure 2: Segmentation of MRI image of Brain tumour

The curve evolution processes are depicted by showing the initial contours (in the left column), and the final contours (in the right column) on the images shown in fig. Intensity in homogeneities can be clearly seen in fig.2. Our method is able to provide a desirable segmentation result for such images.

The estimated bias field $\hat{b}$ by our method can be used for intensity inhomogeneity correction (or bias correction). Given the estimated bias field $\hat{b}$, the bias corrected image is computed as the quotient $I/\hat{b}$
To demonstrate the effectiveness of our method in simultaneous segmentation and bias field estimation, we applied the method to MRI image.

5.1 MR Image Segmentation and Bias Correction

In this subsection, we focus on the application of the method to segmentation and bias correction of brain MR images. We first show the results for 3T MR images in the figure. These images exhibit obvious intensity inhomogeneities. The segmentation results, computed bias fields, bias corrected images, are shown in.

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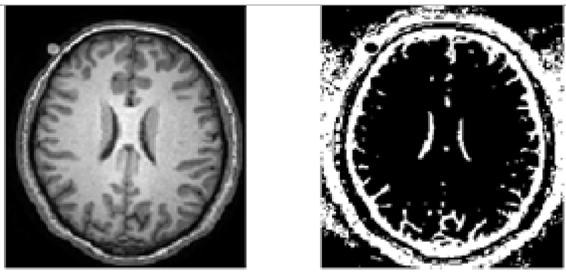
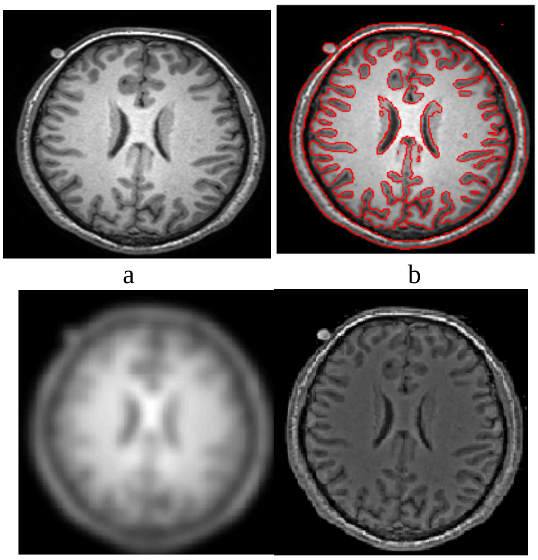


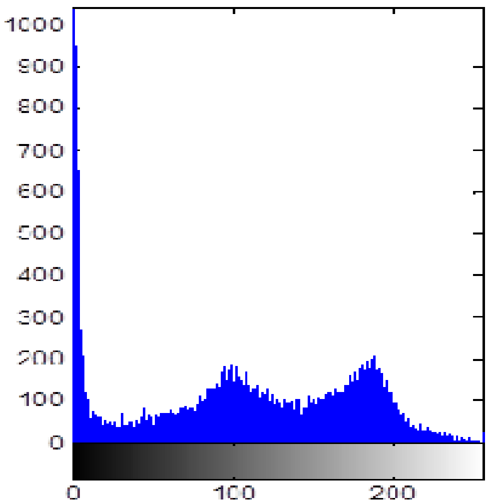
Figure 3
a) Input image, b) Initial feature segmented image



a b
c d

Figure 4
a) Original image b) Segmented Image
c) Blurred image (Estimated Bias field), d) Bias corrected image

It can be seen that the intensities within each tissue become quite homogeneous in the bias corrected images. The improvement of the image quality in terms of intensity homogeneity can be also demonstrated by comparing the histograms of the original images and the bias corrected images.



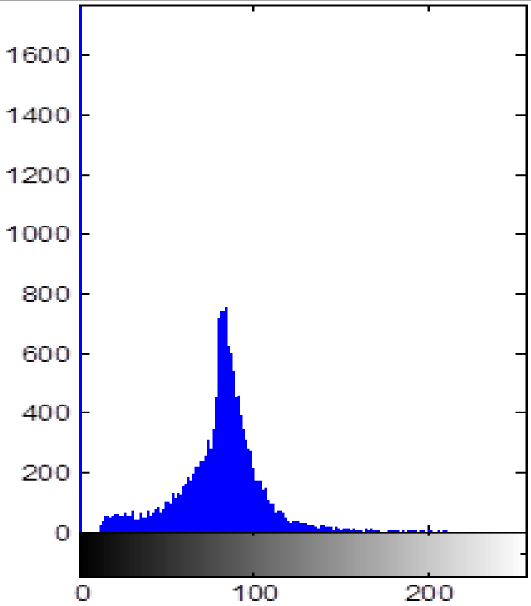


Figure 5: Histograms of Original image (Top), Bias corrected image (below)

The histograms of the original images (left) and the bias corrected images (right) are plotted in the fifth column. There are three well-defined and well-separated peaks in the histograms of the bias corrected image, each corresponding to a tissue or the background in the image. In contrast, the histograms of the original images do not have such well-separated peaks due to the mixture of the intensity distribution caused by the bias.

6. Performance Evaluation

As a level set method, our method provides a contour as the segmentation result. We use the MSE (Mean Square Error), and SNR (Signal to Noise Ratio) plots as the performance Measure and the Graphs are shown below explaining the Variation of MSE and SNR with 7 Values of Sigma and at 7 different initializations.

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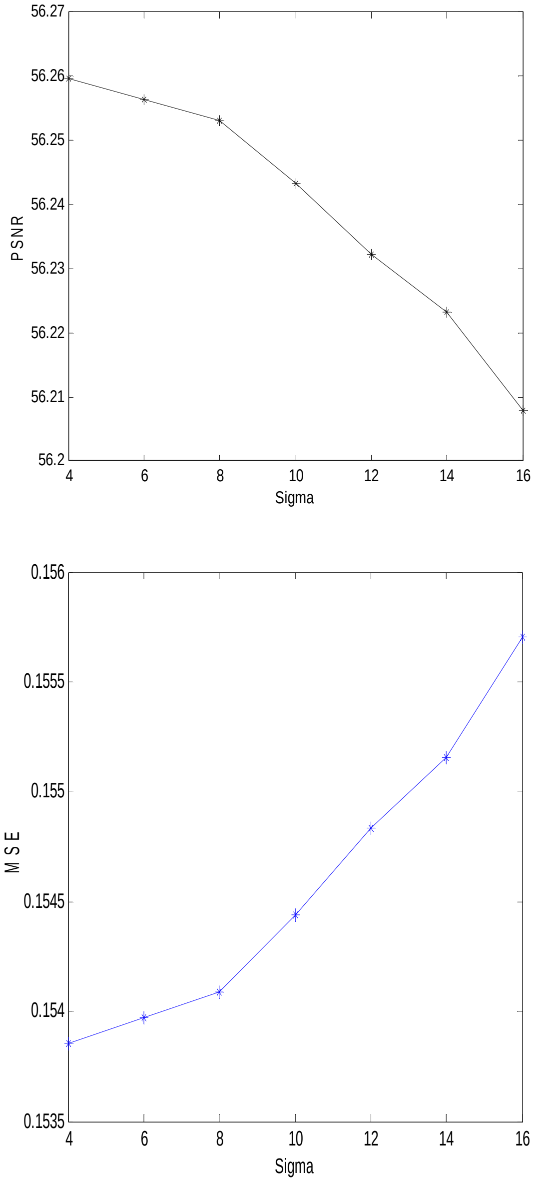

Figure 6: Plots for MSE, PSNR with different sigma values

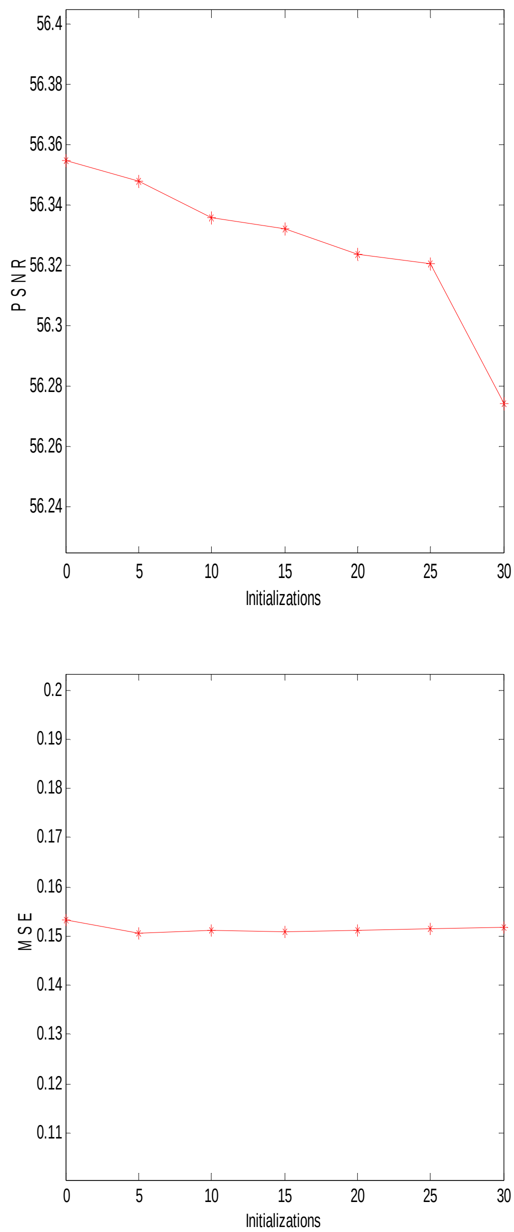

Figure 6: Plots for MSE, PSNR with different initializations

7. Conclusion

A Level Set based segmentation and bias correction of images is implemented in this paper. For the given MRI image with intensity inhomogeneities and a derived local intensity clustering property, we define energy of the level set functions that represent a partition of the image domain and a bias field that accounts for the intensity inhomogeneity. Segmentation and bias field estimation are therefore jointly performed by minimizing the proposed energy functional. This method is evaluated by using MSE and SNR plots and contrast and Brightness are improved.

8. Future Scope

In this paper we implemented the segmentation by considering the intensity inhomogenity of MRI image. There are also articrafts in MRI image which corrupt the

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details of the image such as partial volume effects, for which different tissue types contribute to the intensity of one voxel, RF noise etc., Further this segmentation can be extended to the MRI images with the articrafts mentioned above.

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