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Stress Analysis of Thin Plate with Special Shaped Cutout: A Review

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Abstract: In this paper an effort is made to review on the Stress Analysis of thin plate with special shaped cutout. Plates with variously shaped cutout often used in both modern and classical aerospace, mechanical and civil engineering structure. The understanding of effect of cutout on load bearing capacity and stress concentration of such plate is very important in designing of structure. A number of Analytical, Numerical, experimental technique are available for stress analysis around the different type of cutouts for different condition in an infinite /finite plate made up of different material under different loading condition has been reported in this paper. An attempt has been made in the present work to present an overview of various techniques developed for stress analysis & stress concentration factor.

Keywords: Stress concentration factor, thin plate, special shaped cutout

1. Introduction

In General thin plates are easily manufactured and are widely used for primary structural element in Aerospace mechanical, civil engineering structures, Marine structures. In recent year, different cut out shapes in structural element are need to reduce the weight of the system and provide access to other parts of the structure. It is well known that the presence of cut out or hole in a stressed members creates highly localized stresses at the vicinity of the cut out. This cut out works as stress raisers and may leads to failures of structures /machine components. Hence it is important aspect of stress analysis to predict stress concentration for special shaped cut out like Circular, Elliptical, Triangular, Rectangular, and Square.

2. Stress Concentration Factor

Stress concentration is defined as the localization of high stresses due to the irregularities present in the components and abrupt changes of the cross section. In order to consider the effect of stress concentration and find out localized stress a factor called stress concentration factor is used. It is denoted by K_t [16]. The ratio of maximum stress at the cut out edge to nominal stress is called stress concentration factor

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

Where, σ_{max} =Maximum stress at the hole

σ_{nom} =Nominal stress

For a thin plate with a circular hole at the center, that is subjected to uni-axial far-field tension, σ_n acting along the x-axis, the stresses around the vicinity of the hole which shown in figure 1.



Figure 1: Plate with circular hole subjected to uni-axial stress

3. Stress Analysis of Thin Plate

Different approaches used for stress analysis are Analytical, Numerical and Experimental.

3.1 Analytical Approach

Distribution of stresses in a structure with boundary conditions, i.e. displacements and/or forces on the boundary can be determined by using either the closed form analytical methods or by approximate numerical methods. Boundary value problems can be solved analytically by using constitutive equations based on the elastic or plastic behavior of the material under load. Analytical or close-form solutions can be obtained for simple geometries, constitutive relations and boundary conditions.

Savin's Basic formulation

In this method, by introducing the mapping function constants for required shape of hole and the constants for loading, the stress distribution around the hole is obtained. In Savin's method, the accuracy of the shape of hole depends on the number of terms in the mapping function. Mapping functions for various holes are available with only finite

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number of terms. Stresses are obtained for different number of terms in the mapping function. [3, 10, 11, 15].

Boundary Element Alternating Method

The boundary element alternating method (BEAM) is developed to study the stress concentration of a twodimensional (2D) perforated plate. This method involves the iterative superposition between the boundary element solution of a bounded plate without holes and the analytical solution of an infinite 2D plate with a circular hole subjected to arbitrary normal and shear loading until the required boundary conditions of the original problem are satisfied. A simple and rigorous boundary element alternating technique to analyze the discontinuities in perforated plates. The advantage of the BEAM is no limitation for the numbers, pattern and the gaps of holes. The interaction effects among holes and the boundary effects of finite plate are also accurately evaluated through very limited iterations and very limited number of boundary elements. The computer cost of the present computation is much less than the traditional boundary and finite element method. [4]

Work Energy Method (Modified form of the Ritz Method)

Many Researchers used this method in the field of Plate, shell and solid problem. For plate, geometry is divided into quadrilateral segments, each of which is bounded by four curved/straight edges. Further, the segments are mapped into square using the natural coordinates so that mathematics in the procedure becomes simple and straightforward. A segment is defined in the Cartesian coordinate system using eight geometric nodes, four located at the corner points and the other four at the mid-points of the four edges respectively. Plate equations are based on the first order shear deformation theory known as the "Reissner–Mindlin plate theory. [6]

Lekhnitskii's Solution

All stress components can be expressed in terms of a single stress function called Airy's stress function. Based on Lekhnitskii's theory of elasticity of anisotropic bodies, the stress function can be represented by an analytical function with unknown coefficients.

The unknown coefficients can be determined by applying corresponding boundary conditions of the cutout. Lekhnitskii's solution is limited to circular, elliptical cutouts in orthotropic plates. In order to use this approach to other shape cutouts, establishing a relationship between any cutouts and a circular cutout is necessary. A simple mapping function can be used for modeling different shape cutouts. A wide variety of cutout shapes can be analyzed, once the mapping function is determined for different cutout boundaries. [9, 14].

Muskhelishvili's Complex Variable Method

Based on Muskhelishvili's complex variable method, two modified stress functions which are $\varphi(\zeta)$ and $\psi(\zeta)$ in ζ -plane are proposed. The unknown coefficients of the modified stress functions are determined by enforcing the stress functions to satisfy the stress boundary conditions along the unit circle in ζ -plane and the stress boundary conditions along external boundary in z-plane. Then the stress field of a plate with hole is obtained. Muskhelishvili's complex variable method is useful and handy tool to study two dimensional stress analysis problem. [19, 20, 22, 23].

Two Dimensional Theory of Elasticity

This method of analysis is comparably simple and it produces an exact solution which is its great strength. It Have following steps;

- i) Introduction of Airy stress function.
- ii) Determination of displacement from the stress function.
- iii) Representations of stress function in term of two complex analytical functions.
- iv) Complex representation of displacement and stress. [24]

3.2 Numerical Approach

In Numerical Approach Following Method are include

- i. Boundary Collection
- ii. Finite difference
- iii. Finite Element

Finite Element method is based upon discretization of material surface or space into several small plane or solid elements. The law of distribution of stress over the element is assumed and displacements at boundaries of adjacent element are matched to determine the shape of deformed body. The stresses are calculated making use of stiffness matrix. In recent years, with the advent of advanced software's, the FEA based software ANSYS, COSMOL, DIANA, ABACUS and NASTRAN have been very useful for stress analysis. These software's are preferred by users according to the type of stress analysis, the type of elements to be analyzed and the depth of accuracy required.

Finite Difference is similar technique in which elasticity equilibrium equation is applied and solved by using small but finite distance between adjacent points.

In Boundary collection stress function is determine by satisfying condition along boundary.

Numerical method invariably and theory of elasticity method in several cases depends upon use of computers for arriving at solution. [21]

3.3 Experimental Approach

Among the experimental method three have gain popularity, these methods are;

- i. Photoelasticity
- ii. Brittle coating
- iii. Electrical Strain Gauges

Photoelasticity

The name photoelasticity implies the use of light (photo) and elastically stresses model. This method was earlier used for plane bodies of complicated shape and geometries, particularly for the reason that such geometrical shapes were not amenable to mathematical analysis. Photoelasticity is an experimental method for measurement of stress and strain in which light is either passed through a model or reflected from the surface of loaded body. Photoelastic model is generally

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preferred in situation where and strain information is needed over extended region and thus whole field method.

Photoelastic stress analysis is a full field technique for measuring the magnitude and direction of principal stresses. When polarized light is passed through a stressed transparent model, interference patterns or fringes are formed. These patterns provide immediate qualitative information about the general distribution of stress, positions of stress concentrations and of areas of low stress using the principals of stress optic law.

$$\sigma_1 - \sigma_2 = \frac{Nf_{\sigma}}{h}$$

 σ_1 And σ_2 = Maximum and Minimum principal stresses at the point under consideration,

N=Fringe order,

 f_{σ} = Material Fringe Value, h= Thickness. [21]

Brittle Coating

A brittle coating as the name suggest, is a material forming a thin layer on a base material and is brittle in nature particularly with respect to base material. Due to the brittle nature, the coating may crack when the body is subjected to certain minimum stress. Brittle coating is a Non destructive technique directly applicable on machine part to be analyzed in actual situation. The various type coating material like Stresscoat, Straintec, All-Temp, Glass-Lacquer are now available for application. Coating is normally air sprayed onto the component in the same way as spray paint. Thickness of coating is tried to be maintained uniform in the range of 0.05 to 0.2 mm. The coating is often dried at room temperature but seldom is it dried in oven. [21]

Electrical Strain Gauges

Probably the most ubiquitous and reliable of all the tools of experimental stress analysis is the electrical resistance strain gauges. The principle of its action is that the electrical resistance of a conductor changes proportionally to any strain applied to it. Thus, if a short length of wire were bonded to the structure in such a way that it experiences the same deformation as the structure, then by measuring the change in resistance, the strain can be obtained. [8]

4. Literature Review

E.S.Folias and J.J. Wang [1] presented a series solution for stress field around circular holes in plates with arbitrary thickness is subjected to a uniform tensile load. A wide range of holes diameters to plate thickness was presented.

V.G. Ukadgaonker D.K.N.Rao [2] presented an extension of Becker's solution for elliptical hole problem for unsymmetrical laminates to determine the stress resultant and moment around hole of any shape under arbitrary biaxial loading condition.

V.G. Ukadgaonker D.K.N.Rao [3] adapting Savin's formulation for stress concentration problems in symmetric laminates under inplane loading.

K.T.Chen, K.Ting, W.S.Yang [4] developed Boundary Element Alternating Method to study the stress concentration of two dimensional perforated plate.

Lasko et al. [5] used relaxation element method to determine the stress fields in a plate with three circular cutouts subjected to uniaxial tensile load Singh A.V. and Paul U.K. [6] presented numerical results based on generalized work–energy method for rectangular plates with a circular cutout and circular plates with a rectangular cutout

Hwai-Chung Wu and Bin Mu [7] investigated the SCF for isotropic plates & isotropic and orthotropic cylindrical shells with circular cutout

Rezaeepazhand and Jafari [9, 14] have given the stress distribution around several non circular cut out in isotropic and composite plate using Leknitskii's solution.

V.G. Ukadgaonker and Vyasraj Kakhandki [10] used Savin's complex variable formulation for stress analysis of irregular shaped hole in an orthotropic laminates.

Murat Yazici [11] an Elasto-plastic theoretical analysis of stresses around a square perforated isotropic plate is studies by using Savin's complex elastic equation.

Zheng Yang, Chang-Boo Kim, Chongdu Cho, and Hyeon Gyu Beom [12] studied the sensitivity of stress and strain concentration factor to plate thickness as well as Poisson's ratio by using 3D finite element method.

Yi Yang Jike Liu Chengwu Cai [13] by using U-Transformation technique and FEM solved the stress concentration problem in plate with rectangular hole under biaxial tension.

D.K. Nageswara Rao et al [15] applying Savin's basic formulation for the stress functions for symmetric laminates with a general shape of hole under arbitrary biaxial loading are derived.

Mohsen Mohammadi, John R. Dryden, Liying Jiang [17] analyzes the effect of nonhomogenous stiffness and varying Poisson's ratio upon the stress concentration factor using Frobenius series solution.

Jinho Woo and Won-Bae Na [18] presents stress concentration analysis of perforated plate with not only various cut out and bluntness but also different cut out orientations using ANSYS.

D.S.Sharma [19] gives the stress concentration around circular/elliptical/triangular cut out in infinite composite plate by using Muskhelishvili's complex variable method. The effect of fiber orientation, stacking sequence, loading factor and angle, cut out geometry on SCF around cut out in orthotropic plate is studied.

Milan Batista [20] present Modified Muskhelishvili's complex variable method to calculate the stress distribution around holes of relatively complex shapes in infinite plate subjected to uniform load at infinity.

Dharmendra S. Sharma [22] used Muskhelishvili's complex variable method to present stress distribution around polygonal hole (Triangular, Square, Pentagonal, and Hexagonal). The effect of hole geometry and loading pattern on SCF is studied.

Zuxing Pan, Yuansheng Cheng, Jun Liu [23] presented Muskhelishvili's complex variable method for stress distribution around a rectangular hole in finite plate under uniaxial tension. The effects of hole sizes, hole orientations and

plate's aspect ratios on the stress distribution and stress concentration factor in a finite plate with a rectangular hole subjected to uniaxial tension are studied.

Jae-Hoon Kang [24] had given exact solution for stress, strain, displacement of perforated rectangular plate with central circular hole by two dimensional theory of elasticity using Airy stress function.

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Tawakol A. Enab [25] Stress concentration factors at the root of an elliptic hole in unidirectional functionally graded material (UDFGM) plates under uniaxial and biaxial loads are predicted. ANSYS Parametric Design Language (APDL) was used to build the finite element models for the plates and to run the analysis. A parametric study is performed for several geometric and material parameters such as the elliptic hole major axis to plate width ratio, the elliptical shape factor, the gradation direction of UDFG.

5. Conclusion

Many of work done by the various Researchers in the recent past reveals that there are some Analytical like Muskhelishvili's complex variable method, Two dimensional theory of elasticity, Savin's basic formulation, Leknitskii's solution, Boundary Element Alternating Method, Relaxation element method and Finite element solution for composite/infinite isotropic/orthotropic/FGM plate with hole. According to Zuxing Pan, Yuansheng Cheng, Jun Liu [20], In infinite plate analysis Analytical method are limited to study the stress analysis of an infinite plate with hole but there are lots of cases which do not satisfy the assumption of an infinite plate in practical engineering application .The stress analysis solution of an infinite plate with hole are not suitably applied to the cases of a finite plate with hole in which the effect of outer boundary of the finite plate on the stress field is needed to be considered. Hence, it is necessary to study the stress analysis of finite plate with hole.

In this paper an effort is made to review on the stress analysis of thin plate with special shaped cutout. An attempt has been made in the article to present an overview of various techniques developed for stress analysis of infinite and finite plate. This article justifies the analysis of finite plate, which is vital in stress analysis.

6. Future Scope

Many researcher use analytical and numerical method to solve the stress distribution problem of an infinite plate with different holes, but not much notable work is seen with the finite thin plate with hole like circular, Elliptical, Triangular, Rectangular, and Square. So Experimental, Numerical and Analytical solution concerned with the stress analysis of Isotropic finite thin plate with special shaped cut-out subjected to axial (tension, compression), shear and torsion loading conditions have scope for future work.

References

- [1] E. S Folias and J.-J. Wang On the three-dimensional stress field around a circular hole in a plate of arbitrary thickness Computational Mechanics 1990;6(3):379–391.
- [2] V.G. Ukadgaonker D.K.N.Rao A general solution for stress resultants and moments around holes in unsymmetric laminates Composite Structures 49 (2000) 27-39.
- [3] V.G. Ukadgaonker D.K.N.Rao A general solution for stresses around holes in symmetric laminates under inplane loading Composite Structures 49 (2000) 339-354.

- [4] K.T.Chen, K.Ting, W.S.Yang Stress analysis of twodimensional perforated plates using boundary element alternating method Computers and Structures 75 (2000) 515-527.
- [5] Lasko GV, Deryugin YY, Schmauder S, Saraev D. Determination of stresses near multiple pores and rigid inclusion by relaxation elements. Theoretical and Applied Fracture Mechanics 2000 34:93–100.
- [6] Singh AV, Paul UK. Finite displacement static analysis of thin plate with an opening—a variational approach. International Journal of Solids and Structures 2003; 40:4135–4151.
- [7] Hwai-Chung Wu, Bin Mu On stress concentrations for isotropic/orthotropic plates and cylinders with a circular hole Composites: Part B 34 (2003) 127–134
- [8] James Doyle Modern Experimental Stress Analysis: completing the solution of partially specified problems. John Wiley & Sons, Ltd (2004) 101
- [9] J. Rezaeepazhand M. Jafari Stress analysis of perforated composite plates Composite Structures 71 (2005) 463– 468.
- [10] V.G. Ukadgaonker and Vyasraj Kakhandki Stress analysis for an orthotropic plate with an irregular shaped hole for different in-plane loading conditions—Part 1 Composite Structures 70 (2005) 255–274.
- [11] Murat Yazici Elasto-plastic analysis of stress around square hole Indian Journal of Engineering& Materials Sciences Vol. 14, June 2007, pp. 215-219.
- [12] Zheng Yang, Chang-Boo Kim, Chongdu Cho, Hyeon Gyu Beom The concentration of stress and strain in finite thickness elastic plate containing a circular hole International Journal of Solids and Structures 45 (2008) 713–731.
- [13] Yi Yang Jike Liu Chengwu Cai Analytical solutions to stress concentration problem in plates containing rectangular hole under biaxial tensions Acta Mechanica Solida Sinica, Vol. 21, No. 5, October, 2008.
- [14] J. Rezaeepazhand M.Jafari Stress concentration in metallic plates with special shaped cutout International Journal of Mechanical Sciences 52 (2010) 96–102
- [15] V.B.Bhandari Design of Machine Elements 3rd edition Tata McGraw-Hill (2010) 141-150
- [16] D.K. Nageswara Rao, M. Ramesh Babu, K. Raja Narender Reddy, D. Sunil Stress around square and rectangular cutouts in symmetric laminates Composite Structures 92 (2010) 2845–2859.
- [17] Mohsen Mohammadi, John R. Dryden, Liying Jiang Stress concentration around a hole in a radially inhomogeneous plate International Journal of Solids and Structures 48 (2011) 483–491.
- [18] Jinho Woo & Won-Bae Na Effect Of Cutout Orientation on Stress Concentration of Perforated Plates with Various Cutouts and Bluntness International Journal of ocean system Engineering 1(2)(2011)95-101.
- [19] D.S.Sharma Stress Concentration around Circular/Elliptical/Triangular Cutouts in Infinite Composite Plate Proceedings of the World Congress on Engineering 2011 Vol. III WCE 2011, July 6-8, 2011, London, U.K.
- [20] Milan Batista On the stress concentration around a hole in an infinite plate subject to a uniform load at infinity International Journal of Mechanical Sciences 53 (2011) 254–261.

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- [21] Dr. Abdul Mubeen Experimental stress analysis 2nd edition Dhanpat Rai & Co.(2011-12)
- [22] Dharmendra S. Sharma Stress distribution around polygonal holes International Journal of Mechanical Sciences 65 (2012) 115–124.
- [23] Zuxing Pan, Yuansheng Cheng, Jun Liu Stress analysis of a finite plate with a rectangular hole subjected to uniaxial tension using modified stress functions International Journal of Mechanical Sciences 75 (2013) 265–277.
- [24] Jae-Hoon Kang Exact solutions of stresses, strains, and displacements of a perforated rectangular plate by a central circular hole subjected to linearly varying inplane normal stresses on two opposite edges International Journal of Mechanical Sciences 84 (2014) 18–24.
- [25] Tawakol A. Enab Stress concentration analysis in functionally graded plates with elliptic holes under biaxial loadings Ain Shams Engineering Journal (2014)5, 839–850.

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