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# A Viscous Incompressible Heat Generating Fluid Flow Past an Infinite Porous Plate with Radiation Absorption

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**Abstract:** In this study a viscous incompressible heat generating fluid flow past an infinite vertical porous plate with radiation absorption was investigated. The flow was unsteady and restricted to laminar domain. The equations governing the flow were solved using explicit finite difference method. The influences of the various parameters such as the Eckert number, Grashof number, modified Grashof number, Prandtl number, Schmidt number and Hatman number on the incompressible heat generating fluid past an infinite vertical porous plate in the laminar boundary layers were considered. An analysis of the effects of the parameters on velocity and temperature profiles was done with the aid of graphs and tables. It was found that an increase in mass diffusion parameter Sc, leads to a decrease in both primary and secondary velocity profiles and also concentration profile. However an increase in mass diffusion parameter leads to an increase in the temperature profile. It was also noted that an increase in the viscous dissipative heat Ec, causes an increase in concentration profile. Finally the results obtained are presented using graphs and tables.

Keywords: MHD, Incompressible flow, Viscosity and Radiation absorption

# 1. Introduction

The study of viscous incompressible heat generating fluid past an infinite vertical porous plate has applications in many areas of science and engineering. This includes the MHD power generation and hall accelerator. The influence of magnetic field on the flow of electrically viscous fluid with mass transfer and radiation absorption is also useful in planetary research. Such phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water/ quasi-solid bodies such as earth. In natural processes and industrial applications, many transport processes exists where transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species.

The study of magneto hydrodynamic laminar boundary layer flow of a viscous incompressible heat generating past an infinite vertical porous plate find useful application in many engineering problems such as MHD generator, plasma studies, nuclear reactors, geothermal extractors and boundary layer control in the field of aeronautics and aerodynamics. It serves as the basis of understanding some of the important phenomena occurring heat exchange devices. The influence of magnetic field of the flow of an electrically conducting fluid with radiation absorption, hall and ion slip current is also useful in planetary atmosphere research (Shercliff J.A. 1965).

Kinyanjui M. et al (2001) considered magneto hydrodynamics free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption. The governing equations for the problem were solved by finite different scheme. The influences of the various parameters on the convectively cooled or convectively heated plate in the laminar boundary layer were considered. It was found that an increase in diffusion parameter Sc time, t and removal of the suction causes an increase in velocity  $w_o$  and in concentration profile. Also an increase in the radiation absorption parameter Q, leads to a slight decrease in the primary velocity profile. An increase in hall parameter m causes an increase in the primary velocity profile but leads to a decrease in secondary velocity profile.

Saha L. K. et al (2007) investigated the effect of hall current on the MHD laminar natural convection flow from a vertical permeable flat plate with uniform surface temperature with appropriate transformations the boundary layer equations were reduced to local non similarity equations and the solutions were obtained employing four distinct methods namely; regular perturbation method for small transpiration parameter, asymptotic solutions for large transpiration rate, implicit finite difference method together with Keller-box scheme and the local non similar method for any transpiration rate. Effects of the magnetic field, M, and the hall parameter, m, on the local skin friction and local rate of heat transfer groups were shown graphically for smaller values of the prandtl number, Pr (0.1, 0.01) that represent liquid metals.

Mbeledogu I. U. and Ogulu A. (2007) considered heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. The results obtained showed that the decrease in temperature boundary layer occurs when the prandtl number and the radiation parameter are increased and the flow velocity approaches steady state as the time parameter, t, is increased. The Laplace transfer technique was employed and the leading equations were solved analytically in the complex plate. It was found that radiation affects the temperature and therefore the velocity hence the skin friction. As the radiation parameter is increased it is accompanied by a decrease in the temperature and increase in velocity, when the plate is cooled by convection currents, Gr>0. Increase in the schmidt number results in the rare constant results in the negligible change in the concentration. The flow velocity

approaches steady state conditions as time parameter t, is increased to about 12.

Murali G. et al (2012) considered finite element solution of thermal radiation effect on unsteady MHD flow past a vertical porous plate with variable suction. The nondimensional governing equations were formed with the help of suitable dimensionless governing parameter. The resultant coupled non dimensional governing equations were solved by a finite element method. The velocity, temperature and concentration distributions were derived, discussed numerically and their profiles were shown through graphs. It was observed that, when radiation parameter increases, the velocity and temperature increases in the boundary layer. Also an increase in the magnetic field leads to decrease in the velocity field and rise in the thermal boundary thickness. The velocity increases with an increase in the permeability of the porous medium parameter. Increasing the prandtl number substantially decreases the translational velocity and temperature function. The velocity as well as concentration decreases with an increase in the Schmidt number.

Seddeck M. A. and Faiza A. S (2009) investigated the effects of temperature depending on viscosity and thermal conductivity or unsteady MHD convectional heat transfer past a semi-infinite vertical porous moving plate with variable suction. The governing equations for the flow were transformed into a system of nonlinear ordinary differential equations by perturbation technique and were solved numerically by using shooting method. The results indicated that the velocity increases with increase in variable viscosity, thermal conductivity, the exponential index, porous medium, Grashof number and plate moving velocity, but it decreases as the magnetic field parameter increases. Also the temperature increases as the variable thermal conductivity and the exponential index increases. The surface skin friction decreases as the plate moving velocity increases but it increases as the exponential index parameter increases.

Amkadni M. et al (2008) considered on the exact solution of laminar MHD flow over a stretching steady flat plate. It was shown that in the presence of a vertical inverse-linear magnetic field, we establish a sufficient condition for the existence of exact solution of the problem with the respect to the three parameters; the magnetic parameter M, the suction or injection parameter  $\gamma$  and the stretching parameter  $\xi$ . A pseudo-similarity transformation was employed to reduce the governing partial deferential equation into an ordinary deferential equation. It was shown that to obtain the exact solution of the problem the temperature  $\gamma \log(x)$  must be added to the usual form of the stream function. It was shown that the temperature solutions if  $\gamma > 0$  and M<2 and the exactualy two solution for any M>2.

Ravikumar V. et al (2012) considered heat and mass transfer effect on MHD flow of viscous fluid through nonhomogenous porous medium in presence of temperature dependent on heat source. It was observed that the primary velocity increases with an increase in magnetic parameter M where as it decreases as Grashof number (Gr) increased. An increase in the Schmidt number (Sc) results in the decrease in the concentration.

Das S.S. et al (2009) considered mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. It was found that magnetic parameter and Schmidt number retard velocity of the flow while the Grashof number for the heat and mass transfer, the porosity parameter and the heat some parameter hence accelerating effect on the velocity of the flow field at all points. Further the Prandtl number reduces the temperature and the Schmidt number diminishes the concentration distribution of the flow field at all points.

Mbeledogu I. U. et al (2007) considered unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer, where the viscosity of the fluid V and its thermal conductivity k in this model were assumed to be functions of temperature. Under suitable non dimensionlization the governing non-linear, coupled, partial differential equations were solved employing a perturbation technique based on the assumption that the fluid flow field is made up of a steady part and a transient. It was found that for a single pulse; the temperature boundary layer increases as the radiation parameter and the time period are increased, increase in prandtl number is accompanied by a decrease in the temperature, variation of the exponential index  $\omega$  has little effect on the temperature or the velocity distributions, the velocity increases as Gr, Pr, and M are increased, and the skin friction for a compressible fluid such as air Pr =0.71, is lower than the skin friction for an incompressible fluid such a water Pr = 7.

Takahashi F. et al (2001) investigated effects of boundary layers on magnetic field behavior in an MHD dynamo model. The emphasis was put in an important role of boundary layer which arises for the no-slip boundary condition. The results of computation showed that the dipole field was dominant and that the magnetic field is concentrated in the convection columns. Also the effects of magnetic diffusion were more significant than that of magnetic induction near the spherical surfaces. To investigate a fine structure inside the boundary layers a finite different method in the radial direction in which grid intervals are made variable was used and it was found that the dipole magnetic field is dominant outside the spherical shell both for the no-slip and the stress-free boundary cases, where as there are differences between the structures of magnetic field inside the spherical shell for the two cases; a strong toroidal magnetic field appears, for the no-slip case, at mid and high latitude near the outer surface, due to shear flow there.

Kumarar V. et al (2001) considered MHD flow past stretching permeable sheet where the effects of magnetic, suction injection, linear/nonlinear stretching parameters on the stream function and skin friction are shown graphically and discussed. The results obtained shows that in the presence of a magnetic field, the streamline are steeper and thus make the boundary layer thinner. The skin friction (-

 $\tau_w$ ) is increased. The streamline are affected significantly

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by stretching and cross flow parameters near the wall rather than away from the wall, and the location of backflow advances (b<0). Furthermore, the streamlines near the wall for increasing x get closer, showing the effects of quadratic stretching and linear mass flux.

Rehena Nasrin and Alim M.A. (2012) investigated control volume finite elements simulation of MHD forced and natural convection in a vertical channel with heatgenerating pipe using the Gale kin weighted residual control volume finite element method, the effects of magnetic field and joule heating on combined convection flow and heat transfer characteristics inside an octagonal vertical channel containing a heat generating hollow circular pipe at the centre is performed. The flow enters at the bottom and exists from the top surface. All solid walls of the octagon are considered to be adiabatic. Graphical representation of streamlines, isotherms, average Nassult number and maximum temperature of fluid for different combination of Hartman number (Ha), joule heating parameter (J) and Richardson number (Ri) are displayed. The results indicate that the flow and the thermal fields in the vertical channel depend markedly on the above mentioned parameters. In addition rate of heat is obtained optimum in the absence of both MHD and joule heating effects. It was concluded that, the influence of the aforesaid parameters on velocity field is remarkable. Particularly, the size of vortex devalues due to the hindrances of imposed magnetic field at all convection regions. On the other hand, joule heating plays a significant role on streamlines. The changes of temperature field with the mentioned parameters are not worthy. Mainly, the thermal boundary layer thickness reduces for mounting Ha. The isothermal lines move from the Centre of heat source as J increases. The nature of thermal plume rise from the body changes radically with the escalating RI. The heat transfer rate decreases with rising Ha and J. The maximum temperature of the fluid goes down and goes up for rising values of Ha and J, respectively.

Tezer-Sergim M. and Dost S. (1994) considered boundary element method for MHD channel flow with arbitrary wall conductivity where a boundary element formulation was presented to obtain the solution in terms of velocity and induced magnetic field for the steady MHD flow through a rectangular channel having arbitrary conducting walls, with an external magnetic field applied transverse to the flow. The fluid is viscous, incompressible, and electrically conducting. The boundary element method (BEM) with constant element has been used to cast the problem into the form of an integral equation over the boundary and to obtain a system of algebraic equations for the boundary unknown values only. Computations were carried out for several values of Hartman number  $(1 \le m \le 10)$  and conductivity parameter ( $0 \le \lambda \le \infty$ ). Selected graphs were given showing the behavior of velocity field and induced magnetic field which are the flattening tendency for an increase in M or a decrease in  $\lambda$  and were in agreement with the exact solutions. It was concluded that the BEM solutions can be used to obtain reasonable accurate results for steady MHD channel flow problems with arbitrary shapes and wall conductivity. That was not possible to do analytically even for some special cases. Unfortunately, accuracy decreases with the increasing Hartman number. This is due to rapid variations in velocity and magnetic field near the boundary. Comparing the time taken in obtaining the BEM solution and numerical results either from the exact solutions or from other numerical methods, the BEM solutions consumes negligible time. This was due to the fact that the boundary element does not need domain discretization.

The problem investigated here is the study on a viscous incompressible heat generating fluid past an infinite vertical porous plate with radiation absorption.

# 2. Mathematical Formulation

## **Governing Equations**

General equation governing the flow of electrically conducting fluid in the presence of strong magnetic field are; momentum equation, the equation of continuity, equation of conservation of energy, the concentration equation and Maxwell's equation. The velocity, pressure and temperature at any point of fluid vary smoothly with time and space since the flow is laminar.

These equations are described by partial differential equations expressing the laws of conservation of mass, momentum and energy. The following are the equations;

## **Momentum Equation**

It is based on the Newton's second law of motion which states the total body force and surface forces acting on a system is equal to the rate of momentum of a system. Its general form is given as:

$$\rho\left(\frac{\partial u_{i}}{\partial t}+uj\frac{\partial u_{j}}{\partial x_{j}}\right)=\rho Fi+\frac{\partial \sigma_{ji}}{\partial x_{i}} \tag{1}$$

Since the fluid is viscous, the stress tensor is given by:

$$\sigma ji = -p\sigma ji + \eta \left(\frac{\partial uj}{\partial xj} + \frac{\partial ui}{\partial xi}\right)$$
(2)

Substituting (2) into (1), then the incompressible fluid with constant velocity, the momentum equation becomes

$$\rho\left(\frac{\partial u_{j}}{\partial t} + u_{j}\frac{\partial u_{i}}{\partial x_{j}}\right) = -\frac{\partial\rho}{\partial x_{i}} + \rho v \nabla^{2} u_{j} + F_{i}$$
(3)

Taking into account both the gravitational force g and the electromagnetic force so that the volume density of the external force is given by (Moreau 1990) as

$$Fi = \rho g + J \times B \tag{4}$$

Substituting (3.4) into (3.3) we obtain

$$\rho \left( \frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x_j} \right) = -\frac{\partial \rho}{\partial x_j} + \rho v \nabla^2 u_j - \rho g + J \times B$$
<sup>(5)</sup>

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## The Equation of Continuity

The equation is based on the law which states that mass can neither be heated nor destroyed and is given as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_i} = 0 \tag{6}$$

For an incompressible fluid density is assumed to be constant and equation (3.6) reduces to

$$\frac{\partial u_j}{\partial x_i} = 0 \tag{7}$$

Where j = 1, 2, 3 along the x, y and z axes respectively.

#### **Equation of Conservation of Energy**

The viscous dissipation function  $\phi$  in three dimensions is given by

This equation is derived from the first law of thermodynamics in the form

$$dQ = dE + dW \tag{8}$$

Where E is the internal energy, Q is the heat added to the system and dW is the work done by the system. If heat produced by external forcers is ignored then it's written in tensor form as

$$\rho[\frac{\partial h}{\partial t}] + \frac{\partial(\rho u_j)}{\partial x_j} = \frac{\partial \rho}{\partial t} + \frac{\partial(u_j \rho)}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + \phi$$
(9)

where  $\phi = \tau \frac{\partial u_j}{\partial x_j}$  is the viscous dissipation function.

$$\phi = \mu \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \tag{10}$$

Equation (9) can be simplified by the thermodynamic definition of h,

$$h = E + \frac{p}{\rho} \tag{11}$$

Where E is the specific internal energy

Equation (3.11) in differential form is

$$dh = dE + \frac{dp}{\rho} + p\left(\frac{1}{\rho}\right) \tag{12}$$

The Maxwell's thermodynamics relation is

$$dE = Tds - pd\left(\frac{1}{\rho}\right)$$

Substituting in equation (12) yields

$$dh = Tds - pd\left(\frac{1}{\rho}\right) + pd\left(\frac{1}{\rho}\right) + \frac{dp}{\rho}$$
$$dh = Tds + \frac{dp}{\rho}$$
(13)

Where s is the entropy

Taking s (p, T) and on differentiating yields

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \tag{14}$$

By using the generalized thermodynamics relations

$$\begin{pmatrix} \frac{\partial s}{\partial p} \\ \frac{\partial s}{\partial T} \\ \frac{\partial s}{\partial T}$$

Where  $\beta$  is the coefficient of volumetric expansion.

Substituting equation (15) into equation (14) we obtain

$$ds = \frac{C_p}{T} dT - \frac{\beta}{\rho} dp \tag{16}$$

Substituting equation (3.16) into equation (3.13) results to

$$dh = C_p + \frac{1}{\rho} \left( 1 - \beta T \right) dp \tag{17}$$

Where  $C_p$  is the specific heat capacity at constant pressure

Using Fourier's law of heat conduction given by

$$q_{j} = -k \frac{\partial T}{\partial x_{j}} \tag{18}$$

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Where k is the thermal conductivity

Substituting equation (17) and (18) into equation (9) the energy equation reduces to

$$\rho C_p \frac{DT}{Dt} = k \nabla^2 T + Q^o + \beta T \frac{Dp}{Dt} + Q \qquad (19)$$

Where Q<sup>o</sup> is the dissipation function which is as a result of electromagnetic interactions. By considering electrical dissipation, which is the heat energy produced by the work done by electrical currents and is given by  $\frac{J^2}{\sigma}$  equation (19) becomes

$$\rho C_p \frac{DT}{Dt} = k \nabla^2 T + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 + \frac{J^2}{\sigma} + Q \quad (20)$$

Neglecting the electrical dissipation function and electromagnetic dissipation terms, the energy equation reduces to

$$\rho C_p \frac{DT}{Dt} = k \nabla^2 T + \phi \tag{21}$$

## **The Concentration Equation**

The equation is based on the principal of mass conservation for each species in a fluid mixture, for the fluid flow in consideration the tensor form of the diffusion is

$$\frac{DCj}{Dt} = \frac{\partial J_j}{\partial x_i}$$
(22)

## Maxwell's Equation

This equation provides connection between the electric and magnetic field without considering the properties of the matter, Pai (1962), Moreau (1990). It is summarized in four equations:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial D}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot D = \rho_e$$
(23)

Since the displacement current D is negligible with respect to  $\vec{J}, \frac{\partial D}{\partial t}$  is negligible with respect to  $\vec{J}$  and  $\vec{\nabla} \times \vec{H}$  and since  $\rho_e$  is usually not known, then the last equation of (23) will not be utilized. The Maxwell's equation reduces to the following set [Moreau, (1990)].

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
(24)

Thus

 $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$  is the Faraday's law which states that the

induced electromotive force in any closed circuit is equal to the negative of the time rate of change of the magnetic flux through the circuit.

 $\overrightarrow{\nabla} \cdot D = \rho_e$  is the Coulombs law which states that the magnitude of the electromotive force of interaction between two point charges is directly proportional to the scalar multiplication of the magnitudes of charges and inversely proportional to the square of the differences between them

and

$$\vec{\nabla} \cdot \vec{B} = 0$$
 is the magnetic field continuity

## Ohm's Law

This law characterizes the ability of materials to transport electric charge under the influence of an applied electric field. For electrically conducting material at rest, the current density is given by

$$\vec{J} = \sigma \vec{E}$$
 (25)

In moving electrically conducting fluids the magnetic field

induces a voltage in the conductor of magnitude  $\vec{q} \times \vec{B}$ The generalized Ohm's law is given by

$$\vec{J} = \sigma \left( \vec{E} + \vec{q} \times \vec{B} \right)$$
 (26)

## 3. Approximations and Assumptions

Every mathematical description of natural phenomenon is based on certain approximations and assumptions. In this study the following Assumptions and approximations are made.

- 1. The fluid is incompressible (density is assumed to be constant)
- 2. The fluid flow is laminar

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- 3. There is no external applied electric field, E = 0
- 4. Viscosity  $(\mu)$  is assumed to be constant
- 5. Liquid metals and ionized gases have permeability  $\mu_e$ ,

so that  $\vec{B} = \mu_e \vec{H}$  in any frame of reference.

- 6. Thermal conductivity k is assumed constant.
- 7. The induced magnetic field is negligible.
- 8. The fluid is assumed to be electrically neutral.
- 9. The electrical dissipation is negligible.
- 10. The electric displacement current is zero since the flow velocity is small relative to the speed of light.
- 11. The internal heat generation is assumed to be of the form

$$Q^* = -(T^* - T_{\infty}^{\circ})Q$$

## **Specific Equations**

The x-axis is taken along the plate in vertical upward direction, which is the direction of flow. The y-axis is taken normal to the plate, since the plate is infinite in length and for a two dimensional free convective fluid flow the physical variables are functions of x, y and t. The fluid is permeated with a strong magnetic field.



The continuity equation for the fluid flow under consideration is given by,

$$\frac{\partial V}{\partial y} = 0 \tag{27}$$

Since the fluid particles equal to zero because of no-slip condition. On integration equation gives the constant suction velocity

$$\mathbf{V} = -\mathbf{V}_0 \tag{28}$$

As the fluid is in motion it possesses momentum, hence we consider the equation of momentum.

## **Momentum Equation**

The momentum equation as given in equation (5) is

$$\rho\left[\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right] = -\frac{\partial \rho}{\partial x_j} + \rho \nu \nabla^2 u_j - \rho g + J \times B \quad (29)$$

The velocity profile at various x-positions depends on y coordinates.

$$\left(u_{j}\frac{\partial u_{i}}{\partial x_{j}}\right) = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$
(30)

$$v\nabla^2 u_j = v \frac{\partial^2 u_j}{\partial y^2} \tag{31}$$

Substituting (30) and (31) into (28) gives

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial \rho}{\partial x_j} + \rho v\frac{\partial^2 u_j}{\partial j^2} - \rho g + J \times B$$
(32)

To determine the pressure gradient from equation is evaluated at the edge of the boundary layer  $\rho \rightarrow \rho_{\infty}$  and  $u \rightarrow 0$ . This is because at the boundary layer the velocity of the fluid is at its minimum.

The pressure term in x- direction is,

$$\frac{\partial p}{\partial x} = \rho_{\infty} g$$

which results from the change in elevation and  $\rho_{\infty}$  is the density near the plate.

The body force term in equation (32) along negative xdirection is  $-\rho g$ .Combining the pressure term and body force term yields,

$$-\rho g - (-\rho_{\infty}) = \rho_{\infty} g - \rho g = g(\rho_{\infty} - \rho)$$
(33)

Substituting equation (33) into (32) yields

$$\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} \right] = \rho v \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + g \left[ \rho_{\infty} - \rho \right] + J \times B$$
(34)

If the volumetric coefficient  $\beta$  of thermal coefficient is defined as

$$\beta = \frac{1}{V} \left[ \frac{\Delta v}{\Delta T} \right] = \frac{1}{V} \frac{\partial v}{\partial \rho} \frac{\partial \rho}{\partial T}$$

 $V = \frac{Mass}{Density}$  For unit mass

Thus 
$$\beta = \rho \left[ -\frac{1}{\rho^2} \frac{\partial p}{\partial T} \right] = -\frac{1}{\rho} \frac{\partial \rho}{\partial T}$$
  
 $\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} = -\frac{1}{\rho} \left[ \frac{\rho_{\infty} - \rho}{T_{\infty} - T} \right] = \frac{1}{\rho} \left[ \frac{\rho_{\infty} - \rho}{T - T_{\infty}} \right]$ 
(35)

And the volumetric coefficient of expansion due to concentration gradient of the fluid is given by

$$\beta^* = -\frac{1}{\rho} \left[ \frac{\Delta \rho}{\Delta C} \right] = -\frac{1}{\rho} \left[ \frac{\rho_{\infty} - \rho}{C_{\infty} - C} \right] = \frac{1}{\rho} \left[ \frac{\rho_{\infty} - \rho}{C - C_{\infty}} \right]$$
(36)

On simplifying equation (34) and (35) we get

$$\beta \rho (T - T_{\infty}) = \rho_{\infty} - \rho$$

$$\beta^* \rho (C - C_{\infty}) = \rho_{\infty} - \rho$$
(37)

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The total change in density due to temperature and concentration is given by

$$\Delta \rho = \beta \rho (T - T_{\infty}) + \beta^* (C - C_{\infty})$$
(38)

Substituting equation (3.38) into (3.34) gives

$$\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} \right]$$
  
=  $\rho v \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + \beta \rho g (T - T_{\infty})$   
+  $\beta^* \rho g (C - C_{\infty}) + J \times B$ 

Dividing both sides by  $\rho$  we obtain

$$\begin{bmatrix} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} \end{bmatrix} = v \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \end{bmatrix} + \beta g (T - T_{\infty}) + \beta^* g (C - C_{\infty}) + \frac{J \times B}{\rho}$$
(39)

From Ohm's law  $\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})$  where  $\vec{J}$  is the current density,  $\vec{J} = (\vec{J}_x \vec{J}_y, \vec{J}_z)$  and  $\vec{B}$  is the magnetic induction,  $\vec{B} = (\mu_e \vec{H})$ .From figurer 1  $\vec{B}$  in component form is given as

$$\vec{B}_x = 0, \vec{B}_y = \mu_e H_y \text{ and } \vec{B}_z = 0.$$

Thus

$$\vec{J} = \sigma \begin{vmatrix} ijk \\ uV_0 w \\ 0 \vec{B}_y & 0 \end{vmatrix} = \sigma \left( -w\vec{B}_y i + u\vec{B}_y k \right)$$
(40)

$$\vec{J} = \sigma \vec{B}_y(-wi + uk) \tag{41}$$

Thus the term  $J \times B$  is given by

$$J \times B = \begin{vmatrix} i & j & k \\ \vec{J}_x & 0 & \vec{J}_z \\ 0 & \mu_e H_y & 0 \end{vmatrix}$$
$$J \times B = -i\vec{J}_z \mu_e H_y + k\vec{J}_x \mu_e H_y$$
(42)

From equation of conservation of electric charge  $\nabla \cdot \vec{J} = 0$ , gives  $\vec{J}_y =$  constant, this constant must be zero since  $\vec{J}_y = 0$ at the plane which is electrically non-conducting hence  $\vec{J}_y = 0$  everywhere in the flow.  $\vec{B}_x$  and  $\vec{B}_z$  are equal to zero due to zero due to the geometrical nature of this problem.

Substituting equation (42) into (39) for  $J \times B$  we obtain

$$\begin{bmatrix} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} \end{bmatrix} = v \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \end{bmatrix} + \beta g (T - T_{\infty}) + \beta^* g (C - C_{\infty}) - \frac{\mu_e H_y J_z}{\rho}$$
(43)

and

$$\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} - V_0\frac{\partial w}{\partial y}\right] = v\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] + \frac{\mu_e H_y J_x}{\rho}$$
(44)

From equation (15) the value for  $J_x$  and  $J_z$  are

Substituting (45) into (43) and (44) for  $J_x$  and  $J_z$  gives

$$\begin{bmatrix} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} \end{bmatrix} = v \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \end{bmatrix} + \beta g (T - T_{\infty}) + \beta^* g (C - C_{\infty}) - \sigma \frac{\mu_c^2 H_y^2 u}{\rho}$$
(46)

and

$$\left[\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} - V_0 \frac{\partial w}{\partial y}\right] = v \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \sigma \frac{\mu_e^2 H_y^2 w}{\rho} \quad (47)$$

#### **Energy Equation**

If the energy equation is considered since the fluid possesses energy then

$$\rho C_p \frac{\Delta T}{\Delta t} = k \nabla^2 T + \mu \left[ \frac{\partial u}{\partial y} + \frac{\partial w}{\partial y} \right] + \frac{J^2}{\sigma} + \varphi$$
(48)

Neglecting the electrical dissipation function and electromagnetic terms equation (48) becomes

$$\begin{bmatrix} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} - V_0 \frac{\partial T}{\partial y} \end{bmatrix} = \frac{1}{\rho C_p} \left\{ k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + Q + \mu \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \right\}$$
(49)

#### **Concentration Equation**

The concentration equation is given by

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - V_0 \frac{\partial c}{\partial y} = D \left[ \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right]$$
(50)

#### Non-Dimensiolization

The non-dimensiolization process of the equations is important because the results obtained for a surface experiencing one set of conditions can be applied to a geometrically similar surface experiencing entirely different conditions. These conditions vary with the nature of the fluid, the fluid velocity or the size of the surface. The process also normalizes the boundary layer equations and makes the solution bounded. For example nondimensionalizing velocity such that it varies from 0 to 1.

In order to bring out the essential features of the flow problems in MHD, it is desirable to find important nondimensional parameters which characterize these flow problems. The following are some important parameters used in this study

#### Eckert Number (E<sub>c</sub>)

It is the ratio of kinetic energy of the flow reactive to thermal energy.

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$$Ec = \frac{U^2}{C_p (T_w^+ - T_\infty^+)}$$

#### Grashof Number (Gr)

It occurs in natural convection problems. It Is the ratio of buoyancy forces. The larger it is the stronger is the convectional current.

$$Gr = \frac{\nu g \beta (T_w^+ - T_\infty^+)}{U^3}$$

#### 3.4.3. Prandtl Number (Pr)

It is the measure of the viscous force to the thermal force. It is large when thermal conductivity is less than one and viscosity is large. The number is small when viscosity is less than the one and thermal conductivity is larger.

$$Pr = \frac{\mu C_p}{k}$$

#### **Reynolds Number (Re)**

It is one of the most important parameters of a viscous flow and is measured as the ratio of inertia force to the viscous force. For any flow this number is small then inertia force is negligible. It is large vicious force is ignored and the fluid can be taken as inviscid.

$$Re = \frac{\rho UL}{\mu} = \frac{UL}{\nu}$$

#### Hartman Number, M

It is the ratio of magnetic force to the viscous force

$$M^2 = \frac{\sigma \mu_e^2 H_y^2 \nu}{U^2 \rho}$$

#### Schmidt Number, Sc

This provides a measure of the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers respectively

$$Sc = \frac{v}{D}$$

In this study, all the variables with the superscript (+) plus will represent dimensional variables and non-dimensional variables and non-dimensionalization is based on the following sets of scaling variables.

$$\begin{split} t &= \frac{t^+ U^2}{\nu}, \qquad x = \frac{x^+ U}{\nu}, \quad y = \frac{y^+ U}{\nu}, \quad u = \frac{u^+}{U}, \qquad V_0 = \frac{V_0^+}{U}, \\ w &= \frac{w^+}{U}, \qquad \qquad \theta = \frac{T^+ - T_{\infty}^+}{T_w^+ - T_{\infty}^+}, \qquad C = \frac{C^+ - C_{\infty}^+}{C_w^+ - C_{\infty}^+}, \\ \sigma &= \frac{Q^+ \nu^2}{K U^2} \end{split}$$

Where

U is the characteristic velocity

 $T_w^+ - T_\infty^+$  is the temperature difference between the surface and free stream temperature

 $C_w^+ - C_\infty^+$  is the concentration difference between the concentration at the surface and the free stream concentration.

Equation (46), (47), (48) and (49) can be written using dimensional variables as

$$\begin{aligned} \left[ \frac{\partial u^{+}}{\partial t^{+}} + u^{+} \frac{\partial u^{+}}{\partial x^{+}} - V_{0} \frac{\partial u^{+}}{\partial y^{+}} \right] \\ &= v \left[ \frac{\partial^{2} u^{+}}{\partial x^{+2}} + \frac{\partial^{2} u^{+}}{\partial y^{+2}} \right] + \beta g (T^{+} - T_{\infty}^{+}) \\ &+ \beta^{*} g (C^{+} - C_{\infty}^{+}) - \sigma \frac{\mu_{e}^{2} H_{y}^{2} u^{+}}{\rho} \end{aligned}$$

$$(51)$$

$$\begin{bmatrix} \frac{\partial w^{+}}{\partial t^{+}} + u^{+} \frac{\partial w^{+}}{\partial x^{+}} - V_{0}^{+} \frac{\partial w^{+}}{\partial y^{+}} \end{bmatrix}$$

$$= v \left[ \frac{\partial^{2} w^{+}}{\partial x^{+2}} + \frac{\partial^{2} w^{+}}{\partial y^{+2}} \right] - \sigma \frac{\mu_{e}^{2} H_{y}^{2} w^{+}}{\rho}$$

$$(52)$$

$$\left[ \frac{\partial T^{+}}{\partial t^{+}} + u^{+} \frac{\partial T^{+}}{\partial x^{+}} - V_{0}^{+} \frac{\partial T^{+}}{\partial y^{+}} \right] = \frac{1}{\rho C_{p}} \left\{ k \left[ \frac{\partial^{2} T^{+}}{\partial x^{+2}} + \frac{\partial^{2} T^{+}}{\partial y^{+2}} \right] + Q^{+} + \mu \left[ \left( \frac{\partial u^{+}}{\partial y^{+}} \right)^{2} + \left( \frac{\partial w^{+}}{\partial y^{+}} \right)^{2} \right] \right\}$$

$$(53)$$

$$\frac{\partial c^+}{\partial t^+} + u^+ \frac{\partial c^+}{\partial x^+} - V_0^+ \frac{\partial c^+}{\partial y^+} = D \left[ \frac{\partial^2 c^+}{\partial x^{+2}} + \frac{\partial^2 c^+}{\partial y^{+2}} \right]$$
(54)

Non-dimensionalizing equation (51) we have

$$\begin{split} \frac{\partial u^{+}}{\partial t^{+}} &= \frac{\partial u^{+}}{\partial t} \cdot \frac{\partial t}{\partial t^{+}} = \frac{\partial (uU)}{\partial t} \times \frac{U^{2}}{v} = \frac{U^{3}}{v} \frac{\partial u}{\partial t} \\ u^{+} \frac{\partial u^{+}}{\partial x^{+}} &= u^{+} \frac{\partial u^{+}}{\partial x} \cdot \frac{\partial x}{\partial x^{+}} = uU \frac{\partial (uU)}{\partial x} \times \frac{U}{v} = \frac{uU^{3}}{v} \frac{\partial u}{\partial x} \\ V_{0}^{+} \frac{\partial u^{+}}{\partial y^{+}} &= V_{0}^{+} \frac{\partial u^{+}}{\partial y} \frac{\partial y}{\partial y^{+}} = \frac{V_{0}U\partial (uU)}{\partial y} \times \frac{U}{v} = \frac{V_{0}U^{3}}{v} \frac{\partial u}{\partial y} \\ \frac{\partial^{2}u^{+}}{\partial y^{+2}} &= \frac{\partial}{\partial y^{+}} \left[ \frac{\partial u^{+}}{\partial y^{+}} \right] = \frac{\partial}{\partial y^{+}} \left[ \frac{U^{2}}{v} \frac{\partial u}{\partial y} \right] = \frac{\partial}{\partial y} \left[ \frac{U^{2}}{v} \frac{\partial u}{\partial y} \right] \frac{\partial y}{\partial y^{+}} \\ &= \frac{U^{3}}{v^{2}} \frac{\partial^{2}u}{\partial y^{2}} \\ \frac{\partial^{2}u^{+}}{\partial x^{+2}} &= \frac{\partial}{\partial x^{+}} \left[ \frac{\partial u^{+}}{\partial x^{+}} \right] = \frac{\partial}{\partial x^{+}} \left[ \frac{U^{2}}{v} \frac{\partial u}{\partial x} \right] = \frac{\partial}{\partial x} \left[ \frac{U^{2}}{v} \frac{\partial u}{\partial x} \right] \frac{\partial y}{\partial x^{+}} \\ &= \frac{U^{3}}{v^{2}} \frac{\partial^{2}u}{\partial x^{2}} \\ T^{+} - T_{\infty}^{+} &= (T_{w}^{+} - T_{\infty}^{+})\theta \\ C^{+} - C_{\infty}^{+} &= (C_{w}^{+} - C_{\infty}^{+})C \end{split}$$

Thus we have

$$\frac{U^{3}}{v}\frac{\partial u}{\partial t} + \frac{uU^{3}}{v}\frac{\partial u}{\partial x} + \frac{V_{0}U^{3}}{v}\frac{\partial u}{\partial y}$$

$$= \left[\frac{U^{3}}{v^{2}}\frac{\partial^{2}u}{\partial y^{2}} + \frac{U^{3}}{v^{2}}\frac{\partial^{2}u}{\partial x^{2}}\right] + \beta g(T_{w}^{+} - T_{\infty}^{+})\theta$$

$$+ \beta^{*}g(C_{w}^{+} - C_{\infty}^{+})C - \frac{\sigma\mu_{e}^{2}H_{y}^{2}uU}{\rho}$$

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Dividing through by  $\frac{U^3}{v}$  yields

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} \\ &= \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + \frac{\nu \beta g (T_w^+ - T_w^+) \theta}{U^3} \\ &+ \frac{\nu \beta^* g (C_w^+ - C_w^+) C}{U^3} - \frac{\sigma \mu_e^2 H_y^2 u v}{U^2 \rho} \end{aligned}$$

But 
$$Gr = \frac{\nu\beta g (T_w^+ - T_w^+)}{U^3}$$
,  $Gc = \frac{\nu\beta^* g (C_w^+ - C_w^+)}{U^3}$  and  $M^2 = \frac{\sigma \nu \mu_e^2 H_v^2}{U^2 \rho}$ 

Hence the overall equation reduces to

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} = \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + Gr\theta + GcC - M^2 u \quad (55)$$

Non-dimensionalizing equation (3.52) we obtain

$$\frac{\partial w^{+}}{\partial t^{+}} = \frac{\partial (wU)}{\partial t} \cdot \frac{\partial t}{\partial t^{+}} = \frac{U\partial w}{\partial t} \cdot \frac{U^{2}}{v} = \frac{U^{3}}{v} \frac{\partial w}{\partial t}$$
$$u^{+} \frac{\partial w^{+}}{\partial x^{+}} = \frac{uU\partial w^{+}}{\partial x} \cdot \frac{\partial x}{\partial x^{+}} = \frac{uU(wU)}{\partial x} \times \frac{U}{v} = \frac{uU^{3}}{v} \frac{\partial w}{\partial x}$$
$$V_{0}^{+} \frac{\partial w^{+}}{\partial y^{+}} = \frac{V_{0}U\partial w^{+}}{\partial y} \cdot \frac{\partial y}{\partial y^{+}} = \frac{V_{0}U\partial (wU)}{\partial y} \cdot \frac{U}{v} = \frac{V_{0}U^{3}}{v} \frac{\partial w}{\partial y}$$
$$\frac{\partial^{2}w^{+}}{\partial y^{+2}} = \frac{\partial}{\partial y^{+}} \left[ \frac{\partial w^{+}}{\partial y^{+}} \right] = \frac{\partial}{\partial y} \left[ \frac{U^{2}}{v} \frac{\partial w}{\partial y} \right] \frac{\partial y}{\partial y^{+}} = \frac{U^{3}}{v^{2}} \frac{\partial^{2}w}{\partial y^{2}}$$
$$\frac{\partial^{2}w^{+}}{\partial x^{+2}} = \frac{\partial}{\partial x^{+}} \left[ \frac{\partial w^{+}}{\partial x^{+}} \right] = \frac{\partial}{\partial x} \left[ \frac{U^{2}}{v} \frac{\partial w}{\partial x} \right] \frac{\partial y}{\partial x^{+}} - \frac{U^{3}}{v^{2}} \frac{\partial^{2}w}{\partial x^{2}}$$

Thus equation (52) becomes

$$\frac{U^{3}}{v}\frac{\partial w}{\partial t} + \frac{uU^{3}}{v}\frac{\partial w}{\partial x} - \frac{V_{0}U^{3}}{v}\frac{\partial w}{\partial y} \\ = \left[\frac{U^{3}}{v}\frac{\partial^{2}w}{\partial x^{2}} + \frac{U^{3}}{v}\frac{\partial^{2}w}{\partial y^{2}}\right] - \frac{\sigma\mu_{e}^{2}H_{y}^{2}wU}{\rho}$$

Multiplying through by  $\frac{\nu}{U^3}$  we obtain

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} - V_0 \frac{\partial w}{\partial y} = \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] - \frac{\sigma \mu_e^2 H_y^2 w}{\rho U^2}$$
$$M^2 = \frac{\sigma \mu_e^2 H_y^2 v}{\rho U^2}$$

Thus the equation reduces to

But

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} - V_0 \frac{\partial w}{\partial y} = \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] - M^2 w$$
(56)

Non-dimensionalizing equation (53) we have

$$\frac{\partial T^{+}}{\partial t^{+}} = \frac{\partial T^{+}}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} \cdot \frac{\partial t}{\partial t^{+}} = \frac{U^{2}(T_{w}^{+} - T_{\infty}^{+})}{\nu} \frac{\partial \theta}{\partial t}$$
$$u^{+} \frac{\partial T^{+}}{\partial x^{+}} = uU \frac{\partial T^{+}}{\partial \theta} \cdot \frac{\partial \theta}{\partial x} \cdot \frac{\partial x}{\partial x^{+}} = uU(T_{w}^{+} - T_{\infty}^{+}) \frac{\partial \theta}{\partial x} \cdot \frac{U}{\nu}$$
$$= \frac{uU^{2}(T_{w}^{+} - T_{\infty}^{+})}{\nu} \frac{\partial \theta}{\partial x}$$

$$\begin{split} V_0^+ \frac{\partial T^+}{\partial y^+} &= V_0 U \frac{\partial T^+}{\partial \theta} \cdot \frac{\partial \theta}{\partial y} \cdot \frac{\partial y}{\partial y^+} = V_0 U (T_w^+ - T_w^+) \frac{\partial \theta}{\partial y} \cdot \frac{U}{v} \\ &= \frac{V_0 U^2 (T_w^+ - T_w^+)}{v} \frac{\partial \theta}{\partial y} \\ \frac{\partial^2 T^+}{\partial y^{+2}} &= \frac{\partial}{\partial y^+} \left[ \frac{\partial T^+}{\partial y^+} \right] = \frac{\partial}{\partial y} \left[ \frac{U (T_w^+ - T_w^+)}{v} \right] \frac{\partial y}{\partial y^+} \\ &= \frac{U^2}{v^2} (T_w^+ - T_w^+) \frac{\partial^2 \theta}{\partial y^2} \\ \frac{\partial^2 T^+}{\partial x^{+2}} &= \frac{\partial}{\partial x^+} \left[ \frac{\partial T^+}{\partial x^+} \right] = \frac{\partial}{\partial x} \left[ \frac{U (T_w^+ - T_w^+)}{v} \right] \frac{\partial y}{\partial x^+} \\ &= \frac{U^2}{v^2} (T_w^+ - T_w^+) \frac{\partial^2 \theta}{\partial x^2} \\ \left[ \frac{\partial u^+}{\partial y^+} \right]^2 &= \left[ \frac{\partial u^+}{\partial y} \cdot \frac{\partial y}{\partial y^+} \right]^2 = \left[ U \frac{\partial u}{\partial y} \cdot \frac{U}{v} \right]^2 = \frac{U^4}{v^2} \left[ \frac{\partial u}{\partial y} \right]^2 \\ \left[ \frac{\partial w^+}{\partial y^+} \right]^2 &= \left[ \frac{\partial w^+}{\partial y} \cdot \frac{\partial y}{\partial y^+} \right]^2 = \left[ U \frac{\partial w}{\partial y} \cdot \frac{U}{v} \right]^2 = \frac{U^4}{v^2} \left[ \frac{\partial w}{\partial y} \right]^2 \\ Q^+ &= \frac{K U^2 \delta}{v^2} \end{split}$$

Substituting them back in equation (53) yields

$$\begin{aligned} \frac{U^2(T_w^+ - T_\infty^+)}{\nu} \frac{\partial \theta}{\partial t} + \frac{uU^2(T_w^+ - T_\infty^+)}{\nu} \frac{\partial \theta}{\partial x} \\ + \frac{V_0 U^2(T_w^+ - T_\infty^+)}{\nu} \frac{\partial \theta}{\partial y} \\ = \frac{k}{\rho C_p} \left[ \frac{U^2}{\nu^2} (T_w^+ - T_\infty^+) \frac{\partial^2 \theta}{\partial y^2} \\ + \frac{U^2}{\nu^2} (T_w^+ - T_\infty^+) \frac{\partial^2 \theta}{\partial x^2} \right] + \frac{k U^2 \delta}{\rho C_p \nu^2} \\ + \frac{\mu}{\rho C_p} \left[ \frac{U^4}{\nu^2} \left[ \frac{\partial u}{\partial y} \right]^2 + \frac{U^4}{\nu^2} \left[ \frac{\partial w}{\partial y} \right]^2 \right] \end{aligned}$$

Dividing through by  $\frac{U^2(T_w^+ - T_\infty^+)}{v}$  results to

$$\frac{\partial\theta}{\partial t} + u \frac{\partial\theta}{\partial x} - V_0 \frac{\partial\theta}{\partial y} = \frac{k}{\rho C_p} \left[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] + \frac{k}{\rho C_p v (T_w^+ - T_w^+)} + \frac{U^2 \mu}{\rho C_p (T_w^+ - T_w^+)} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right]$$
  
But  $Pr = \frac{\mu C_p}{\rho C_p} = \frac{\rho v C_p}{\rho C_p}$ 

But 
$$Pr = \frac{Pre_p}{k} = \frac{Pre_p}{k}$$
  
 $Ec = \frac{U^2}{C_p(T_w^+ - T_w^+)}$  and  
 $Q = \frac{\delta K U^2}{v^2}$ 

Hence we have

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} - V_0\frac{\partial\theta}{\partial y} = \frac{1}{Pr} \left[\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right] + \frac{\delta}{Pr}\theta + E_c \left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$
(57)

Non-dimensionalizing equation (54) we have

$$\frac{\partial c^{+}}{\partial t^{+}} = \frac{\partial c^{+}}{\partial c} \cdot \frac{\partial c}{\partial t} \cdot \frac{\partial t}{\partial t^{+}} = \frac{U^{2}}{v} (C_{w}^{+} - C_{\infty}^{+}) \frac{\partial c}{\partial t}$$
$$u^{+} \frac{\partial c^{+}}{\partial x^{+}} = uU^{2} \frac{\partial c^{+}}{\partial c} \cdot \frac{\partial c}{\partial x} \cdot \frac{\partial x}{\partial x^{+}} = \frac{uU}{v} (C_{w}^{+} - C_{\infty}^{+}) \frac{\partial c}{\partial x}$$
$$V_{0}^{+} \frac{\partial c^{+}}{\partial y^{+}} = V_{0}U^{2} \frac{\partial c^{+}}{\partial c} \cdot \frac{\partial c}{\partial y} \cdot \frac{\partial x}{\partial y^{+}} = \frac{V_{0}U}{v} (C_{w}^{+} - C_{\infty}^{+}) \frac{\partial c}{\partial y}$$

$$\frac{\partial^2 c^+}{\partial y^{+2}} = \frac{\partial}{\partial y^+} \left[ \frac{\partial c^+}{\partial y^+} \right] = \frac{\partial}{\partial y} \left[ \frac{U}{v} \left( C_w^+ - C_w^+ \right) \frac{\partial c}{\partial y} \right] \frac{\partial y}{\partial y^+}$$
$$= \frac{U^2}{v^2} \left( C_w^+ - C_w^+ \right) \frac{\partial^2 c}{\partial y^2}$$
$$\frac{\partial^2 c^+}{\partial x^{+2}} = \frac{\partial}{\partial x^+} \left[ \frac{\partial c^+}{\partial x^+} \right] = \frac{\partial}{\partial x} \left[ \frac{U}{v} \left( C_w^+ - C_w^+ \right) \frac{\partial c}{\partial x} \right] \frac{\partial x}{\partial x^+}$$
$$= \frac{U^2}{v^2} \left( C_w^+ - C_w^+ \right) \frac{\partial^2 c}{\partial x^2}$$

Substituting back in equation (54) we obtain

$$\frac{J^{2}}{v}(C_{w}^{+}-C_{\infty}^{+})\frac{\partial c}{\partial t} + \frac{uU^{2}}{v}(C_{w}^{+}-C_{\infty}^{+})\frac{\partial c}{\partial x}$$
$$-\frac{V_{0}U^{2}}{v}(C_{w}^{+}-C_{\infty}^{+})\frac{\partial c}{\partial y}$$
$$=\frac{DU^{2}}{v^{2}}(C_{w}^{+}-C_{\infty}^{+})\left[\frac{\partial^{2}c}{\partial x^{2}}+\frac{\partial^{2}c}{\partial y^{2}}\right]$$

Dividing through by  $\frac{U^2(C_w^+ - C_\infty^+)}{v}$  gives

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - V_0 \frac{\partial c}{\partial y} = \frac{D}{v} \left[ \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right]$$

Using Schmidt number  $Sc = \frac{v}{D}$  meaning  $\frac{1}{Sc} = \frac{D}{v}$  thus the equation reduces to

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - V_0 \frac{\partial c}{\partial y} = \frac{1}{Sc} \left[ \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right]$$
(58)

## 4. Methodology

Due to the non-linear nature of equations, an interactive procedure that is more accurate and flexible is employed. A numerical method of finding a solution is therefore employed in order to satisfy basic requirements such as consistency, stability and convergence. A method is said to be convergent if, as more grid points are taken or step size decreased, the numerical solution converge to the exact solution. A method is stable if the effect of any single fixed round off error is bounded. Lastly a method is consistent if the truncation error tends to zero as the step size decreases. The numerical error arises because in most computations we cannot exactly compute the difference solution as we encounter round off error. If the effects of the round off error remains bounded as the mesh point tend to infinity with fixed step size then the difference method is said to be stable. In order to approximate equations (29) to (32) by a set of finite difference equations, we define a suitable mesh point.

#### The Numerical Method

#### A Mesh Point

In order to give a relationship between the partial derivatives in the differential equation and the function value at the adjacent nodal points we use a uniform mesh. Let x-y plane be divided into a network of uniform rectangular cells of width  $\Delta y$  and height  $\Delta x$  as shown below, j and i refer to y and x respectively.



Figure 2: Mesh point

Let  $\Delta y$  represent increment in y and  $\Delta x$  represent increment in x then  $y = j\Delta y$  and  $x = i\Delta x$ . The finite difference approximation of the partial derivatives appearing in equation (55) to (58) are obtained by Taylor series expansion of the dependent variable about a grid point (j, i) as,

$$\phi(j-1,i) = \phi(j,i) - \phi'(j,i)\Delta y + \frac{1}{2}\phi''(j,i)(\Delta y)^{2} - \frac{1}{6}\phi'''(j,i)(\Delta y)^{3} + \cdots$$
(59)

$$\phi(j+1,i) = \phi(j,i) + \phi'(j,i)\Delta y + \frac{1}{2}\phi''(j,i)(\Delta y)^{2} + \frac{1}{6}\phi'''(j,i)(\Delta y)^{3} + \cdots$$
(60)

On eliminating  $\phi$  from equation (55) to (58) by subtraction we get

$$\phi'(j,i) = \frac{\phi(j+1,i) - \phi(j-1,i)}{2\Delta y} + \text{Hot}$$
 (61)

On eliminating  $\phi'$  from equation (55) to (58) by adding them we obtain

$$\phi' = \frac{\phi(j+1,i) - 2\phi(j,i) + \phi(j-1,i)}{(\Delta y)^2} + \text{Hot}$$
(62)

Similarly the central difference formulae for the first and second derivatives are

$$\phi'(j,i) = \frac{\phi(j,i+1) - \phi(j,i-1)}{2\Delta x} + \text{Hot}$$
(63)  
$$\phi' = \frac{\phi(j,i+1) - 2\phi(j,i) + \phi(j,i-1)}{2\Delta x} + \text{Hot}$$
(64)

$$\phi' = \frac{\phi(j,i+1) - 2\phi(j,i) + \phi(j,i-1)}{(\Delta x)^2} + \text{Hot}$$
(64)

In this study we use subscripts to indicate spatial points and superscripts to indicate time.

 $T_{(j,i)}^{n+1} = (y_j, x_i, t_{n+1})$ . Let the mesh point variable at time  $t_n$  be denoted by  $\phi_{(j,i)}^n$ . The forward difference for the first order derivatives with respect to time is given by

$$\phi_{(j,i)}^{'n} = \frac{\phi_{(j,i)}^{n+1} - \phi_{(j,i)}^{n}}{\Delta t} + \text{Hot}$$
(65)

Substituting finite difference equations for the first and second derivatives in equations

(55) to (58) the final set of governing equations is

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$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - V_0 \frac{\partial u}{\partial y} = \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + Gr\theta + GcC - M^2 u$$

becomes

$$\begin{aligned} \frac{u_{(j,i)}^{n+1} - u_{(j,i)}^n}{\Delta t} + u_{(j,i)}^n \left[ \frac{u_{(j,i+1)}^n - u_{(j,i-1)}^n}{2\Delta x} \right] \\ &- V_0^n \left[ \frac{u_{(j+1,i)}^n - u_{(j-1,i)}^n}{2\Delta y} \right] \\ &= \left[ \frac{u_{(j,i+1)}^n - 2u_{(j,i)}^n + u_{(j,i-1)}^n}{(\Delta x)^2} \right] \\ &+ \left[ \frac{u_{(j+1,i)}^n - 2u_{(j,i)}^n + u_{(j-1,i)}^n}{(\Delta y)^2} \right] + G_r \theta_{(j,i)}^n \end{aligned}$$

Making  $u_{(j,i)}^{n+1}$  the subject

$$\begin{split} u_{(j,i)}^{n+1} &= \\ u_{(j,i)}^{n} + \left[ -u_{(j,i)}^{n} \left[ \frac{u_{(j,i+1)}^{n} - u_{(j,i-1)}^{n}}{2\Delta x} \right] + V_{0}^{n} \left[ \frac{u_{(j+1,i)}^{n} - u_{(j-1,i)}^{n}}{2\Delta y} \right] + \\ \left[ \frac{u_{(j,i+1)}^{n} - 2u_{(j,i)}^{n} + u_{(j-1,i)}^{n}}{(\Delta x)^{2}} \right] + \\ \left[ \frac{u_{(j+1,i)}^{n} - 2u_{(j,i)}^{n} + u_{(j-1,i)}^{n}}{(\Delta y)^{2}} \right] + G_{r} \theta_{(j,i)}^{n} + \\ G_{c} C_{(j,i)}^{n} - M^{2} u_{(j,i)}^{n} \right] \Delta t \qquad (3.66) \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} - V_{0} \frac{\partial w}{\partial y} = \left[ \frac{\partial^{2} w}{\partial x^{2}} + \frac{\partial^{2} w}{\partial y^{2}} \right] - M^{2} w \text{ becomes} \\ \frac{w_{(j,i)}^{n+1} - w_{(j,i)}^{n}}{\Delta t} + u_{(j,i)}^{n} \left[ \frac{w_{(j,i+1)}^{n} - w_{(j,i-1)}^{n}}{2\Delta x} \right] \\ - V_{0}^{n} \left[ \frac{w_{(j,i+1)}^{n} - 2w_{(j,i)}^{n} + w_{(j,i-1)}^{n}}{(\Delta x)^{2}} \right] \\ + \left[ \frac{w_{(j+1,i)}^{n} - 2w_{(j,i)}^{n} + w_{(j,i-1)}^{n}}{(\Delta y)^{2}} \right] \\ - M^{2} w_{(j,i)}^{n} \end{split}$$

Making  $w_{(j,i)}^{n+1}$  the subject  $w_{(j,i)}^{n+1} =$ 

$$\begin{split} w_{(j,i)}^{n} &= \\ w_{(j,i)}^{n} + \left[ -u_{(j,i)}^{n} \left[ \frac{w_{(j,i+1)}^{n} - w_{(j,i-1)}^{n}}{2\Delta x} \right] + V_{0}^{n} \left[ \frac{w_{(j+1,i)}^{n} - w_{(j-1,i)}^{n}}{2\Delta y} \right] + \\ \left[ \frac{w_{(j,i+1)}^{n} - 2w_{(j,i)}^{n} + w_{(j,i-1)}^{n}}{(\Delta x)^{2}} \right] + \\ \left[ \frac{w_{(j+1,i)}^{n} - 2w_{(j,i)}^{n} + w_{(j-1,i)}^{n}}{(\Delta y)^{2}} \right] - M^{2} w_{(j,i)}^{n} \right] \Delta t \end{split}$$

$$(67)$$

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} - V_{0} \frac{\partial \theta}{\partial y} = \frac{1}{P_{r}} \left[ \frac{\partial^{2} \theta}{\partial x^{2}} + \frac{\partial^{2} \theta}{\partial y^{2}} \right] + \frac{\delta}{P_{r}} \theta + E_{c} \left[ \left( \frac{\partial u}{\partial y} \right)^{2} + \left( \frac{\partial w}{\partial y} \right)^{2} \right]$$
becomes

$$\begin{split} \frac{\theta_{(j,i)}^{n+1} - \theta_{(j,i)}^{n}}{\Delta t} + u_{(j,i)}^{n} \left[ \frac{\theta_{(j,i+1)}^{n} - \theta_{(j,i-1)}^{n}}{2\Delta x} \right] \\ &- V_{0}^{n} \left[ \frac{\theta_{(j+1,i)}^{n} - \theta_{(j-1,i)}^{n}}{2\Delta y} \right] \\ &= \frac{1}{Pr} \left[ \frac{\theta_{(j,i+1)}^{n} - 2\theta_{(j,i)}^{n} + \theta_{(j,i-1)}^{n}}{(\Delta x)^{2}} \right] \\ &+ \frac{1}{Pr} \left[ \frac{\theta_{(j+1,i)}^{n} - 2\theta_{(j,i)}^{n} + \theta_{(j-1,i)}^{n}}{(\Delta y)^{2}} \right] \\ &+ \frac{\delta}{Pr} \theta_{(j,i)}^{n} + Ec \left[ \frac{u_{(j+1,i)}^{n} - u_{(j-1,i)}^{n}}{2\Delta y} \right]^{2} \\ &+ Ec \left[ \frac{w_{(j+1,i)}^{n} - w_{(j-1,i)}^{n}}{2\Delta y} \right]^{2} \end{split}$$

Making  $\theta_{(j,i)}^{n+1}$  the subject  $a^{n+1}$ 

$$\begin{split} \theta_{(j,i)}^{n+1} &= \\ \theta_{(j,i)}^{n} + \left[ -u_{(j,i)}^{n} \left[ \frac{\theta_{(j,i+1)}^{n} - \theta_{(j,i-1)}^{n}}{2\Delta x} \right] + V_{0}^{n} \left[ \frac{\theta_{(j+1,i)}^{n} - \theta_{(j-1,i)}^{n}}{2\Delta y} \right] + \\ \frac{1}{P_{r}} \left[ \frac{\theta_{(j,i+1)}^{n} - 2\theta_{(j,i)}^{n} + \theta_{(j,i-1)}^{n}}{(\Delta x)^{2}} \right] + \\ \frac{1}{P_{r}} \left[ \frac{\theta_{(j+1,i)}^{n} - 2\theta_{(j,i)}^{n} + \theta_{(j,i-1)}^{n}}{(\Delta y)^{2}} \right] + \\ Ec \left[ \frac{u_{(j+1,i)}^{n} - u_{(j-1,i)}^{n}}{2\Delta y} \right]^{2} + \\ Ec \left[ \frac{w_{(j+1,i)}^{n} - u_{(j-1,i)}^{n}}{2\Delta y} \right]^{2} \right] \Delta t (68) \\ \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - V_{0} \frac{\partial c}{\partial y} = \frac{1}{S_{c}} \left[ \frac{\partial^{2} c}{\partial x^{2}} + \frac{\partial^{2} c}{\partial y^{2}} \right] \text{ becomes} \\ \frac{C_{(j,i)}^{n+1} - C_{(j,i)}^{n}}{\Delta t} + u_{(j,i)}^{n} \left[ \frac{C_{(j,i+1)}^{n} - C_{(j,i-1)}^{n}}{2\Delta x} \right] \\ - V_{0}^{n} \left[ \frac{C_{(j+1,i)}^{n} - C_{(j,i-1)}^{n}}{2\Delta y} \right] \\ = \frac{1}{S_{c}} \left[ \frac{C_{(j,i+1)}^{n} - 2C_{(j,i)}^{n} + C_{(j,i-1)}^{n}}{(\Delta x)^{2}} \right] \\ + \frac{1}{S_{c}} \left[ \frac{C_{(j+1,i)}^{n} - 2C_{(j,i)}^{n} + C_{(j-1,i)}^{n}}{(\Delta y)^{2}} \right] \end{split}$$

Making  $C_{(j,i)}^{n+1}$  the subject

$$C_{(j,i)}^{n+1} = C_{(j,i)}^{n} + \left[ -u_{(j,i)}^{n} \left[ \frac{C_{(j,i+1)}^{n} - C_{(j,i-1)}^{n}}{2\Delta x} \right] + V_{0}^{n} \left[ \frac{C_{(j+1,i)}^{n} - C_{(j-1,i)}^{n}}{2\Delta y} \right] + \frac{1}{Sc} \left[ \frac{C_{(j,i+1)}^{n} - 2C_{(j,i)}^{n} + C_{(j,i-1)}^{n}}{(\Delta x)^{2}} \right] + \frac{1}{Sc} \left[ \frac{C_{(j+1,i)}^{n} - 2C_{(j,i)}^{n} + C_{(j-1,i)}^{n}}{(\Delta y)^{2}} \right] \Delta t$$

$$(69)$$

The computations are performed using small values in  $\Delta x$ and  $\Delta y$ . Thus in this research we set  $\Delta x = 0.012$  and  $\Delta y = 0.25$ . The x-axis along the infinite vertical plate then x varies from 0 to infinity. Fixing y=3.1 for which j=31 as corresponding to  $y = \infty$  therefore set  $u_{(31,i)}^n = v_{(31,i)}^n =$  $C_{(31,i)}^n = \theta_{(31,i)}^n = 0$  because u, v, C and  $\theta$  tend to zero around y=3.1.

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The initial conditions are

At 
$$y = 0$$
,  $u_{(0,i)}^0 = 1$ ,  $\theta_{(0,i)}^0 = 1$ ,  $w_{(0,i)}^0 = 1$   $C_{(0,i)}^0 = 1$   
 $y > 0$ ,  $u_{(j,i)}^0 = 1$ ,  $\theta_{(j,i)}^0 = 1$ ,  $w_{(j,i)}^0 = 1$   $C_{(j,i)}^0 = 1$ 

k > 0 and all i the boundary conditions takes the form

At 
$$y = 0$$
,  $u_{(0,i)}^n = 1$ ,  $\theta_{(0,i)}^n = 1$ ,  $w_{(0,i)}^n = 1$   $C_{(0,i)}^n = 1$   
 $x = 0$ ,  $u_{(i,0)}^n = 1$ ,  $\theta_{(i,0)}^n = 1$ ,  $w_{(i,0)}^n = 1$   $C_{(i,0)}^n = 1$ 

In this computation the Prandtl number is taken as 0.71 which corresponds to air, magnetic parameter  $M^2 = 5.0$ signifying strong magnetic field, Grashof number Gr = 1, modified Grashof number Gc = 5, d = 5, V = 5 and Ec =100. To ensure stability and convergence of the finite difference method, the program is run using smaller values of  $\Delta x$  and  $\Delta y$ .

## 5. Observation and Discussion of Results

A java program was written and run for various values of velocities, temperatures and concentrations for finite difference equation (3.66) to (3.69) for various values of Schmidt parameter (10 and 50) and also Eckert parameter (10 and 100). The primary velocity (u), secondary velocity (w), temperature  $(\theta)$  and concentration (C) profiles are presented as shown in the figures and the tables below.

## 5.1 Discussion of the Results

6

7

Table 1: Primary velocity profile			
	Sc=10	Sc=50	
0	0	0	
1	-1.75E-06	-1.75E-06	
2	-5.26E-18	-5.26E-18	
3	-1.55E-29	-1.55E-29	
4	-4.43E-41	-4.43E-41	
5	-1.24E-52	-1.24E-52	

-3 32E-64

-8.56E-76

-3 32E-64

-8.56E-76





From table 1 and figure 3, it was observed that an increase in mass diffusion parameter Sc leads to a decrease in primary velocity profile as shown by the curves Sc=10 and Sc=50. This is because the mass diffusion parameter Sc is directly proportional to shear stresses and inversely proportional to the product of velocity profile, density which is assumed constant and mass diffusivity, therefore an increase in mass diffusion parameter leads to a decrease in the velocity profile. However as the fluid flows away from the plate, the trend reverses as the Sc parameter decreases and velocity profile starts increases since the two are inversely proportional

Table 2:	Secondary	Velocity Profile
----------	-----------	------------------

	Sc=10	Sc=50
0	0	0
1	-1.04E-02	-8.45E-03
2	-1.14E-02	-9.39E-03
3	-1.23E-02	-1.04E-02
4	-1.31E-02	-1.14E-02
5	-1.34E-02	-1.23E-02
6	-1.30E-02	-1.31E-02
7	-1.23E-02	-1.34E-02
8	-1.02E-02	-0.0134
9	-7.95E-03	-1.30E-02



Figure 4: Secondary Velocity Profile (w-graph)

From table 2 and figure 4, it was observed that an increase in mass diffusion parameter Sc causes a decrease in the secondary velocity profile as shown by the curves Sc=10 and Sc=50. This is because the mass diffusion parameter is directly proportional to dynamic viscosity which is a measure of shear stresses and inversely proportional to the product of velocity profile, density which is assumed to be constant and mass diffusivity, therefore an increase in mass diffusion causes a decrease in secondary velocity profile

Table 3: Temp	berature Profile
---------------	------------------

	Sc=10	Sc=50
0	0.00E+00	0
1	8.94E-03	8.94E-03
2	9.86E-03	9.86E-03
3	1.09E-02	1.09E-02
4	1.19E-02	1.19E-02
5	1.27E-02	1.27E-02
6	1.33E-02	1.33E-02
7	0.013467	0.013467
8	0.013236	0.013236

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**Figure 5:** Temperature profile ( $\theta$ -graph)

From table 3 and figure 5, it was observed that an increase in mass diffusion parameter Sc causes an increase in the temperature profile as shown by the curves Sc=10 and Sc=50. The mass diffusion parameter Sc gives the ratio of kinematic viscosity and mass diffusivity thus when mass diffusivity is increased, the temperature also increases since the two are directly proportional.

Table 4: Concentration profile with Sc varying

	Sc=10	Sc=50
0	0	0
1	1.84E-13	3.67E-13
2	9.74E-20	7.80E-19
3	5.16E-26	1.65E-24
4	2.71E-32	3.48E-30
5	1.39E-38	7.16E-36
6	6.95E-45	1.43E-41
	3.35E-51	



Figure 6.1: Concentration profile with Sc varying (cgraph)

From table 4 and figure 6.1, it was observed that a decrease in mass diffusion parameter Sc causes an increase in the concentration profile. This is because Sc is directly proportional to the dynamic viscosity which is a measure of shear stresses and inversely proportional to product of velocity profiles, density which is assumed to be constant and mass diffusivity. Since the mass diffusion parameter Sc is inversely proportional to mass diffusivity, then a decrease in Sc results into an increase in concentration profile. However as the fluid flows away from the plate the trend reverses and there is an increase in Sc which results into a decrease in the concentration profile.

Fable 5:	Concentration	profile	with	Ec	varying
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	Ec=10	Ec=100
0	0	0
1	1.84E-13	1.84E-13
2	9.74E-20	9.74E-20
3	5.16E-26	5.16E-26
4	2.71E-32	2.71E-32
5	1.39E-38	1.39E-38
6	6.95E-45	6.95E-45



Figure 6.2: Concentration profile with Ec varying (c-graph)

From table 5 and figure 6.2, it was observed that as the Eckert parameter which provides a measure of kinetic energy to the flow relative to thermal energy decreases, the concentration profile increases. This is because a decrease in the Ec leads to an increase in thermal energy because they are inversely proportional hence an increase in the concentration of the fluid which results to an increase in the concentration profile. However, as fluid flows away from the plate, the trend reverses and this result into an increase in the Ec parameter which in turn reduces the concentration profile.

# 6. Conclusions and Recommendations

## Conclusion

An analysis of effects of various parameters on the velocities, temperature and concentration profiles on a viscous incompressible heat generating fluid flow past an infinite porous plate with radiation absorption has been carried out. In all the cases considered, the applied magnetic field was resolved into two components and our work restricted to laminar boundary layer. The equations governing the flow considered in our problem are non-linear therefore in order to obtain their solutions; an efficient finite difference method has been used as outlined in chapter three. The results obtained for various values of Schmidt parameter Sc and Eckert Ec parameter were presented graphically and in table form.

In this study the results obtained for mass diffusion parameter Sc, for Sc=10 and Sc=50 was investigated on

velocity profiles, temperature profile and concentration profile. In all the cases it was noted that an increase in mass diffusion parameter leads to a decrease in both primary and secondary profiles and also concentration profile. However an increase in mass diffusion parameter leads to an increase in the temperature profile.

It was noted that an increase in radiation absorption leads to a decrease in primary velocity profile in the presence of heating of the plate by free convection currents. However in the presence of cooling of the plate by free convection currents, increase in radiation absorption leads to an increase in velocity profiles. It was also noted that an increase in the viscous dissipative heat Ec, causes an increase in concentration profile.

# 7. Recommendations

This work considered a viscous incompressible heat generating fluid flow past an infinite porous plate with radiation absorption. The flow was restricted to laminar boundary layer. It is recommended that this work be extended by considering the following;

- i) Varied viscosity and thermal conductivity
- ii) Compressible fluid
- iii) Periodic suction
- iv) Fluid flow in the turbulent boundary layer
- v) Investigating the exact implications of the decrease and increase in the parameters

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