Stress Analysis of Finite Plate with Special Shaped Cutout

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Abstract: ***Plates with various shaped cutout are often used in both modern and classical aerospace mechanical and civil engineering structure. The understanding of the effect of load bearing capacity and stress concentration of such plate is very important in designing of structure. The main objective of this paper is for stress analysis of finite plate with special shaped cut out for stress distribution and Stress Concentration Factor (SCF). An Experimental investigation is taken to study the stress analysis of plate with special shaped cut out. The results based on Experimental analysis are compared with result obtained using finite element analysis (FEA).***

Keywords: Stress concentration factor, Finite plate, special shaped cutout, Photoelasticity

1. Introduction

In General thin plates are easily manufactured and are widely used for primary structural element in aerospace mechanical, civil engineering structures, marine structures. In recent year, different cut out shapes in structural element are need to reduce the weight of the system and provide access to other parts of the structure.

It is well known that the presence of cut out or hole in a stressed members creates highly localized stresses at the vicinity of the cut out. This cut out works as stress raisers and may leads to failures of structures/machine components. Hence it is important aspect of stress analysis to predict stress concentration for special shaped cut out like circular, triangular, square and rectangular.

2. Stress Concentration Factor

Stress concentration is defined as the localization of high stresses due to the irregularities present in the components and abrupt changes of the cross section. In order to consider the effect of stress concentration and find out localized stress a factor called stress concentration factor is used. It is denoted by K_t [9]. The ratio of maximum stress at the cut out edge to nominal stress is called stress concentration factor

$K_t = \sigma_{max} / \sigma_{nom}$

Where,
σ_{max} = Maximum stress at the hole
σ_{nom} = Nominal stress

For a finite plate with a circular hole at the center, that is subjected to uni-axial far-field tension, σ_n is acting along the x-axis, the stresses around the vicinity of the hole which shown in fig. 1.

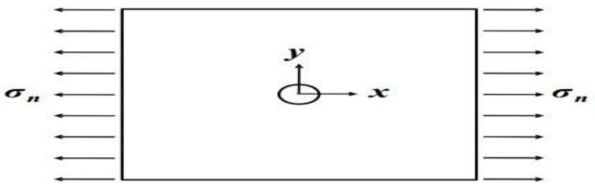

Figure 1: Plate with circular hole subjected to uni-axial stress

3. Literature Review

V.G. Ukadgaonker D.K.N.Rao [1] presented an extension of Becker's solution for elliptical hole problem for unsymmetrical laminates to determine the stress resultant and moment around hole of any shape under arbitrary biaxial loading condition.
V.G. Ukadgaonker D.K.N.Rao [2] adapting Savin's formulation for stress concentration problems in symmetric laminates under inplane loading.
K.T.Chen, K.Ting, W.S.Yang [3] developed Boundary Element Alternating Method to study the stress concentration of two dimensional perforated plate.
Lasko et al. [4] used relaxation element method to determine the stress fields in a plate with three circular cutouts subjected to uniaxial tensile load
Singh A.V. and Paul U.K. [5] presented numerical results based on generalized work–energy method for rectangular plates with a circular cutout and circular plates with a rectangular cutout
Rezaeepazhand and Jafari [6,8] have given the stress distribution around several non circular cut out in isotropic and composite plate using Leknitskii's solution.
Murat Yazici [7] an Elasto-plastic theoretical analysis of stresses around a square perforated isotropic plate is studies by using Savin's complex elastic equation.
Mohsen Mohammadi, John R. Dryden, Liying Jiang [10] analyzes the effect of nonhomogenous stiffness and varying Poisson's ratio upon the stress concentration factor using Frobenius series solution.
Jinho Woo and Won-Bae Na [11] presents stress concentration analysis of perforated plate with not only various cut out and bluntness but also different cut out orientations using ANSYS.

D.S.Sharma [12] gives the stress concentration around circular/elliptical/triangular cut out in infinite composite plate by using Muskhelishvili's complex variable method. The effect of fiber orientation, stacking sequence, loading factor and angle, cut out geometry on SCF around cut out in orthotropic plate is studied.
Milan Batista [13] present Modified Muskhelishvili's complex variable method to calculate the stress distribution around holes of relatively complex shapes in infinite plate subjected to uniform load at infinity.
Dharmendra S. Sharma [15] used Muskhelishvili's complex variable method to present stress distribution around polygonal hole (Triangular, Square, Pentagonal, and Hexagonal). The effect of hole geometry and loading pattern on SCF is studied.
Zuxing Pan, Yuansheng Cheng, Jun Liu [16] presented Muskhelishvili's complex variable method for stress distribution around a rectangular hole in finite plate under uniaxial tension. The effects of hole sizes, hole orientations and plate's aspect ratios on the stress distribution and stress concentration factor in a finite plate with a rectangular hole subjected to uniaxial tension are studied.
Tawakol A. Enab [17] Stress concentration factors at the root of an elliptic hole in unidirectional functionally graded material (UDFGM) plates under uniaxial and biaxial loads are predicted. ANSYS Parametric Design Language (APDL) was used to build the finite element models for the plates and to run the analysis. A parametric study is performed for several geometric and material parameters such as the elliptic hole major axis to plate width ratio, the elliptical shape factor, the gradation direction of UDFG.

Many of work done by the various Researchers in the recent past reveals that there are some Analytical like Muskhelishvili's complex variable method, Two dimensional theory of elasticity, Savin's basic formulation, Leknitskii's solution, Boundary Element Alternating Method, Relaxation element method and Finite element solution for composite/infinite isotropic/orthotropic/FGM plate with hole. According to Zuxing Pan, Yuansheng Cheng, Jun Liu [20], In infinite plate analysis Analytical method are limited to study the stress analysis of an infinite plate with hole but there are lots of cases which do not satisfy the assumption of an infinite plate in practical engineering application .The stress analysis solution of an infinite plate with hole are not suitably applied to the cases of a finite plate with hole in which the effect of outer boundary of the finite plate on the stress field is needed to be considered. Hence, it is necessary to study the stress analysis of finite plate with hole.

4. Experimental Procedure

4.1 Photoelasticity

The name photoelasticity implies the use of light (photo) and elastically stresses model. This method was earlier used for plane bodies of complicated shape and geometries, particularly for the reason that such geometrical shapes were not amenable to mathematical analysis. Photoelasticity is an experimental method for measurement of stress and strain in which light is either passed through a model or reflected from the surface of loaded body. Photoelastic model is generally preferred in situation where and strain information is needed over extended region and thus whole field method.

Photoelastic stress analysis is a full field technique for measuring the magnitude and direction of principle stresses. When polarized light is passed through a stressed transparent model, interference patterns or fringes are formed. These patterns provide immediate qualitative information about the general distribution of stress, positions of stress concentrations and of areas of low stress using the principles of stress optic law.

$$\sigma_1 - \sigma_2 = \frac{N f_\sigma}{h}$$

Where,
σ_1And σ_2 =Maximum and Minimum principle stresses at the point under consideration.
N=Fringe order
Fσ =Material Fringe Value
h= Thickness. (14)

4.2 Experimental Setup

4.2.1 Circular Polariscope

The technique of photoelasticity depends upon unique phenomenon known as birefringence, of transparent material, particularly the plastics. Birefringence implies that the plastic sheet is optically orthotropic in the sense that at any point two axes can be identified as slow and fast axes along which plate has two different values of refractive indices. The circular polariscope consists of a light source, a polarizer, a quarter-wave plate oriented at 45° with respect to the polarizer, a specimen, a second quarter-wave plate, an analyzer that is always crossed with respect to the polarizer.(14)

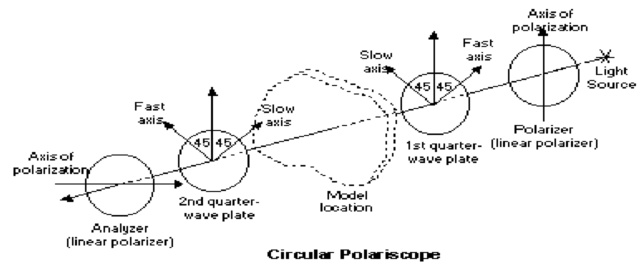

Figure 2: Schematic Representation of Circular Polariscope

Figure 3: Circular Polariscope

4.3 Material and Specimen Dimensions

To study the stress distribution in a plate with cutout a Photoelastic test is done on Araldite model uniformly loaded in one direction. The models are prepared with dimensions 200 mm×100mm×5mm. For photoelastic test Four cutout shapes are considered i.e. circular, square, triangular and rectangular. For square and triangular cutout Circumscribed circle is used. and for rectangular cut out consider L/H ratio 3, 4, 5 where L is length and H is the width of the rectangular cut out. The Diameter of circular cutout is 10,20,30 mm. The location of cutout is at the centre of plate. The plate is fixed at Lower one edge and the loading condition 981N (100kg) is applied at upper edge.

The Mechanical properties of the test specimen material Araldite Resin CY 230 and Hardener HY 951 shown in table 1

Table 1: Mechanical Properties of Araldite Resin

Name	*Trade Name*	*Young's Modulus(Mpa)*	*Poisson's ratio*
Epoxy resin	Araldite CY 230 and Hardener HY-951	2570.22	0.38

4.4 Casting Procedure for Preparation of Sheet

Araldite CY-230 along with hardener HY-951 is used for casting the sheets. For every 100 cc of araldite 10.5 cc of hardener is mixed. The resin is heated in oven up to 50° C to 80°C for about one hours to remove all air bubbles and moisture. Then it is cooled down slowly to the room temp. The hardener is added slowly by stirring the mixture continuously. The mixture should be stirred in one direction for ten minutes till it is transparent, clear and homogeneous. The mould is completely filled by the mixture, i.e. up to the top surface. The mould is kept at this position for proper curing at room temperature. For easy removal of the sheet from the mould, the curing time of sixteen to eighteen hours is sufficient. After curing time the sheet is removed from the mould carefully. The sheet in this stage is slightly plastic. So it is kept on the perfect flat transparent glass for further curing. The total curing time is about one week.

4.5 Preparation of Photoelastic Model with cutout

The drawing of plate is prepared on AUTOCAD to the actual size of plate with circular, triangular, square Rectangular cutout. Then the drawing is pasted on the Photoelastic sheet and it is cut by using model cutter, providing 2 to 3 mm allowance. Finally model is finished to the required size by filing and using fine emery paper. The required holes are drilled in the model for proper mounting in the loading fixture. The model of plate with circular, triangular, square, rectangular cutout is used for analysis.

4.6 Calibration of Photoelastic Material

The photoelastic material is calibrated by making a circular disc of 60 mm dia. out of the same sheet. The disc is loaded in increments under the diametral compression on Circular polariscope to find the material fringe value(Fσ). The fringe order at the centre of disc and corresponding load are recorded. The photoelastic material Plate model is found to have a stress fringe value equal to 13.4131 N/mm^2, as shown in Table 2.

Figure 4: The Calibration of photoelastic material

Table 2: Stress fringe value of photoelastic material

Sr. No	Load Kg	N	Fσ=8P/πDN	Fσ
1	10	0	0	13.4131 N/mm
2	30	1	12.4904	
3	60	2	12.4904	
4	100	3	13.8783	
5	135	4	14.0517	
6	170	5	14.1558	

4.7 Procedure for SCF in photoelasticity

Photoelastic method has been convincingly applied for the determination of SCF machine and structure parts. The stress concentration at any point in a photoelastic model becomes visible with concentration of fringe around any point. Since, the maximum stress would always occur at the boundary of a geometrical discontinuity the fringe order will directly give one of the principle stresses as other principle stresses would not exist. Thus Maximum principle stress are calculate by

$$\sigma_{max} = \frac{N_A F_\sigma}{h}$$

$$\sigma_{nom} = \frac{N_B F_\sigma}{h}$$

or Nominal stress =P/w h

$$K = \frac{\sigma_{max}}{\sigma_{nom}} = \frac{N_A}{N_B}$$

Where,
N_A=Fringe order near discontinuity
N_B= Fringe order at cross section (14)

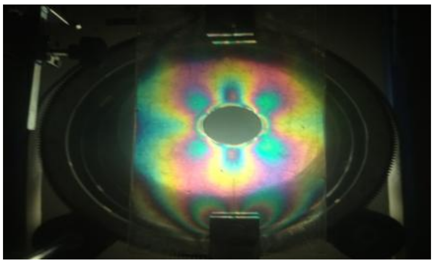

Figure 5: Isochromatic fringe pattern for circular hole

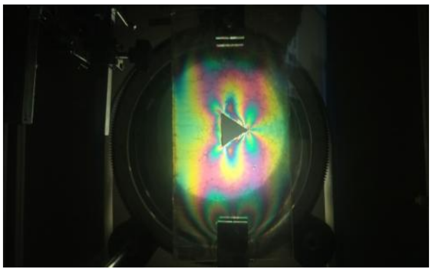

Figure 6: Isochromatic fringe pattern for Triangular hole

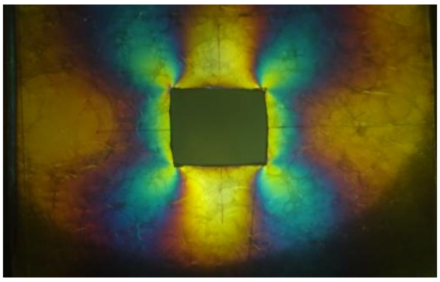

Figure 7: Isochromatic fringe pattern for Square hole

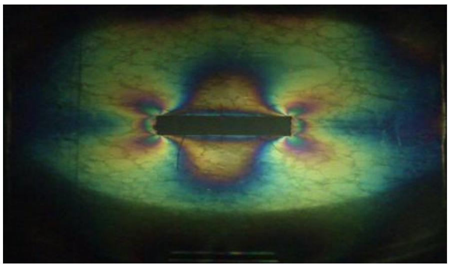

Figure 8: Isochromatic fringe pattern for Square hole

Table 3: Photoelasticity Result at 981 N (100kg)

Shape	Dimension	Fringe order (N)	σ_1-σ_2=NFσ/h (Mpa)	SCF
Circle	10 mm Dia	2	5.3652	2.46
	20 mm Dia	2.35	6.3041	2.57
	30 mm Dia	2.35	6.3041	2.24
Triangle	10 mm Dia	2	5.3652	2.73
	20 mm Dia	2	5.3652	2.73
	30 mm Dia	2.35	6.3041	3.21
Square	10 mm Dia	1.82	4.8823	2.48
	20 mm Dia	2	5.3652	2.73
	30 mm Dia	2	5.3652	2.73
Rectangle	L/H=3	1.82	4.8823	2.48
	L/H=4	2	5.3652	2.73
	L/H=5	2.35	6.3041	3.21

5. Finite Element Analysis

General purpose finite element software ANSYS^{TM} 14.5 is used for modeling, analysis and post processing of Finite plate with circular, triangular square & rectangular opening under axial Tension. Modeling of plate involves generation of a rectangle of size 200 mm x 100 mm and thickness 5mm. For the analysis of plate,8 NODE 183 SOLID Element is considered shown in fig.9. This element is defined by 8 nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element (plane stress, plane strain and generalized plane strain) or as an axisymmetric element. This element has plasticity, hyper elasticity, creep, stress stiffening, large deflection, and large strain capabilities For Finite Element Analysis keeping 981 N(100kg) load constant as in Photoelasticity analysis. The Loading & Boundary condition as shown in fig.10

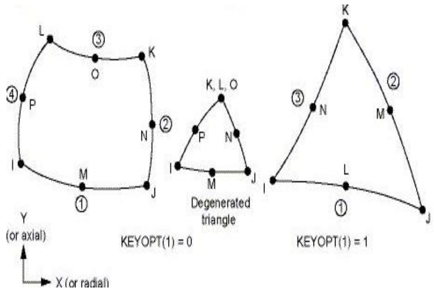

Figure 9: PLANE183 Geometry

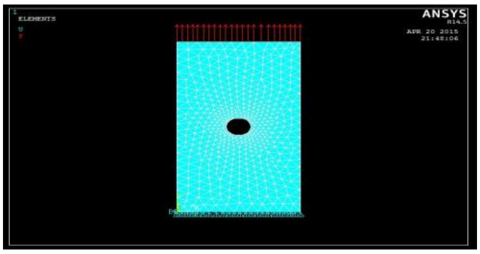

Figure 10: Loading, Boundary and Meshing of test specimen

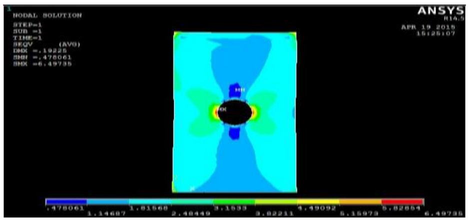

Figure 11: Plate with Circular hole d = 30mm

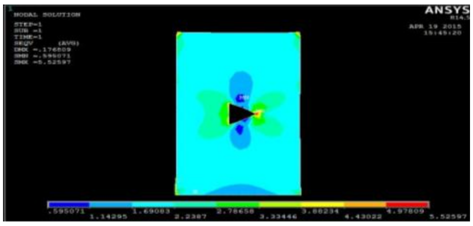

Figure 12: Plate with Triangular hole 30mm dia

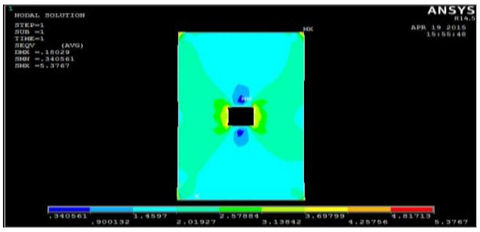

Figure 13: Plate with Square hole 30mm dia

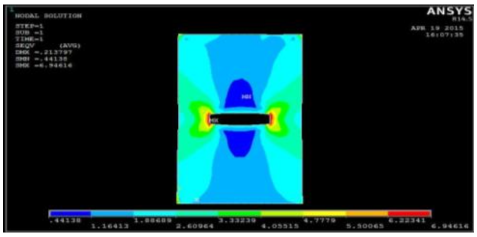

Figure 14: Plate with Rectangular hole L/H=5

Table 4: FEA Result at 981N (100kg)

Shape	*Dimension*	σ_{max} *(Mpa)*	*SCF*
Circle	10 mm Dia	5.8815	2.69
	20 mm Dia	6.0631	2.47
	30 mm Dia	6.4973	2.31
Triangle	10 mm Dia	5.3283	2.71
	20 mm Dia	5.3347	2.71
	30 mm Dia	5.5259	2.80
Square	10 mm Dia	5.3278	2.71
	20 mm Dia	5.3492	2.72
	30 mm Dia	5.3767	2.74
Rectangle	L/H=3	5.3575	2.73
	L/H=4	5.7247	2.91
	L/H=5	6.9461	3.54

6. Result and Discussion

6.1 Circular hole

Fig. 15 and 16 present the Max. principle stress and SCF result from photoelasticity and FEA for circular hole. In case of circular hole, Max. Principle stress increases as the diameter of hole varies from 10 to 30 mm and SCF decrease as the size of hole increase.

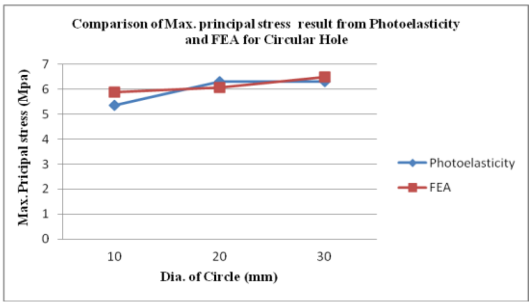

Figure 15: Comparison of Max. principle result from photoelasticity and FEA for circular hole

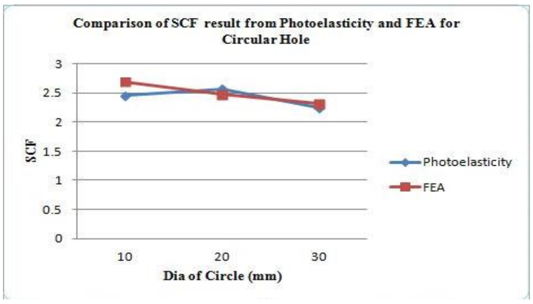

Figure 16: Comparison of SCF result from photoelasticity and FEA for Circular hole

6.2 Triangular Hole

Fig. 17 and 18 present the Max. principle stress and SCF result from photoelasticity and FEA for triangular hole. In this case, Max. principle stress and SCF increase as the dia. of circumscribed circle increases from 10mm to 30 mm.

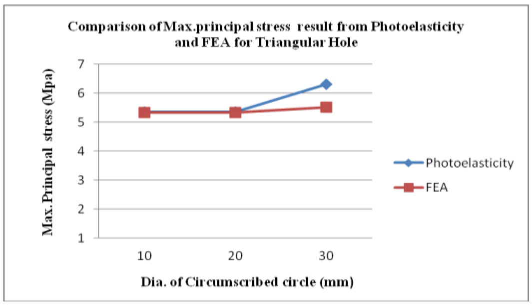

Figure 17: Comparison of Max. principle stress result from photoelasticity and FEA for Triangular hole

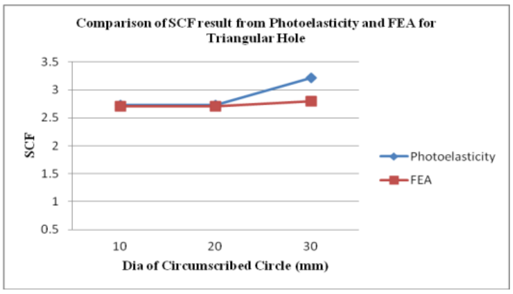

Figure 18: Comparison of SCF result from photoelasticity and FEA for Triangular hole

6.3 Square Hole

Fig.19 and 20 present the Max. principle stress and SCF result from photoelasticity and FEA for Square hole. for square hole, the Max. principle stress and SCF increase as the dia. of circumscribed circle increases from 10mm to 30 mm

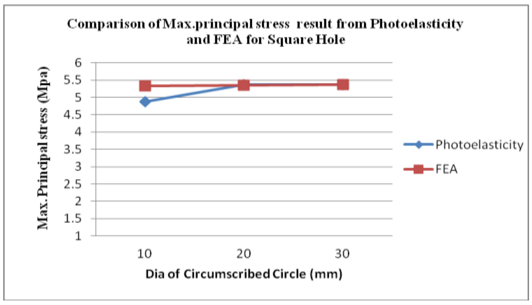

Figure 19: Comparison of Max. principle stress result from photoelasticity and FEA for Square hole

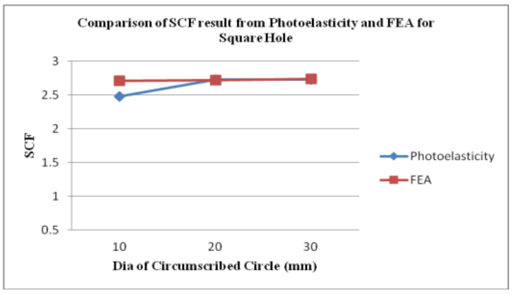

Figure 20: Comparison of SCF result from photoelasticity and FEA for Square hole

6.4 Rectangular Hole

Fig. 21 and 22 present the Max. principle stress and SCF result from photoelasticity and FEA for rectangular Hole. In case of rectangular hole, as L/H ratio increase from 3 to 5, Max. principle stress and SCF increases gradually.

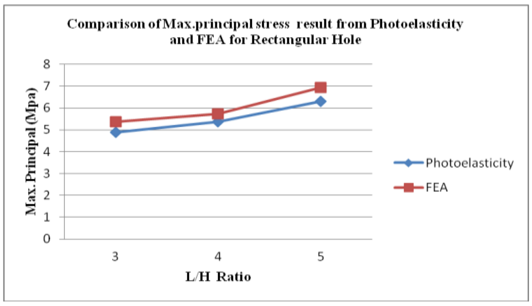

Figure 21: Comparison of Max. principle stress result from photoelasticity and FEA for Rectangular hole

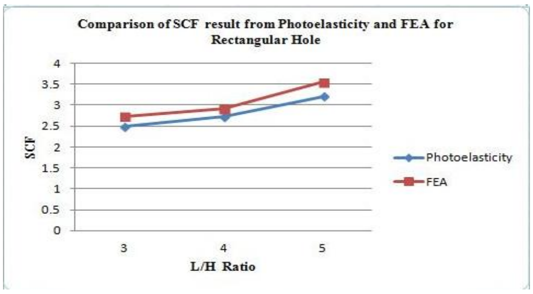

Figure 22: Comparison of SCF result from photoelasticity and FEA for Rectangular hole

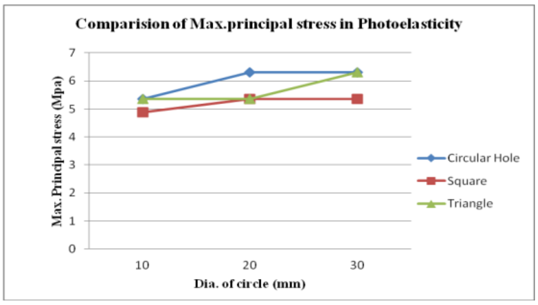

Figure 23: Comparison of Max. principle stress result from photoelasticity for circular, triangular, square hole

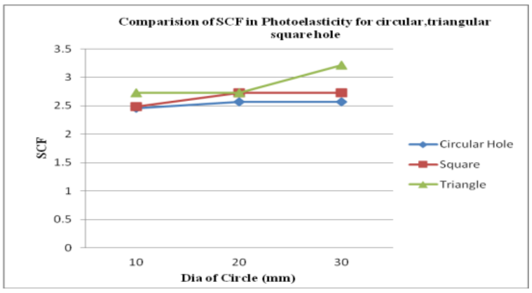

Figure 24: Comparison of SCF result from photoelasticity for circular, triangular, square hole

7. Conclusion

The high stress concentration at the edge of cut out is of practical importance in designing of engineering structures. The SCF_S of these type of cutout determined experimentally using photoelasticity and numerically using Finite Element Analysis (FEA).The result presented herein indicated that the stress concentration factor of plate with hole can significantly changed using proper cut out shape and size. From experimental and numerical analysis it is found that the SCF for circular cut out is less than triangular, square and rectangular cut out. The SCF for square cut out is less than Triangular cut out. The triangular cut out has highest stress concentration Factor. From graph it is observed that the stress concentration for plate with cut out by experimentally and numerically, are in good agreement for various cut out shape and size.

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