An Investigation on Cracked Plate for Stress Intensity Factor for Selected Configurations under Different Loading Modes

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Abstract: In this paper an effort is made to find stress intensity factor near the crack tip. Cracks in plates with different configurations often occurs in both modern and classical aerospace, mechanical and civil engineering structure. The understanding of effect of loading mode and crack configuration on load bearing capacity of such plate is very important in designing of structure and in damage tolerance analysis. A number of Analytical, Numerical, and Experimental techniques are available for stress analysis to determine the stress intensity factor near crack tip under different loading modes in an infinite/finite plate made up of different materials. In this paper stress intensity factors are determined for four key geometric configurations. These configurations were developed by using finite element method packages. Two dimensional finite plates under uniaxial loading with either: edge crack, both edge cracks, crack approaching circular hole, crack approaching triangular hole are considered for investigation.

Keywords: Stress intensity factor, finite plate, different configurations, and photoelasticity, loading modes

1. Introduction

In the lifetime of mechanical structures they are subjected to unfavorable changes in their structural properties mainly caused due to fatigue, environmental degradation, wear and errors in design and construction, overloads, unanticipated result from impacts. Aluminum and steel sheet metal plates are widely used in industrial applications such as aviation, automotive, ship-building industries etc. Plate’s structures are highly sensitive to crack formation and crack growth and the outcome of this can affect the performance and reliability. Notches, holes, and other mechanical defects that are unavoidable structural component acts as a stress concentration zone which initiates the formation of cracks.

The knowledge of acuteness or severity of cracks is necessary in order to predict fatigue crack growth rate, critical crack length, and fatigue life of component. According to the linear elastic fracture mechanics (LEFM)stress intensity factor is the key parameter which determines the severity of cracks as it reflects the effect of loading (mode-I, mode-II, mode-III), crack size and crack shape. Damage tolerance principle is the basis for modern structural design which requires tight inspection and maintenance plans, which adds the cost to the product. This increases the cost of ownership of these structures. But loosening inspection frequencies without compromising safety is highly needed throughout the service life. Damage tolerance analysis of structures is one of the fundamental tool in managing safety. The primary input to damage tolerance analysis is stress intensity factor which is used to determine crack growth life and critical crack length. Hence it is important aspect of stress analysis to predict stress intensity factor for different loading modes.

2. Stress Intensity Factor

The stress intensity factor is used in fracture mechanics to predict the stress state ("stress intensity") near the tip of a crack caused by a remote load or residual stresses. The magnitude of SIF depends on sample geometry, the size and location of the crack, and the magnitude and the modal distribution of loads on the material.

\[ K_i = \sigma \beta \sqrt{a} \]

Where,
\( \sigma \) = applied stress,
\( \beta \) = geometrical factor (dimensionless),
\( a \) = crack length,
For centre crack, length= 2a,
For edge crack, length= ‘a’

Modes of loading
Mode-I loading
Mode-II loading
Mixed mode loading

3. Literature Review

Veronique Lazarus, Jean-Baptiste Leblond, Salah-Eddine Mouchrif [1] evaluates the stress intensity factor (SIF) along the crack front after rotation by using Muskhelishvili’s complex potentials formalism and conformal mapping.

M. Gosza et.al. [2] used interaction energy integral method for computation of SIF along crack fronts.
B. Bachir Bouiadjra et al. [3] used FEM to compute SIF for repaired cracks with bonded composite patches, in mode-I and mixed mode.

Bo Cerup Simonsen, Rikard T. Ornqvist [4] presented a combined experimental and numerical procedure for development of model in large scale shell structure.


J. H. Chang, D.J. Wu et al. [7] Presented numerical procedure based on the concept of the J_k integrals for computation of mixed mode SIF for curved cracks.


Yongming Liu et al. [9] Developed the solution for threshold SIF using a critical plane based multiaxial fatigue theory and Kitwaga diagram.

Nagaraj K. Arakere et al. [10] Investigated on mixed mode SIF for foam material using numerical (ANSYS & FRANC3D) and experimental approach.


M. Beghinia et al. [19] provided a simplified approach for evaluating SIF for inclined edge kinked crack by using analytical weight function.

Calvin Rans et al. [22] Determined SIF in cracked skin panels containing bonded stiffening elements by using analytical method.


Rui Zhang, Lingfeng He [21] used Digital Image Correlation (DIC) method for determination of SIF in mixed mode.

Paulo J. Tavares et al. [23] Presented hybrid methodology for the determination of the stress intensity factor (SIF) parameter, which entails combining experimental and numerical procedures to compute the SIF based of linear elastic fracture-mechanics concepts.

R. Evans, A. Clarke et al. [24] determined SIF for an edge crack, a crack approaching a hole, or a crack propagating from a hole after ligament failure by using The Stress Check commercial FE software package, (Version 8.0.1) in mode-I loading.

4. Experimental Procedure

3.1 photoelasticity

The name photoelasticity implies the use of light (photo) and elastically stresses model. This method was earlier used for plane bodies of complicated shape and geometries, particularly for the reason that such geometrical shapes were not amenable to mathematical analysis. Photoelasticity is an experimental method for measurement of stress and strain in which light is either passed through a model or reflected from the surface of loaded body. Photoelastic model is generally preferred in situation where and strain information is needed over extended region and thus whole field method.

Photoelastic stress analysis is a full field technique for measuring the magnitude and direction of principal stresses. When polarized light is passed through a stressed transparent model, interference patterns or fringes are formed. These patterns provide immediate qualitative information about the general distribution of stress, positions of stress concentrations and of areas of low stress using the principals of stress optic law.

\[ \sigma_1 - \sigma_2 = \frac{Nf}{h} \]

\[ \sigma_1, \sigma_2 \text{= maximum and minimum principal stresses at the point under consideration} \]

\[ N \text{= Fringe order} \]

\[ f = \text{Material Fringe Value} \]

\[ h = \text{Thickness} \]

3.2 Experimental setup

3.2.1 Circular Polariscope

The technique of photoelasticity depends upon unique phenomenon known as birefringence, of transparent material, particularly the plastics.

Birefringence implies that the plastic sheet is optically orthotropic in the sense that at any point two axes can be identified as slow and fast axes along which plate has two different values of refractive indices.

The circular polariscope consists of a light source, a polarizer, a quarter-wave plate oriented at 45° with respect to the polarizer, a specimen, second quarter-wave plate, and an analyzer that is always crossed with respect to the polarizer.

![Circular Polariscope](image)

3.3 Specimen Geometries

In this experiment four different geometrical configurations were used for analysis.

3.3.1 Specimen Material

The specimen material used is epoxy resin having trade name Araldite CY-230 with 10 percent hardener.
### Table 1: Material Properties

<table>
<thead>
<tr>
<th>Name</th>
<th>Trade Name</th>
<th>Young’s modulus (Mpa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoxy resin</td>
<td>Araldite CY-230</td>
<td>2570.22</td>
<td>0.38</td>
</tr>
</tbody>
</table>

This material shows the property of birefringence necessary for the photoelasticity experiments.

#### 3.3.2 Specimen dimensions

To study the stress distribution in a plate with different configurations photoelastic test is done on Araldite model uniformly loaded in one direction. The models are prepared with dimensions 200 mm×100 mm×5 mm. For photoelastic test four different configurations are considered i.e. edge crack, both edge crack, crack approaching circular hole, crack approaching triangular hole. For square and triangular cutout Circumscribed circle is used. The crack length for all the specimen is used in incremental format as 10, 20, 30 mm. The diameter of hole in third configuration is taken as 20 mm and for fourth configuration triangle inscribed in a 30 mm diameter hole is used. The plate is fixed at lower one edge and the loading conditions are applied at the upper edge.

#### 3.3.3 Specimen configurations

**3.3.3.1 Edge Crack**

The problem considered here is a finite width plate with an edge crack, as defined in Fig. 1 (a). The geometric parameters are: crack length a, plate width w, and half plate height h.

**3.3.3.2 Both Edge Crack**

The problem considered here is a finite width plate with both edge crack as defined in Fig. 4. The geometric parameters are crack length a, plate width w, and half plate height h.

**3.3.3.3 Crack Approaching a Circular Hole**

The geometry and notation for a crack approaching a hole are shown in Fig. 6 (a). The geometric parameters are: hole Radius R, crack half-length a, distance between the crack centre and the edge of the hole b, distance between the centre of the hole and the centre of the crack c, interaction zone s, half plate height h, and plate width w.

**3.3.3.4 Crack approaching a triangular Hole**

Specimens were cut from a sheet of epoxy material which is casted according to the standard procedure. In this specimen the crack was developed with the help of hacksaw blade. In order to study the effect of crack geometry and width of the plate, different crack length to width ratio (a/w) were taken for the observation.

#### 3.3.4 Both Edge Crack

The problem considered here is a finite width plate with both edge crack as defined in Fig. 4. The geometric parameters are crack length a, plate width w, and half plate height h.

**3.3.4 Casting Procedure for Preparation of Sheet**

Araldite CY-230 along with hardener HY-951 is used for casting the sheets. For every 100 cc of araldite 10.5 cc of hardener is mixed. The resin is heated in oven up to 50°C to 80°C for about one hours to remove all air bubbles and moisture. Then it is cooled down slowly to the room temp. The hardener is added slowly by stirring the mixture continuously. The mixture should be stirred in one direction for ten minutes till it is transparent, clear and homogeneous. The mould is completely filled by the mixture, i.e. up to the top surface. The mould is kept at this position for proper curing at room temperature. For easy removal of the sheet from the mould, the curing time of sixteen to eighteen hours is sufficient. After curing time the sheet is removed from the mould carefully. The sheet in this stage is slightly plastic. So...
it is kept on the perfect flat transparent glass for further curing. The total curing time is about one week.

### 3.3.5 Calibration of Photoelastic Material

The photoelastic material is calibrated by making a circular disc of 60 mm dia. out of the same sheet. The disc is loaded in increments under the diametral compression on Circular polariscope to find the material fringe value ($F_{\sigma}$). The fringe order at the centre of disc and corresponding load are recorded. The photoelastic material Plate model is found to have a stress fringe value equal to $13.4131 \text{ N/mm}^2$, as shown in Table 2.

**Figure 8: Circular disk under diametral compression**

**Table 2: Stress fringe value of photoelastic material**

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Load Kg</th>
<th>N</th>
<th>$F_{\sigma} = \frac{8P}{\pi DN}$</th>
<th>$F_{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>1</td>
<td>12.4904</td>
<td>12.4904</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>2</td>
<td>12.4904</td>
<td>12.4904</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>3</td>
<td>13.8783</td>
<td>13.8783</td>
</tr>
<tr>
<td>5</td>
<td>135</td>
<td>4</td>
<td>14.0517</td>
<td>14.0517</td>
</tr>
<tr>
<td>6</td>
<td>170</td>
<td>5</td>
<td>14.1558</td>
<td>14.1558</td>
</tr>
</tbody>
</table>

### 3.3.6 Procedure for SIF determination in photoelasticity

Photoelastic method has been convincingly applied for the determination of SIF for machine and structure parts. Since, the maximum stress would always occur at the boundary of a geometrical discontinuity, the stress concentration at the tip of crack becomes infinity hence to determine severity of crack stress intensity factor is determined. Fringe order will directly give the value of stress intensity factor by using formula,

$$K = \frac{N F_{\sigma}}{h} \frac{\sqrt{\pi r}}{\sin \theta}$$

Where, $K$ – Stress intensity factor  
$N$ – Fringe order  
$F_{\sigma}$ – Material fringe value  
$h$ – Plate thickness  
$r$ - Distance from the crack tip  
$\theta$ – Angle between two fringes

**Figure 9: Isochromatic fringe pattern for edge crack**

**Figure 10: Isochromatic fringe pattern for both edge crack**

**Figure 11: Isochromatic fringe pattern for crack approaching circular hole**

**Figure 12: Isochromatic fringe pattern for crack approaching a triangular hole**
5. Experimental Results

Table 4: Experimental results for edge cracks

<table>
<thead>
<tr>
<th>Edge Crack</th>
<th>$a/w = 0.1$</th>
<th>$a/w = 0.2$</th>
<th>$a/w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(N) K1</td>
<td>K2</td>
<td>K3</td>
<td>Load(N) K1</td>
</tr>
<tr>
<td>490.5</td>
<td>3.52</td>
<td>0.76</td>
<td>1.91</td>
</tr>
<tr>
<td>981</td>
<td>7.34</td>
<td>2.7</td>
<td>3.14</td>
</tr>
</tbody>
</table>

Table 5: Experimental results for both edge cracks

<table>
<thead>
<tr>
<th>Both Edge Crack</th>
<th>$a/w = 0.1$</th>
<th>$a/w = 0.2$</th>
<th>$a/w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(N) K1</td>
<td>K2</td>
<td>K3</td>
<td>Load(N) K1</td>
</tr>
<tr>
<td>490.5</td>
<td>4.08</td>
<td>1.91</td>
<td>2.35</td>
</tr>
<tr>
<td>981</td>
<td>9.22</td>
<td>3.06</td>
<td>5.07</td>
</tr>
</tbody>
</table>

Table 6: Experimental results for crack app. Circular hole

<table>
<thead>
<tr>
<th>Crack approaching a circular hole</th>
<th>$a/w = 0.1$</th>
<th>$a/w = 0.2$</th>
<th>$a/w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(N) K1</td>
<td>K2</td>
<td>K3</td>
<td>Load(N) K1</td>
</tr>
<tr>
<td>490.5</td>
<td>4.42</td>
<td>1.91</td>
<td>2.96</td>
</tr>
<tr>
<td>981</td>
<td>9.1</td>
<td>3.51</td>
<td>4.52</td>
</tr>
</tbody>
</table>

Table 7: Experimental results for crack app. triangular hole

<table>
<thead>
<tr>
<th>Crack approaching a triangular hole</th>
<th>$a/w = 0.1$</th>
<th>$a/w = 0.2$</th>
<th>$a/w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(N) K1</td>
<td>K2</td>
<td>K3</td>
<td>Load(N) K1</td>
</tr>
<tr>
<td>490.5</td>
<td>4.47</td>
<td>1.91</td>
<td>2.96</td>
</tr>
<tr>
<td>981</td>
<td>9.26</td>
<td>3.51</td>
<td>4.52</td>
</tr>
<tr>
<td>1275</td>
<td>14.1</td>
<td>4.52</td>
<td>6.57</td>
</tr>
</tbody>
</table>

6. Finite Element Analysis

ANSYS Workbench 14.5 software package is used for the modeling, analysis and post processing of the crack. Modeling of cracked plate is done using the ANSYS design modeler. Meshing of plate is done by using tetrahedron element with size 3 mm. crack tip areas are finely meshed with element size 1 mm. Cracks are developed by using Fracture tab available in the model window. The specimens were loaded by using appropriate boundary conditions under three different loading modes. The solution for SIF is obtained by using Fracture Tool available in the solution window. For the analysis four different configurations with varying a/w ratio were used. Three different loads were applied for each loading mode( i.e.490.5N,981N,1471N)

Figure 13: plate with edge crack

Figure 14: plate with both edge crack

Figure 15: plate with crack app. Circular hole

Figure 16: plate with crack app. Triangular hole

5.1 FEA Results

Table 8: FEA results for edge cracks

<table>
<thead>
<tr>
<th>Edge Crack</th>
<th>$a/w = 0.1$</th>
<th>$a/w = 0.2$</th>
<th>$a/w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(N) K1</td>
<td>K2</td>
<td>K3</td>
<td>Load(N) K1</td>
</tr>
<tr>
<td>490.5</td>
<td>3.63</td>
<td>1.77</td>
<td>3.63</td>
</tr>
<tr>
<td>981</td>
<td>7.26</td>
<td>1.59</td>
<td>3.42</td>
</tr>
<tr>
<td>1471</td>
<td>14.36</td>
<td>2.38</td>
<td>5.13</td>
</tr>
</tbody>
</table>

Table 9: FEA results for both edge cracks.

<table>
<thead>
<tr>
<th>Both Edge Crack</th>
<th>$a/w = 0.1$</th>
<th>$a/w = 0.2$</th>
<th>$a/w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(N) K1</td>
<td>K2</td>
<td>K3</td>
<td>Load(N) K1</td>
</tr>
<tr>
<td>490.5</td>
<td>4.66</td>
<td>1.93</td>
<td>1.77</td>
</tr>
<tr>
<td>981</td>
<td>9.33</td>
<td>3.06</td>
<td>3.4</td>
</tr>
<tr>
<td>1471</td>
<td>14.4</td>
<td>4.93</td>
<td>5.11</td>
</tr>
</tbody>
</table>

Table 10: FEA results for crack approaching circular hole.

<table>
<thead>
<tr>
<th>Crack approaching a circular hole</th>
<th>$a/w = 0.1$</th>
<th>$a/w = 0.2$</th>
<th>$a/w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(N) K1</td>
<td>K2</td>
<td>K3</td>
<td>Load(N) K1</td>
</tr>
<tr>
<td>490.5</td>
<td>4.65</td>
<td>1.23</td>
<td>1.76</td>
</tr>
<tr>
<td>981</td>
<td>9.11</td>
<td>2.46</td>
<td>3.32</td>
</tr>
<tr>
<td>1471</td>
<td>13.07</td>
<td>3.7</td>
<td>5.3</td>
</tr>
</tbody>
</table>
7. Result and Discussion

The values of the SIF obtained through the experimental results were validated by using the ANSYS software. Here because of space limitations, presenting the comparison graphs for edge crack, both edge crack, crack approaching a circular hole & crack approaching a triangular hole at 490.5 N load.

7.1 Edge Crack

Table 11: FEA results for crack approaching triangular hole.

<table>
<thead>
<tr>
<th>Load(N)</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
<th>Load(N)</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
<th>Load(N)</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
</tr>
</thead>
<tbody>
<tr>
<td>490.5</td>
<td>4.71</td>
<td>1.35</td>
<td>1.79</td>
<td>490.5</td>
<td>6.44</td>
<td>2.29</td>
<td>1.04</td>
<td>490.5</td>
<td>8.05</td>
<td>2.86</td>
<td>0.88</td>
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<td>981</td>
<td>9.42</td>
<td>2.71</td>
<td>3.59</td>
<td>981</td>
<td>12.81</td>
<td>4.58</td>
<td>2.09</td>
<td>981</td>
<td>16.17</td>
<td>5.68</td>
<td>1.76</td>
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<tr>
<td>1471</td>
<td>14.13</td>
<td>4.06</td>
<td>5.38</td>
<td>1079</td>
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<td>6.67</td>
<td>3.13</td>
<td>1079</td>
<td>24.16</td>
<td>8.15</td>
<td>2.64</td>
</tr>
</tbody>
</table>

7.2 Both Edge Crack

![Figure 19: Comparison of experimental & FEA results for both edge crack.](image)

7.3 Crack approaching circular hole

![Figure 20: comparison of experimental & FEA results for crack approaching circular hole.](image)
8. Conclusion

In damage tolerance analysis theory it is important to determine the severity of the cracks. The severity of crack is determined by using the stress concentration factor. In an attempt of determination of SIF by using photoelasticity and FEM analysis it is found that the stress intensity factor is greater in case of crack approaching a circular and triangular hole. Hence we can say that larger discontinuity near the crack increases the severity of the crack. Also from the results it is found that the value of SIF is always greater in mode-I loading. As load increases the value of SIF increases in all the cases. From the graphs shown in results and discussion experimental and numerical results are in a good agreement.

From experimental observations it is also found that crack approaching a triangular hole shows has the most severity in mode-I loading (Fig.22). In mode- II and mode-III loading both edge crack shows the most severity (Fig.23, Fig.24).

7.4 Crack approaching a triangular hole
References


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