International Journal of Scientific Engineering and Research (IJSER)
www.ijser.in
ISSN (Online): 2347-3878, Impact Factor (2014): 3.05

Replacement Model with Linear Trend Running Cost and Forecasted Inflation

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Abstract: ***The current paper is focused in development of best replacement strategy to determine an age of machine at which the replacement is most economical. It is assumed that the annual maintenance cost of machine follows linear trend. Using trend equation along with parameters like money value, forecasted inflation, nominal interest rate and real interest rate are taken into consideration in model development and the replacement period to machine is suggested.***

Keywords: Replacement, Inflation, Forecasting, Interest rates, Money value

1. Introduction

The replacement problems are concerned with the situation that arises when the efficiency of item decreases, failure or breakdown occurs. The decrease of efficiency or breakdown may be either gradual or sudden. The objective of replacement is to decide best policy to determine an age at which the replacement is most economical instead of continuing at increased maintenance costs. The problem of replacement is encountered in the case of both men and machines. It is possible to estimate the chances of failure of various ages. The fundamental objective of replacement is to direct the organization for maximizing its profit (or minimizing the cost). Formulation of problem is done for a situation "Replacement model considering forecasted inflation" and a case study is conducted considering the machine maintenance data

2. Forecasting Technique

Forecasts are estimates of the occurrence, timing or magnitude of future events and forecasting is a technique, which is used for estimating. Forecasting gives operations manager a rational basis for planning and scheduling activities, even though the actual demand is quite uncertain. Every day the managers have to take decisions in the face of uncertainty, without knowing what would happen in future. The manager strives to reduce this uncertainty and make better estimates of what is likely to happen in future. This is what forecasting aims to accomplish. Thus, whether it is inventory control, marketing strategy formulation, financial planning, production planning or any other such area of operations, managers have to employ the tool of forecasting.

3. Introduction to Nominal Interest Rate, Real Interest Rate and Inflation

The amount charged, expressed as a percentage of principal, by a lender to a borrower for the use of assets. Interest rates are typically noted on an annual basis, known as the annual percentage rate (APR). The assets borrowed could include, cash, consumer goods, large assets, such as a machine. Interest is essentially a rental, or leasing charge to the borrower, for the asset's use. In the case of a large asset, like a machine or building, the interest rate is sometimes known as the "lease rate".

Nominal interest rate or nominal rate of interest refers to the interest rate on an investment or loan without adjusting for inflation. The nominal interest rate is simply the interest rate stated on the loan or investment agreement. If one makes a loan at a high nominal interest rate, this does not guarantee a real profit. Real interest rate refers to an interest rate that has been adjusted to remove the effects of inflation to reflect the real cost of funds to the borrower, and the real yield to the lender. The real interest rate of an investment is calculated as the amount by which the nominal interest rate is higher than the inflation rate.

Real Interest Rate = Nominal Interest Rate - Inflation (Expected or Actual).

In ordinary language inflation means a process of rising prices. Different Economists have defined the term inflation in different ways. According to Irving Fisher "Inflation occurs when the supply of money actively bidding goods and services increases faster than the available supply of goods". Inflation leads to Inflationary spiral. When prices rise, workers demand higher wages. Higher wages leads to higher costs. Higher costs lead to higher prices. Higher prices again lead to higher wages, higher costs and so on. Thus prices, wages, costs chase each other leading to hyperinflation. Recession is just opposite of inflation and is a state of declining prices due to reduction of total expenditure of the community and this occurs when demand is less than supply of goods and services. According to Quantity of money theorists, inflation is caused by excessive issue of money. According to demand and supply theorists, it is caused by total demand exceeding the total supply of goods and services.

3.1 Relation between Real, Normal Interest Rate and Inflation

Irving Fisher theory of interest rates relates the nominal interest rate “i” to the rate of inflation “ϕ” and the real interest rate “r”. The real interest rate “r” is the interest rate after adjustment for inflation. It is the interest rate that lenders have to have to be willing to loan out their funds. The relation Fisher posted between these three rates is:

$(1 + i) = (1 + r) (1 + \phi) = 1 + r + \phi + r\phi$

This is equivalent to $i = r + \phi (1 + r)$

Thus, according to this equation, if ‘ϕ’ increases by 1% the nominal interest rate increases by more than 1%. This means that if ‘r’ and ‘ϕ’ are known then ‘i’ can be determined. On the other hand, if ‘i’ and ‘ϕ’ are known then ‘r’ can be determined by the relationship is

$1+r = (1+i) / (1+\phi)$ or $\mathbf{r = (i - \phi) / (1+\phi)}$

3.2 Regression Model with Trigonometric function to accommodate cyclical fluctuation of Prices of Items/Inflation.

In the current model it is assumed that inflation which plays vital role in calculation of real interest rate is assumed to be a regression model with trigonometric function to accommodate cyclical fluctuation of prices. Forecasting of inflation is carried out for 20 years based on the regression equation generated.

A model for a seasonal variation might include transcendental functions. The cycle of the model is as below . The model might be used to represent data for the four seasons of the year.

$\phi = b_0 + b_1x + b_2 \sin (2t \pi /4) + b_2 \cos (2t \pi /4)$

Regression equation with trigonometric function is:

$\phi = b_0 + b_1x + b_2 \sin (2t \pi /4) + b_2 \cos (2t \pi /4)$

Here ‘ϕ’ is inflation, ‘x’ is a variable and ‘t’ period, b_0,b_1,b_2 are the coefficients.

Table 1: Quarterly data of inflation for two years

Year (t)	0	1/4	1/2	3/4	1	5/4	3/2	7/4	2
ϕ	4.0	4.35	4.2	4.15	4.4	4.75	4.6	4.45	4.8

Following set of equations are used to obtain the values of coefficients of regression model

$\phi = b_0 + b_1x + b_2 \sin (2t \pi /4) + b_2 \cos (2t \pi /4)$ (1)
$\sum \phi = nb_0 + b_1 \sum x + b_2 \sum \sin (T \pi /2) + b_2 \sum\cos (2t \pi /4)$ (2)
$\sum (\phi x) = b_0 \sum x + b_1 \sum x^2 + b_2 \sum x. \sin (2 \pi t/4) + b_2 \sum x. \cos (2t \pi /4)$ (3)
$\sum (\phi x^2) = b_0 \sum x^2 + b_1 \sum n^3 + b_2 \sum x^2. \sin (2 \pi t/4) + b_2 \sum x^2. \cos (2t \pi /4)$ (4)

When time origin is taken between middle of years the equation reduces to

$\sum \phi = n b_0 + b_2 \sum \sin (2 \pi t /4) + b2 \sum \cos (2 \pi t /4)$ (5)
$\sum (\phi x) = b_1 \sum x^2$ (6)
$\sum (\phi x^2) = b_0 \sum x^2 + b_2 \sum x^2. \sin (2\pi t /4) + b_2 \sum x^2. \sin (2\pi t /4)$ (7)

by solving above equations we get the values of regression coefficients

$b_0 = 4.413, b_1 =0.0817, b_2 = -0.022$

The final regression equation is : $F_n = 4.413+0.082x-0.022\sin(2\pi t/4)-0.022 \cos(2\pi t/4)$

Table 2: Forecasted Inflation for 20 years

x	t (Period in years)	ϕ
24	7.00	6.342
28	8.00	6.618
32	9.00	6.938
36	10.00	7.302
40	11.00	7.622
44	12.00	7.898
48	13.00	8.218

x	t (Period in years)	ϕ
-4	0.00	4.00
0	1.00	4.40
4	2.00	4.80
8	3.00	5.062
12	4.00	5.338
16	5.00	5.658
20	6.00	6.022

x	t (Period in years)	ϕ
52	14.00	8.582
56	15.00	8.902
60	16.00	9.178
64	17.00	9.498
68	18.00	9.862
72	19.00	10.182
76	20.00	10.458

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4. Replacement Problem with Linear Trend Running Cost

The replacement problems are concerned with the situation that arises when the efficiency of item decreases, failure or breakdown occurs. The decrease of efficiency or breakdown may be either gradual or sudden. The situation which demands the replacement of items are;

1. The old item has become inefficient or require expensive maintenance
2. The old item has failed due to accident or otherwise and does not work at all, or the old item is expected to fail shortly
3. A better design of equipment has been developed or due to obsolescence

The objective of replacement is to decide best policy to determine an age at which the replacement is most economical instead of continuing at increased maintenance costs. The problem of replacement is encountered in the case of both men and machines. It is possible to estimate the chances of failure of various ages. The fundamental objective of replacement is to direct the organization for maximizing its profit (or minimizing the cost). In the current paper It is assumed that running cost of the item follows linear trend with governing relation as:

$R(n) = a_0 + a_1 n$

'n' is time period, a_0, a_1 are parameters or coefficients. The model can be fitted to the data by using ordinary least square method so that

Following set of equations are used to obtain the values of coefficients of regression model

$R = a_0 + a_1 t$ (1)
$\sum R = ma_0 + a_1 \sum t$ (2)
$\sum(R\,n) = a_0 \sum t + a_1 \sum t^2$ (3)

The following yearly maintenance cost (in rupees) of a machine is used to get trend equation of running cost. The machine is purchased at a total cost of C = Rs.4100 and immediately after usage for first year its salvage value is reduced to Rs.1900 and it remains the same in all subsequent periods.

Table 3: Running cost of Machine

Year (n)	1	2	3	4	5	6	7	8
R(n)	500	520	550	570	600	620	640	670

Year (n)	9	10	11	12	13	14	15
R(n)	690	720	740	760	790	810	840

by solving above equations we get the final regression equation as **R_n=668 + 35.78 (n)** and using this trend, running cost are forecasted and used in replacement decision making.

Table 4: Calculation of average annual cost with money value (Forecasted running cost, inflation and real rate of interest)

1	2	3	4	5	6	7	8
Period n	Inflation ϕ_n	Real interest rate r_n	Present worth factor v	Discount factor v^n	Discount factor v^{n-1}	Dividing discount factor $\sum v^{n-1}$	Maintenance Cost (Forecasted) R_n
1	3.4	3.773	0.964	0.964	1.000	1.000	703.780
2	3.8	3.375	0.967	0.936	0.967	1.967	739.560
3	4.04	3.167	0.969	0.911	0.940	2.907	775.340
4	4.37	2.911	0.972	0.892	0.918	3.824	811.120
5	4.69	2.691	0.974	0.876	0.899	4.724	846.900
6	5	2.500	0.976	0.862	0.884	5.608	882.680
7	5.33	2.318	0.977	0.852	0.872	6.479	918.460
8	5.65	2.158	0.979	0.843	0.861	7.340	954.240
9	6.03	1.987	0.981	0.838	0.854	8.195	990.020
10	6.29	1.881	0.982	0.830	0.846	9.040	1025.800
11	6.94	1.645	0.984	0.836	0.849	9.890	1061.580
12	**7.26**	**1.542**	**0.985**	**0.832**	**0.845**	**10.735**	**1097.360**
13	7.58	1.448	0.986	0.830	0.842	11.576	1133.140
14	7.91	1.357	0.987	0.828	0.839	12.416	1168.920

9	10	11	12	13
Maintenance Cost with money value $R_n v^{n-1}$	Cumulative Maintenance Cost with money value $\sum R_n v^{n-1}$	Salvage with money value $S_n v^n$	Total annual cost $TC = (C - S_n v^n + \sum R_n v^{n-1})$	Average annual cost $TC/\sum v^{n-1}$
703.780	703.780	1830.924	2972.856	2972.856
715.415	1419.195	1777.962	3741.232	1901.659
728.473	2147.668	1730.356	4517.312	1553.994
744.226	2891.893	1693.998	5297.895	1385.276
761.567	3653.460	1663.789	6089.671	1289.181
780.161	4433.620	1638.364	6895.256	1229.643
800.497	5234.118	1618.464	7715.653	1190.854
821.777	6055.895	1601.689	8554.206	1165.379
845.821	6901.716	1591.632	9410.084	1148.324
867.435	7769.151	1577.016	10292.135	1138.480
901.778	8670.929	1587.870	11183.059	1130.777
927.318	**9598.247**	**1581.197**	**12117.050**	**1128.768**
953.641	10551.888	1576.207	13075.681	1129.517
981.051	11532.939	1573.284	14059.655	1132.416

Table 5: Calculation of average annual cost without money value

Year **n**	Forecasted Maintenance cost **R_n**	Cumulative Maintenance Cost **$\sum R_n$**	Salvage value **S_n**	Loss due to resale **(C-S)**	Total cost **$TC=(C+\sum R_n - S)$**	Average cost per year **TC / n**
1	703.780	703.78	1900	2200	2903.78	2903.78
2	739.560	1443.34	1900	2200	3643.34	1821.67
3	775.340	2218.68	1900	2200	4418.68	1472.893
4	811.120	3029.8	1900	2200	5229.8	1307.45
5	846.900	3876.7	1900	2200	6076.7	1215.34
6	882.680	4759.38	1900	2200	6959.38	1159.897
7	918.460	5677.84	1900	2200	7877.84	1125.406
8	954.240	6632.08	1900	2200	8832.08	1104.01
9	990.020	7622.1	1900	2200	9822.1	1091.344
10	1025.800	8647.9	1900	2200	10847.9	1084.79
11	**1061.580**	**9709.48**	**1900**	**2200**	**11909.48**	**1082.68**
12	1097.360	10806.84	1900	2200	13006.84	1083.903

5. Results and Conclusion

When money value with inflation is considered, it is observed from the table 4, that the average annual cost of machine is decreasing gradually up to 12th year and from 13th year it is increasing. So it is advisable to replace the machine at the end of 12th year. But when money value is not considered the replacement period comes to 11th period (table 5). In general real time decision making depends on various uncontrollable parameters. In the current work of replacement of machine, only some macroeconomic parameters are considered with an assumption that the running cost has linear trend, but there is a lot of scope to develop a robust model considering different patterns of running cost, mileage of machine , technological changes, government policies (taxes etc.,).

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