

# Design of Optimized Speed Controller of DC Motor Using Chopper

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**Abstract:** The objective of the present work is to design an optimized filter to be used as an automatic speed controller of DC motors to make it work efficiently in a man-less environment. The optimization has been achieved by Modulus Hugging Approach for the design of the speed controller transfer function. The stability criterion is graphically analyzed and tested for closed loop speed control operation of the DC motor. The proposed methodology has been tested and verified in both MATLAB Simulink environment and programming. The proposed methodology can be implemented for industrial automation purposes e.g. automatic speed adjustment of conveyor belts for sudden change of weights of its carrying loads, automatic speed adjustment of the rolling shaft of the plate roller machine while the diameter of the bobbin consisting sheets are gradually increasing, etc.

**Keywords:** Optimized Filter, Speed Control, Modulus Hugging Approach, PI Controller, Separately Excited DC Motor

## 1 Introduction

DC drives, because of their straightforwardness, ease of application, dependability and satisfactory cost, have long been a backbone of industrial applications. DC drives are less complex as compared to AC drive systems and are normally less expensive for low horsepower ratings. DC motors have a long tradition of being used as adjustable speed machines and a wide range of options have evolved for this purpose. Also developments of high performance motor drives are very essential for industrial applications which must provide excellent control of speed for acceleration and deceleration.

The speed of separately excited DC motor can be controlled from below and up to rated speed using chopper as a converter. DC motor can be operated in two modes - Motoring and Braking. In motoring, it converts electrical energy to mechanical energy, which supports its motion. In braking, it works as a generator, converting mechanical energy to electrical energy, and thus opposes the motion. It can provide motoring and braking operation in both forward and reverse direction. DC regenerative drives are available for applications requiring continuous regeneration for overhauling loads. In addition to the stated advantages, they also provide excellent control of speed for acceleration and deceleration; hence they have long been the primary means of electric traction. They are also used for mobile equipment, such as golf carts trolleys, quarry and mining applications. DC motors are conveniently portable and well fit to special applications, like industrial equipments and machineries that are not easily run from remote power sources [1].

## 2 Modulus Hugging Approach

The process of making the output as close to the input as possible, in order to obtain unity gain for a wide range of frequency, is termed as Modulus Hugging Approach (MHA).

A controller designed by modulus hugging approach for the optimization of a control loop, we begin by considering the transfer function of the closed loop system. The dynamic performance of a control system is good if the controlled variable very rapidly reaches the value required by the

command variable. In terms of the frequency characteristic; this means a frequency range as wide as possible over which the modulus lays very near to 1 i.e. a wide bandwidth. At the instant of the input step, the controlled variable cannot correspond to the command variable, but very rapidly after, when lower frequencies come into effect, the modulus of the frequency characteristic comes very close to 1. Then the error between the command variable and the controlled variable quickly becomes zero.

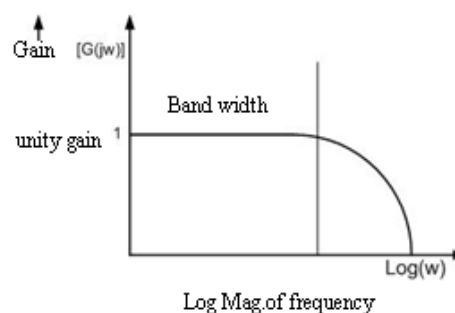


Figure 1: Gain vs. frequency waveform

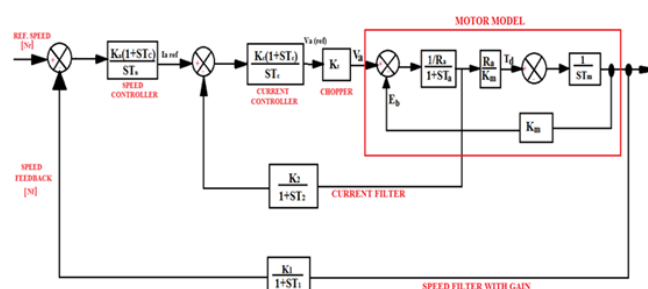


Figure 2: Complete layout for DC motor speed control

In this paper the proposed methodology has been implemented for designing the current controller, described later on.

## 3 Complete Layout for DC Motor Speed Control

The chopper firing circuit receives signal from controller and then chopper gives variable voltage to the armature of the

motor for achieving desired speed. There are two control loops, one for controlling current and another for speed.

The controller used is Proportional-Integral type which removes the delay and provides fast control. Modeling of separately excited DC motor is done. The complete layout of DC drive mechanism is obtained. The designing of current and speed controller is carried out. The optimization of speed controller is done using modulus hugging approach, in order to get stable and fast control of DC motor. After obtaining the complete model of DC drive system, the model is simulated in MATLAB Simulink environment, and then the stability criterion is analyzed in MATLAB programming. The complete layout for the proposed technique has been depicted in Figure 2.

## 4 Controller Design

The controller used in a closed loop provides a very easy and common technique of keeping motor speed at any desired set point speed under changing load conditions. This controller can also be used to keep the speed at the set-point value when, the set-point is ramping up or down at a defined rate. For example, if the error speed is negative, this means the motor is running slow so that the controller output should be increased and vice-versa [2], [3].

The control action can be imagined at first sight as something simple like if the error speed is negative, then multiply it by some scale factor generally known as gain and set the output drive to the desired level. But this approach is only partially successful due to the following reason: if the motor is at the set-point speed under no load there is no error speed so the motor free runs. If a load is applied, the motor slows down and a positive error speed is observed. Then the output increases by a proportional amount to try and restore the desired speed. However, when the motor speed recovers, the error reduces drastically and so does the drive level. The result is that the motor speed will stabilize at a speed below the set-point speed at which the load is balanced by the product of error speed and the gain. This basic technique discussed above is known as "proportional control" and it has limited use as it can never force the motor to run exactly at the set-point speed [4].

When the machine is made to run from zero speed to a high speed then motor has to go to specified speed. But due to electromechanical time constant motor will take some time to speed up. But the speed controller used for controlling speed acts very fast. Speed feedback is zero initially. So this will result in full controller output and hence converter will give maximum voltage. So a very large current flow at starting time because back emf is zero at that time which sometime exceeds the motor maximum current limit and can damage the motor windings. Hence there is a need to control current in motor armature. To solve the above problem we can employ a current controller which will take care of motor rated current limit [5], [6]. The applied voltage will now not dependent on the speed error only but also on the current error. We should ensure that the applied voltage is in such a way that machine during positive and negative torque, does not draw more than the rated current. So, an inner current loop hence current controller is required.

### 4.1 Current Controller Design

We need to design current controller for the extreme condition when back emf is zero that is during starting period because at that time large current flows through the machine. The model of the current controller is shown in Figure 3.

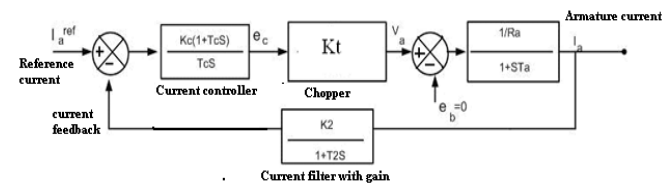


Figure 3: Block Model for Current Controller Design

Transfer function of the above block model is given by,

$$\frac{I_a(s)}{I_a(s)(ref)} = \frac{\frac{K_c(1+sT_c)K_t}{(sT_c)(1+sT_a)R_a}}{1 + \frac{K_c(1+sT_c)K_tK_2}{(sT_c)(1+sT_a)(1+sT_2)R_a}} \quad (1)$$

In the above equation,  $T_c$  is the Current Controller parameter which should be chosen in such a way so that it cancels the largest time constant in the transfer function in order to reduce the order of the system [7]-[9]. In this way, the response time will be reduced, and hence faster. Let us assume that

$$T_c = T_a$$

Now, putting this value in equation (1),

$$\begin{aligned} \frac{I_a(s)}{I_a(s)(ref)} &= \frac{K_c(1+sT_a)K_t(1+sT_2)}{(sT_a)(1+sT_a)(1+sT_2)R_a + K_c(1+sT_a)K_tK_2} \\ \Rightarrow \frac{I_a(s)}{I_a(s)(ref)} &= \frac{K_cK_t(1+sT_2)}{(sT_a)(1+sT_2)R_a + K_cK_tK_2} \\ \Rightarrow \frac{I_a(s)}{I_a(s)(ref)} &= \frac{K_cK_t(1+sT_2)}{(sTaRa + s^2TaT_2Ra) + K_cK_tK_2} \end{aligned}$$

Dividing both numerator and denominator by  $T_a R_a$ ,

$$\frac{I_a(s)}{I_a(s)(ref)} = \frac{\frac{K_cK_t}{TaRa}(1+sT_2)}{s^2 T_2 + s + \frac{K_cK_tK_2}{TaRa}} \quad (2)$$

Now, let

$$K_d = \frac{K_cK_t}{TaRa} \quad (3)$$

Where,  $T_2$  denotes filter lag.

Dividing the above equation by  $T_2$ , we get,

$$\frac{I_a(s)}{I_a(s)(ref)} = \frac{\frac{K_d}{T_2}(1+sT_2)}{s^2 + \frac{s}{T_2} + \frac{K_dK_2}{T_2}} \quad (4)$$

Hence the Characteristic Equation may be written as,

$$s^2 + \frac{s}{T_2} + \frac{K_dK_2}{T_2} \approx s^2 + 2\epsilon\omega s + \omega^2$$

Here,  $\omega = \sqrt{\frac{K_dK_2}{T_2}}$  and  $\epsilon = \frac{1}{2\sqrt{K_dK_2T_2}}$

This is a second order system. So for proper response,  $\zeta$  should be approximately 0.707.

$$\begin{aligned} \text{Therefore, } \frac{1}{\sqrt{2}} &= \frac{1}{2\sqrt{K_d K_2 T_2}} \\ \Rightarrow K_d &= \frac{1}{2K_2 T_2} = \frac{K_c K_t}{T_a R_a} \\ K_c &= \frac{T_a R_a}{2K_t K_2 T_2} \end{aligned} \quad (5)$$

From equation (3), we have,

$$\frac{I_a(s)}{I_a(s)(\text{ref})} = \frac{K_d(1+sT_2)}{s^2 T_2 + s + K_d K_2}$$

And from equation (5), we have,

$$\begin{aligned} K_d &= \frac{1}{2K_2 T_2} \quad \text{and} \quad K_d K_2 = \frac{1}{2T_2} \\ \frac{I_a(s)}{I_a(s)(\text{ref})} &= \frac{\frac{1}{2K_2 T_2}(1+sT_2)}{s^2 T_2 + s + \frac{1}{2T_2}} \\ \frac{I_a(s)}{I_a(s)(\text{ref})} &= \frac{\frac{1}{K_2}(1+sT_2)}{2s^2 T_2^2 + 2sT_2 + 1} \end{aligned} \quad (6)$$

From the above transfer function, it can be observed that the Zero present in the numerator can result in an overshoot. It corresponds to a time lead, and hence to cancel its effect, a time lag filter may be used [10]. The small delay introduced by the filter will not affect the response much as the current loop time constant is much higher than filter time constant. Thus by incorporating such a filter, the following transfer function will be obtained as,

$$\frac{I_a(s)}{I_a(s)(\text{ref})} = \frac{\frac{1}{K_2}}{2s^2 T_2^2 + 2sT_2 + 1}$$

## 4.2 Speed Controller Design

The block model of speed controller is shown in Figure 4.

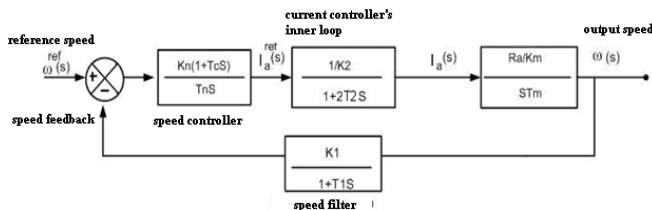


Figure 4: Block model for Speed Controller design

Representing the above block model in transfer function form, we have,

$$\frac{\omega(s)}{\omega(s)(\text{ref})} = \frac{\frac{K_n R_a}{K_2 K_m T_m T_n} \left[ \frac{(1+sT_n)}{(1+2T_2s)s^2} \right]}{\left[ 1 + \left( \frac{K_n R_a}{K_2 K_m T_m T_n} \right) \left( \frac{(1+sT_n)}{(1+2T_2s)s^2} \right) \left( \frac{K_1}{1+sT_1} \right) \right]}$$

To cancel the effect of the largest time constant of the system  $T_2$ , let us assume that  $T_n = 2T_2$ .

$$\begin{aligned} \frac{\omega(s)}{\omega(s)(\text{ref})} &= \frac{\frac{K_n R_a}{s^2(K_2 K_m T_m T_n)}}{1 + \left( \frac{K_n R_a}{K_2 K_m T_m T_n} \right) \left( \frac{K_1}{1+sT_1} \right) \left( \frac{1}{s^2} \right)} \\ \frac{\omega(s)}{\omega(s)(\text{ref})} &= \frac{K_n R_a(1+sT_1)}{s^2 K_2 K_m T_m T_n (1+sT_1) + K_n R_a K_1} \end{aligned} \quad (7)$$

In the above equation, it can be observed that the 's' term is absent in the denominator; which implies that the damping constant of the system is zero. As a result, the system becomes unstable and of oscillatory nature. To overcome this shortcoming, a transfer function with unity gain is required [11], [12]. Thereby, for obtaining such a transfer function, the Modulus Hugging Approach is used as an improvement to the conventional methods.

The Modulus Hugging Approach is one of the best classical optimization methods for designing a speed controller. It determines the controller parameters in order to achieve optimum working of the control system.

Now considering equation (7), multiplying both numerator and denominator by  $(1+sT_n)$ ,

$$\frac{\omega(s)}{\omega(s)(\text{ref})} = \frac{K_n R_a(1+sT_1)(1+sT_n)}{s^2 K_2 K_m T_m T_n (1+sT_1)(1+sT_n) + K_n R_a K_1 (1+sT_n)}$$

$$\begin{aligned} \text{The term } (1+sT_1)(1+sT_n) &= (1+sT_1)(1+2T_2s) \\ &= 1+sT_1+2T_2s+2T_1T_2s^2 = 1+s.(2T_2+T_1)+2T_1T_2s^2 \end{aligned}$$

Since  $T_1$  and  $T_2$  are very small, their product is negligible. So the above expression can be written as,

$$(1+sT_1)(1+sT_n) = 1+s(2T_2+T_1)$$

$$\text{Let } 1+s(2T_2+T_1) = 1+\delta s. \quad \delta = 2T_2+T_1$$

$$\begin{aligned} \text{Assuming } K_o &= \frac{K_n R_a}{K_2 K_m}, \\ \frac{\omega(s)}{\omega(s)(\text{ref})} &= \frac{K_n R_a(1+sT_1)(1+sT_n)}{s^2 K_2 K_m T_m T_n (1+\delta s) + K_n R_a K_1 (1+sT_n)} \\ \Rightarrow \frac{\omega(s)}{\omega(s)(\text{ref})} &= \frac{K_n R_a(1+sT_1)(1+sT_n)}{s^3 K_2 K_m T_m T_n \delta + s^2 K_2 K_m T_m T_n + K_n R_a K_1 (1+sT_n)} \end{aligned}$$

Dividing numerator and denominator by  $K_2 K_m$ , we get,

$$\frac{\omega(s)}{\omega(s)(\text{ref})} = \frac{\frac{K_n R_a}{K_2 K_m} (1+sT_1)(1+sT_n)}{s^3 T_m T_n \delta + s^2 T_m T_n + s K_o K_1 T_n + K_o K_1} \quad (8)$$

Considering a standard 3<sup>rd</sup> order system,

$$G(j\omega) = \frac{b_0 + j\omega b_1}{a_0 + j\omega a_1 + (j\omega)^2 a_2 + (j\omega)^3 a_3} \quad (9)$$

For low frequency,  $b_0 = a_0$  and  $b_1 = a_1$ .

$$|G(j\omega)| = \frac{b_0^2 + \omega^2 b_1^2}{\sqrt{a_0^2 + \omega^2(a_1^2 - 2a_0a_2) + \omega^4(a_2^2 - 2a_1a_3) + \omega^6(a_3^2)}}$$

By Modulus hugging principle,  $|G(j\omega)| = 1$ , for which the coefficients of  $\omega^2$  and  $\omega^4$  are made equal to zero.

$$\therefore a_1^2 = 2a_0a_2 \quad \text{and} \quad a_2^2 = 2a_1a_3. \quad (10)$$

Comparing equations (8) and (9),

$$a_0 = K_o K_1, \quad a_1 = K_o K_1 T_n, \quad a_2 = T_m T_n, \quad a_3 = T_m T_n \delta$$

Substituting these values in equation (10),

$$(K_o K_1 T_n)^2 = 2K_o K_1 T_m T_n \quad \text{and} \quad K_o K_1 T_n = 2T_m \quad (11)$$

$$(T_m T_n)^2 = 2(K_o K_1 T_n)(T_m T_n \delta) \quad \text{and} \quad T_m = 2K_o K_1 \delta \quad (12)$$

Equating equations (11) and (12), we get,

$$\frac{K_0 K_1 T_n}{2} = 2K_0 K_1 \delta \quad \text{and} \quad T_n = 4\delta = 4(2T_2 + T_1)$$

From equation (12),

$$T_m = 2K_0 K_1 \delta = 2\left(\frac{K_n R_a}{K_2 K_m}\right) K_1 \delta \quad (\text{Since } K_o = \frac{K_n R_a}{K_2 K_m})$$

$$K_n = \frac{T_m K_m K_2}{2K_1 R_a \delta}$$

Now, modifying the terms of the denominator in equation (8),

$$S^3 T_m T_n \delta = s^3 (2K_0 K_1 \delta) (4\delta) = 8s^3 K_0 K_1 \delta^3$$

$$S^2 T_m T_n = s^2 (2K_0 K_1 \delta) (4\delta) = 8s^2 K_0 K_1 \delta^2$$

$$s K_0 K_1 T_n = s (K_0 K_1) (4\delta) = 4s K_0 K_1 \delta$$

From equation (8),

$$\frac{\omega(s)}{\omega(s)_{\text{ref}}} = \frac{\frac{K_n R_a}{K_2 K_m} (1+sT_1)(1+sT_n)}{s^3 T_m T_n \delta + s^2 T_m T_n + s K_0 K_1 T_n + K_0 K_1}$$

The terms  $(1+sT_1)(1+sT_n)$  in the numerator can be cancelled using filters.

$$\therefore \frac{\omega(s)}{\omega(s)_{\text{ref}}} = \frac{K_0}{8s^3 K_0 K_1 \delta^3 + 8s^2 K_0 K_1 \delta^2 + 4s K_0 K_1 \delta + K_0 K_1}$$

Dividing both numerator and denominator by  $K_0$ , we get;

$$\frac{\omega(s)}{\omega(s)_{\text{ref}}} = \frac{1}{8s^3 K_1 \delta^3 + 8s^2 K_1 \delta^2 + 4s K_1 \delta + K_1} \quad (13)$$

## 5 Modeling of Different Components of the Loop Control

### 5.1 Separately Excited DC Motor

The equivalent circuit of a dc motor armature is based on the fact that the armature winding has a resistance  $R_a$  self-inductance  $L_a$ , and an induced emf. This is shown in Figure 5 [13]-[16].

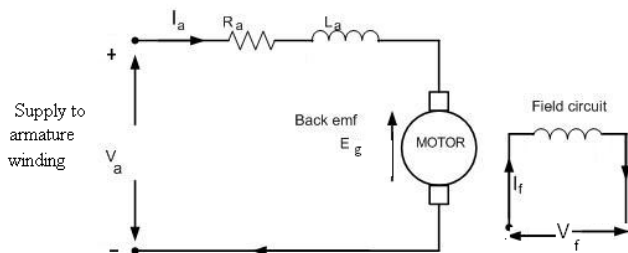


Figure 5: Separately Excited DC motor model

The armature equation is shown below,

$$V_a = E_g + I_a R_a + L_a (dI_a/dt) \quad (14)$$

Now the torque equation will be given by,

$$T_d = J d\omega/dt + B\omega + T_L \quad (15)$$

Therefore, new torque equation will be given by,

$$T_d = J d\omega/dt + T_L \quad (16)$$

Taking field flux as  $\Phi$  and (Back EMF Constant)  $K_v$  as equation for back emf of motor will be,

$$E_g = K \Phi \omega \quad (17)$$

$$\text{Also, } T_d = K \Phi I_a \quad (18)$$

From motor's basic armature equation, after taking Laplace Transform on both sides, we will get,

$$I_a(s) = (V_a - E_g)/(R_a + L_a s)$$

Now, taking equation (15) into consideration, we have,

$$\Rightarrow I_a(s) = (V_a - K\Phi\omega)/R_a(1+L_a s/R_a)$$

$$\text{And, } \omega(s) = (T_d - T_L)/J s = (K\Phi I_a - T_L)/J s$$

Also, the armature time constant will be given by,

$$T_a = L_a/R_a \text{ (Armature Time Constant)}$$

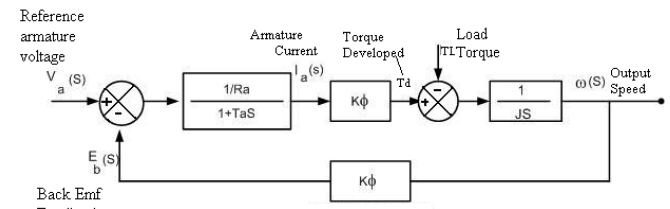


Figure 6: Block Model of Separately Excited DC Motor

After simplifying the above motor model, the overall transfer function is expressed as,

$$\omega(s)/V_a(s) = [K\Phi/R_a]/J s(1+T_a s)/[1+(K^2\Phi^2/R_a)/J s(1+T_a s)]$$

Further simplifying the above transfer function will yield,

$$\omega(s)/V_a(s) = (1/k\Phi)/[sT_m(1+sT_a)+1] \quad (19)$$

Assuming,  $T_m = J R_a / (k\Phi)^2$  as electromechanical time constant. Then the above transfer function can be written as below,

$$\omega(s)/V_a(s) = (1/k\Phi)/[sT_m(1+sT_a)+1] \quad (20)$$

Let us assume that during starting of motor, load torque,  $T_L=0$  and applying full voltage  $V_a$ . Also assuming negligible armature inductance, the basic armature equation can be written as,

$$V_a = K\Phi\omega(t) + I_a R_a$$

At the same time Torque equation will be,

$$T_d = J d\omega/dt = K\Phi I_a \quad (21)$$

Putting the value of  $I_a$  in above armature equation,

$$V_a = K\Phi\omega(t) + (J d\omega/dt) R_a / K\Phi$$

Dividing on both sides by  $K\Phi$ ,

$$V_a/K\Phi = \omega(t) + J R_a (d\omega/dt)/(K\Phi)^2 \quad (22)$$

$(V_a/K\Phi)$  is the value of motor speed under no load condition. Therefore,

$$\omega(\text{no load}) = \omega(t) + J R_a (d\omega/dt)/(K\Phi)^2 = \omega(t) + T_m (d\omega/dt)$$

Therefore,

$$J = T_m (K\Phi)^2 / R_a \quad (23)$$

From motor torque equation, we have,

$$\omega(s) = K\Phi I_a(s)/J s - T_L/J s \quad (24)$$



From equations (21) & (22) above, we have,

$$\omega(s) = [(Ra / Km) Ia(s) - TL Ra / (Km)^2] (1/Tm(s))$$

Now, Replacing  $K\Phi$  by  $Km$  in equation (18), we will get,

$$\omega(s)/Va(s) = (1/Km) / (1 + sTm + s^2TaTm) \quad (25)$$

$Tm$ , ( $T_a \ll T_m$ ) Simplifying,

$$1 + sTm + s^2TaTm \approx 1 + s(Ta + Tm) + s^2TaTm \\ = (1 + sTm)(1 + sTa)$$

The largest time constant will play main role in delaying the system when the transfer function is in time constant form. To compensate that delay due to largest time constant we can use PI controller as speed controller. It is because the zero of the PI controller can be chosen in such a way that this large delay can be cancelled. In Control system term a time delay generally corresponds to a lag and zero means a lead, so the PI controller will try to compensate the whole system. Hence, the equation can be written as,

$$t.f = \omega(s)/Va(s) = (1/Km)/((1 + sTm)(1 + sTa)) \quad (26)$$

Hence the dc motor can be replaced by the transfer function obtained in the above equation in the DC drive model shown in figure earlier.

## 5.2 Representation of Chopper

A DC Chopper is a static power electronic device, wherein it converts an unregulated DC input voltage to a controlled DC output voltage.

Its transfer function is given by,

$$\frac{V_a(s)}{V_a(s)(ref)} = \frac{K_t}{1+sT_t} \quad (27)$$

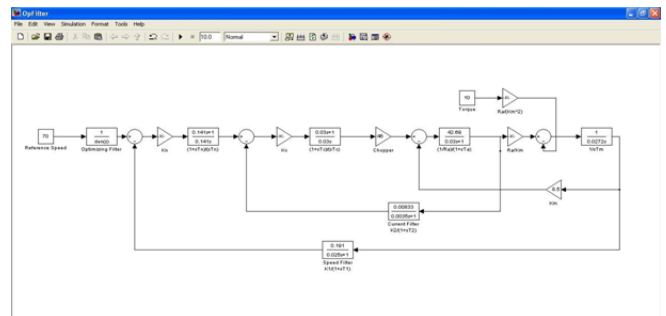
As the chopping frequency is increased, the time constant  $T_t$  decreases. In this manner,  $T_t$  becomes so small that it can be neglected. Henceforth, a chopper can be represented by a constant gain  $K_t$ , since it does not suffer any time delay during its operation.

## 6 Simulation Results

A separately excited DC motor is used in analysis and simulation, having name plate rating of, 300KW, 460V (DC), 52.3 rad/sec. Following parameter values are associated with it;

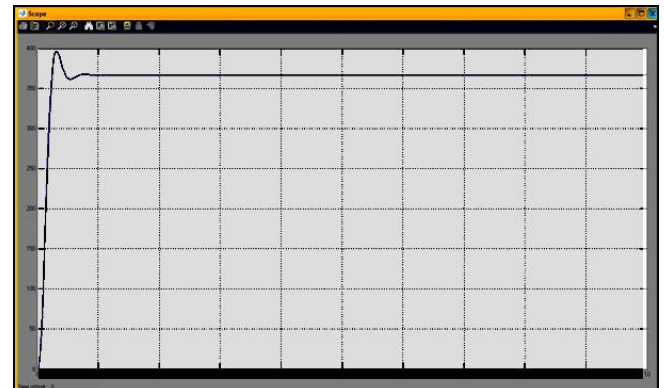
Moment of inertia,  $J = 84 \text{ Kg-m}$ , Rated current = 690 A, Maximum current limit = 1200 A, Armature inductance,  $L_a = 0.7026 \text{ mH}$ , Armature resistance,  $R_a = 0.02342 \text{ ohm}$ , Back emf constant = 8.5 volt-sec/rad. Speed feedback filter time constant  $T_1 = 25 \text{ ms}$ , Current feedback filter time constant  $T_2 = 3.5\text{ms}$ .

The MATLAB simulation of the complete model is shown in Figure 7.



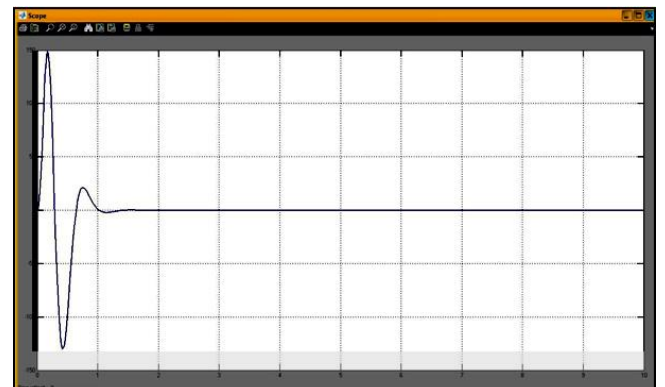
**Figure 7:** Simulink model for speed control of separately excited DC motor using chopper with optimizing filter

The speed output response of the motor is shown in Figure 8.



**Figure 8:** Speed response at reference speed same as rated speed at full Load

The output error response of the motor is shown in Figure 9.



**Figure 9:** Error in Speed Response at reference speed same as rated speed at full Load

From the simulation results, it can be observed that the Simulink model with optimizing filter incorporating MHA shows a reduced overshoot by a great extent. The dynamic response of the output speed is better, as the transient and over shoot in the beginning is very small. Again, the speed is maintained constant even after loading condition. Thus it has been observed that the dynamic response of the DC motor is achieved by incorporating modulus hugging approach.

## 7 Discussion

In order to evaluate the proposed method, the stability analysis of closed loop speed control of DC motor is examined by Routh-Hurwitz Criterion.

The overall transfer function of closed loop speed control of motor without using modulus hugging approach is given from equation (7),

$$\frac{\omega(s)}{\omega(s)(ref)} = \frac{KnRa(1+sT_1)}{s^2K_2K_mT_mT_n(1+sT_1)+KnRaK_1}$$

The characteristic equation is,

$$S^3K_2K_mT_mT_nT_1 + S^2K_2K_mT_mT_n + K_nR_aK_1 = 0$$

The above equation can be written as following after replacing their values,

$$6.16 \times 10^{-6}s^3 + 2.4 \times 10^{-4}s^2 + 3 \times 10^{-2} = 0$$

The Routh-Array is formed as,

$S^3$	$6.16 \times 10^{-6}$	0
$S^2$	$2.4 \times 10^{-4}$	$3 \times 10^{-2}$
$S^1$	$-7.7 \times 10^{-4}$	0
$S^0$	$3 \times 10^{-2}$	0

In the first column of Routh-Array, there are two sign changes indicating that there are two closed loop poles on the RHS of the S-Plane. Hence the system is unstable.

The overall transfer function of closed loop speed control of motor using modulus hugging approach is given from equation (13):

$$\frac{\omega(s)}{\omega(s)(ref)} = \frac{1}{8s^3K_1\delta^3 + 8s^2K_1\delta^2 + 4sK_1\delta + K_1}$$

The characteristic equation is:

$$8s^3K_1\delta^3 + 8s^2K_1\delta^2 + 4sK_1\delta + K_1 = 0$$

The above equation can be rewritten as following by replacing their values:

$$5 \times 10^{-5}s^3 + 1.564 \times 10^{-3}s^2 + 0.0244s + 0.191 = 0$$

The Routh-Array is formed as:

$S^3$	$5 \times 10^{-5}$	0.0244
$S^2$	$1.564 \times 10^{-3}$	0.191
$S^1$	0.0183	0
$S^0$	0.191	0

From the Routh-Array it can be observed that all the elements of the first column are positive and have the same sign. So it can be concluded that all the closed loop poles lie on the LHS of the S-Plane. Henceforth, the system is stable.

## 8 Conclusion

In this paper, the block diagram representation of speed control of DC motor with the help of chopper is evaluated. The proposed method expresses the total DC motor into a single transfer function. Designing of PI type speed controller as well as the current controller is carried out. Using Modulus Hugging Approach, the optimization of speed controller is

achieved. A DC motor specification is taken and corresponding parameters are found out from derived design approach. MATLAB simulations are carried out with and without optimizing filter. The closed loop system stability is then determined by Routh-Hurwitz Criterion, which yields that the system with optimizing filter is stable. The systems response shows the fast rise time, fast setting time as well as fast recovering time.

## Nomenclature

$T_L$	:	Load torque in Nm.
$T_d$	:	The torque developed in Nm.
$J$	:	Moment of inertia in Kg/m <sup>2</sup> .
$B$	:	Friction coefficient of the motor.
$\omega$	:	Angular velocity in rad/sec.
$V_a$	:	The armature voltage in Volts.
$E_g$	:	The motor back emf in Volts.
$I_a$	:	The armature current in Amperes.
$R_a$	:	Is the armature resistance in Ohms
$L_a$	:	The armature inductance in Henry.

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